Suggested Approach for Developing Mathematical Process in Applied Technology High School Based on Standards of the National Council of Teachers of Mathematics (NCTM)

رؤية مقترحة لتطوير العمليات الرياضياتية في ثانوية التكنولوجيا التطبيقية استنادا إلى معايير المجلس الوطني لمعلمي الرياضيات (NCTM)

By

Hisham Hanfy Hassan Ayob

Dissertation submitted in partial fulfillment of

Master of Education (International Management and Policy)

Faculty of Education

Dissertation Supervisor

Professor Clifton Chadwick

April – 2012

شكر وإهداء

﴿ وَقُلِ اعْمَلُوا فَسَيَرَى اللَّـهُ عَمَلَكُمْ وَرَسُولُهُ وَالْمُؤْمِنُونَ ﴾ (التوبه: ١٠٥)

الحمد لله والصلاة والسلام على من لا نبي بعده، أما بعد:

فلا يسعني إلا أن أتوجه إلى المولى جل وعلا بالحمد والشكر الخالص الجزيل، على ما أنعم وأجزل، وامتن به وتفضل، فله الحمد في الأولى والآخرة، على ما أعان وسهل، وأسبل من النعم ويسر، وما لطف به وقدر، له الحمد على أن أعانني على إتمام هذا البحث الذي أسأل العلي القدير أن يجعله عملا خالصا لوجهه الكريم.

ثم الشكر لوالدي الكريمين، فكم حرصا على تعليمي منذ نعومة أظفاري، وتشجيعي بشتى الوسائل، حتى غرسا حب العلم في فؤادي، ولم يبخلا عليّ بدعواتهما المستمرة، واقول لهم انتم وهبتمونى الحياة والنشاة على شغف الاطلاع وأسأل الله أن يرحمهما كما ربياني صغيرا.

والشكر موصول لرياحين حياتى. زوجتي الكريمة التي ضحت كثيرا، وشدت من أزري، وأولادي الذين تجرعوا مرارة غيابي رغم وجودي بينهم.

ثم الشكر والعرفان لفضيلة الدكتور كليفتون شادويك (Clifton Chadwick).الذي تفضل بالإشراف على هذا البحث، على ما أحاطني به من كريم اهتمامه، وخلاصة فكره وعلمه، وثمين وقته، فكان لي موجها ومصوبا، ومشجعا، فله الشكر الوفير، والاحترام والتقدير، وأسأل الله تعالى أن يمن عليه بالصحة والعافية، وأن يجزيه عني خير الجزاء.

والشكر موصول لأخي وصديقي العزيز فضيلة الدكتور إكرامي مرسال. الذي لم يضن علي بعلمه ولا وقته. كما أتوجه بالشكر والعرفان لكل من ساهم في اتمام هذا العمل المتواضع.

وفي الختام أسأل الله تعالى أن يغفر لي تقصيري، ويتجاوز عن خطئي، فهذا الجهد المتواضع – لا شك- لا يخلو من الخطأ والنقص والتقصير، فإن أصبت فمن الله تعالى، وإن أخطأت فمن نفسي والشيطان، كما أسأله تعالى أن ينفعني بما علمني، وأن يوفقني لما يحب ويرضى.

الطالب: هشام حنفي أيوب

Abstract

The title of this study is "Suggested Approach for developing Mathematical Process in Applied Technology High School Based on the Standards of National Council of Teachers of Mathematics (NCTM)". The aim of this study is to analyze the content of Mathematics curriculum at the Applied Technology High School (ATHS) in Dubai in order to determine the reasons behind the students' low achievements in the ATHS exams and International Examinations.

The Applied Technology High School (ATHS) is a governmental school."It offers career – based technical education (CTE) in English at the secondary and tertiary levels. The institute five campuses are located in UAE at Abu Dhabi, Al Ain, Dubai, RAK, and Fujairah with its directorate located in Abu Dhabi. The institute was found in 2005 through royal decree of His Highness Sheikh Khalifa Bin Zayed Al Nahyan president of United Arab Emirates, Ruler of Abu Dhabi as a corporate body with full financial and administrative independence." (IAT, 2009)

The researcher analyzed the content of Mathematics at the Applied Technology High School (ATHS) based on the standards of national council of teacher of math (NCTM) and the results of the analysis showed that the content of the curriculum was compatible with international standards.

The educational process consists of three dimensions; the curriculum, the learner and the teacher. Since Applied Technology High School follows a specific system in the selection of students as enrolled students must pass the entrance exam, so the enrolled students have the minimum requirement to study at ATHS. And since the analysis of the curriculum content showed that the ATHS curriculum is compatible with the international standards, so the problem behind the students' low performance most probably is the teaching process (problem solving, mathematical reasoning, and mathematical communication).

Based on the analysis results the researcher suggested an approach to develop the teacher's skills in teaching with mathematical process to enhance the students' achievement in the ATHS and international exams. Also the researcher suggested an observation tool to follow up the teachers' performance in the classroom.

Key words: content analysis, international standards, high school math curriculum.

ملخص

عنوان هذه الدراسة هو: "رؤية مقترحة لتطوير العمليات الرياضياتية في ثانوية التكنولوجيا التطبيقية استنادا إلى معايير المجلس الوطني لمعلمي الرياضيات (NCTM)".

والهدف من هذه الدراسة هو تحليل محتوى منهج الرياضيات في ثانوية التكنولوجيا التطبيقية(ATHS) في دبي، لتحديد الأسباب الكامنة وراء الأداء المنخفض للطلاب في مادة الرياضيات، بما فيها الاختبارات التحصيلية والاختبارات الدولية.

ثانوية التكنولوجيا التطبيقة (ATHS) هي مدرسة حكومية، تقدم تعليما تكنولوجيا باللغة الإنجليزية، للمرحلتين (الثانوية والجامعية) ، تلبية لاحتياجات سوق العمل (CTE). عدد فروع المعهد في دولة الإمارات العربية المتحدة خمسه، تتوزع كل منها في إمارة أبو ظبي والعين، ودبي، ورأس الخيمة، والفجيرة.

تم إنشاء المعهد في عام 2005 من خلال مرسوم ملكي من صاحب السمو الشيخ خليفة بن زايد آل نهيان رئيس دولة الإمارات العربية المتحدة، حاكم إمارة أبوظبي كهيئة ذات شخصية اعتبارية مع كامل الاستقلال المالي والإداري (معهد التكنولوجيا التطبيقية 2009).

قام الباحث بتحليل محتوى منهج الرياضيات في ثانوية التكنولوجيا التطبيقية (ATHS)، استنادا إلى معايير المجلس الوطني للمعلمي الرياضيات (NCTM)، وأظهرت نتائج التحليل أن محتوى المناهج الدراسية بثانوية التكنولوجيا التطبيقية تتوافق مع المعايير الدولية.

العملية التعليمية تتكون من ثلاثة أبعاد هي: المناهج الدراسية، والمتعلم والمعلم. تتبع ثانوية التكنولوجيا التطبيقة نظاما شديدا في قبول الطلاب، بحيث يتحتم على الطالب أن يتجاوز اختبار القبول بنوعيه الكتابي والشفوي، لذا تجد لدى الطلبة المقبولين الحد الأدنى من الإمكانات العلمية للدراسة في ATHS .

ولقد أبانت نتائج تحليل محتوى المناهج الدراسية أن منهج ATHS متوافق مع المعايير الدولية، وبالتالي فإن المشكلة وراء انخفاض درجات الطلاب تكمن على الأرجح في عملية التدريس فيما يختص بالعمليات الرياضياتيه (حل المشكلات،التفكير الرياضي، والاتصالات الرياضية).

واستنادا إلى تحليل النتائج -أيضا- توصل الباحث إلى اقتراح نهج لتطوير مهارات المعلم في التدريس باستخدام العمليات الرياضياتية لتعزيز إنجاز الطلاب في الاختبارت التحصيلية والاختبارات الدولية، كما اقترح الباحث أداة رصد لمتابعة أداء المعلمين في الفصول الدراسية.

الكلمات المفتاحية: تحليل المحتوى، والمعايير الدولية، منهج الرياضيات بالمرحلة الثانوية.

Table of Content

Content Page

Table of Tables

Table of Figures

Abbreviation

IAT	Institute of Applied Technology
ATHS	Applied Technology High School
UAE	United Arab Emirates
USA	United States of America
NCTM	National Council for Teachers of Mathematics
PPP	Public Private Partnership
CTE	Career – based Technical Education
SACS	Southern Association of Colleges and Schools
SAT	Scholastic Aptitude Test

Chapter I: Introduction

Content **Page**

Introduction

The textbook reflect the curriculum content and it has a big impact on the student's achievement, so it has received more attention from many researchers and educators, and from time to time the curriculum developer try to enhance the new generations of textbook.

Textbook remains the most important source of learning, student's evaluation, user-friendly, and low-cost compared with other technological alternatives, it also provides the minimum required curriculum content and can control the four elements: objectives, knowledge content, evaluation and activities, and also it is easy to develop graphics pictures and make it fun and exciting, and interesting.

Therefore it is important to review Textbook content continually, because it shows us how the power of the most important tools in learning and teaching used by the teacher and student, especially after the changing view of educators to textbook, we have today recommended that the teacher and student and each interested must have point of view for development and improvement" (Abdullah, 1991: 10).

Contemporary educational trends confirm the need for an educational system which is characterized by total quality processes and innovate outputs, so it is important to continuously review the methods and tools for educational evaluation and development. So as to achieve good continues and comprehensive evaluation that match the requirements of the continuous development of human communities and building creative individuals and conscious of the problems together, and needs, and requirements for its growth and progress.

It is necessary to develop the Student's cognitive structure as one of the main aspects to achieve overall quality in the educational process. The educational system looks at the learner's personality without neglecting any aspect of the personality to be a balanced, and to be able to deal with the challenges and obstacles posed by contemporary life.

Many researchers and specialists in textbook analysis and evaluation tried to analyze and evaluate written mathematics' contexts depending on adopted tools and

standards, (Robert, 1980), (NCSS, 1994), (El-qa'oud, 1990), (DeRoche, 1981), (Lorant&Wilt, 1998), (Ungurait, 2001).

In 1986 a committee of directors of the National Council of Teachers of Mathematics (NCTM) established a working group to develop standards for teaching and learning mathematics in order to improve the quality of school mathematics and evaluate the curriculum (Michael, 2001). This document for curriculum and content standards called School Mathematics Curriculum and Evaluation Standards, was the first effort by the educational organization specializing in mathematics, reflected in this document are visions and attitudes and perceptions of those interested in mathematics school teachers, as well as researchers and educators (Olsan, & Berk , 2001).

"The NCTM had previously produced a landmark of standards documents- *Curriculum and evaluation standards for school mathematics (1989), Professional standards for teaching mathematics (1991), Assessment standards for school mathematics (1995), and principles and standards for school mathematics (2000)* These four documents represented a historically important first attempt by a professional organization to develop and articulate explicit extensive goals for teachers and policymakers. Since their release, they have given focus, coherence, and new ideas to efforts to improve mathematics education". (NCTM, 2000)

"Principles and standards reflect input and influence from many different sources. Educational research serves as the basis for many of the proposals and claims made throughout this document about what it is possible for students to learn about certain content areas at certain levels and under certain pedagogical conditions. The content and processes emphasized in Principles and Standards also reflect society's need for mathematical literacy, past practice in mathematics education, and the values and expectations held by teachers, mathematics educators, mathematicians in the writing group".(NCTM, 2000)

However, these criteria have developed a standard vision of how under which mathematics learning and teaching and evaluation, and formed the outline of the Platform for balanced based on the theory and application (Abed and Al-Khatib, 2002), while there has been redefining the role of both student and teacher, change the

role of the student than the recipient of negative information to an active participant in the educational process, and the teacher's role changed from a carrier of knowledge to the facilitator for acquiring the educational experience.

The Institute of Applied Technology (IAT) obtained accreditation from AdvancED, so the curriculum and assessment unit in Institute of Applied Technology faces many challenges in realizing the vision for improving mathematics ability for students. For example, how can all students achieve high-quality mathematics education? How can teachers of mathematics learn what they need to practice on before beginning the scholastic year? Are assessments matching with instructional goals?

"A school is successful in meeting the AdvancEd standards when it implements a curriculum based on clear and measurable expectations for student learning that provide opportunities for all students to acquire requisite knowledge, skills, and attitudes. Teachers use proven instructional practices that actively engage students in the learning process. Teachers provide opportunities for students to apply their knowledge and skills to real world situations. Teachers give students feedback to improve their performance". (AdvancEd, 2007)

Although Applied Technology High School (ATHS) uses an international curriculum (Glenco Algebra) with approved content, students' achievement is very low, so we need research on how to deliver the content to students.

The current Study aims to analyze the Applied Technology High School (ATHS) mathematics curriculum based on national standards of teachers of mathematics (NCTM) to determine the strengths and weakness in order to make changes to improve quality in the educational system in Applied Technology High School (ATHS).

The problem

Applied Technology High School math curriculum consists of two levels: core math for all students, and engineering math for engineering science cluster and energy cluster. During the academic year 2010/2011 low achievement in the core math exams in *Applied Technology High School (ATHS) – Dubai* was observed. In addition, the big gap in the SAT Reasoning Test Average between ATHS students' average was approximately (361) and international average was approximately (531)for males. There is a need to analyze the content of ATHS mathematics curriculum based on an international standards such as standards of the National Council of Teachers of Mathematics (NCTM) in 2000, because the Institute Applied Technology has obtained international accreditation from Southern Association of Colleges and Schools (SACS) in USA.

More procedurally, the problem of this study can be identified in the next main questions;

- Does the ATHS math curriculum meet the international standards of mathematics content (NCTM standards)?
- Based on the result of the content analysis, what is the suggested procedure to improve standards and teaching?

The Importance

- Provide information to what extent the current math curriculum meets international standards of mathematics in secondary school.
- Provide suggestions to help curriculum developers to identify the aspects that must be reconsidered in order to improve the math curriculum.
- Present classroom observation sheet based on process standards to determine Teacher's performance in the class room.

Study Tools

The study conducted based on the List of international content and process standards of mathematics (NCTM Standards, 2000), and SAT Reasoning Test Standards.

Study Sample

The study relied mainly on Applied Technology High School math curriculum from grade 9 till grade 12 for the academic year 2011/2012.

Methodology & Statistics Treatment

In this study, the researcher used a descriptive analytical approach to answer the study's questions, by identifying a list of international standards used in the analysis, then analyze the ATHS math curriculum from grade 9 till grade 12 based on this list.

Procedures

- Reviewing the literatures related to curriculum analysis and international standards of mathematics education.
- Identifying a list of international standards of mathematics education (NCTM Standards, 2000).
- Analyzing the ATHS math curriculum from grade 9 till 12
- Discussing the analysis results.
- Identifying the reasons behind the students' low achievement in math.
- Presenting suggestions to solve these problems
- Providing recommendations for further studies.

Terminology

NCTM Standards:

"Ambitious and comprehensive set of goals for mathematics instruction, The first five Standards present goals in the mathematical content areas of number and operations, algebra, geometry, measurement, and data analysis and probability. The second five describe goals for the processes of problem solving, reasoning and proof, connections, communication, and representation. Together, the Standards describe the basic skills and understandings that student will need to function effectively in the twenty-first century". (NCTM, 2000)

Chapter II: Literature Review

Content **Page**

Introduction

The educational system in Emirate society is facing many challenges, which have led to the increasing need to develop this educational system and to enable each individual to acquire the knowledge and skills that help him to develop himself and play main role in his society.

These challenges include;

- Deepening of accounting and accountability principle in the education system.
- Linking reward and punishment to academic performance in education.
- Reflecting new concepts such as continuing education for life, and sustainable human development and education for the future on education (education in critical thinking).
- The high development in teaching methods, methods, and the diversity of sources of teaching and learning.
- Transferring the focus center of the educational process from education to learning and from teacher to learner.
- Shifting from measuring inputs to focus on results.

Therefore, a standards-based reform has become a driving force for many of the educational policy, establishment which stresses the need to increase the level of student performance, and provide the opportunity for each student to learn appropriate content up to the level of performance required.

According to this vision, the standards not only emerged in education, but also spread strongly in recent years, so that sometimes we call this decade, a standardized-decade .(Kamal Abdel-Hamid, 2004: 115)

Based on analyzing number of standards' definitions, we can define the standards as " what students should know and able to do in domains of mathematics, or statements that describe what students should do after studying". (NCTM, 2000)

So in this study we focus on educational standards movement, its nature, its importance, and the foundations based upon, both in the educational system in general or in teacher preparation programs in particular.

The historical View of educational standards:

The United States is the most important countries that paid high attention to the standards approach in education, which has been taken as a trend to reform the American educational system and teacher preparation programs.

The standards approach, determining levels of acceptance performance, and academic curriculum for all students is not a new vision to American education system, it has historical old roots backing to publication of the report prepared by a "the Committee of Ten" (1894) which called for the establishment of academic disciplines suitable for all new students.

In 1918, The "National Education Association" has been formed and led the work in a group of progressive educators, who denied the work of the "Committee of Ten", and issued their report on " The Cardinal Principles", which called for the organization of secondary education in order to better education in the twentieth century, as well as the adoption of new issues on the present life concerns of students. (Jeffrey: 2005)

However, there are many researchers in the field of education who believe that the beginning of the movement of modern educational standards due to the publication of the famous report "A Nation at Risk " in 1983, and caused a significant change in the educational reform discourse, as well as raising serious concern of American society on the future and the quality of educational system , and the extent of accreditation in this system, as it believes that educational institutions too weak to face the danger that facing the nation, because of the weakness of these institutions.(Marzano, R: 1998: 1).

The report of "A Nation at Risk" presented important recommendations to reform the American education system, including: the need to development schools, colleges and universities to high standards, more measurable, and those colleges and universities raised the requirements to join. As well as the report recommended the

need to develop teacher preparation, and make the teaching profession more respected, through the preparation of teachers based on prevail educational standards, so that they can teach efficiently, and must evaluate colleges and universities that offer programs for teacher preparation by matching graduates to those standards. (The National Commission on Excellence in Education: 1983)

Thus, the need for capable effective education for improving the life of Americans led to the need for a comprehensive assessment of the goals and policies of educational systems in the performance of these systems, and then work on the establishment of educational standards, to evaluate and develop the educational system.

Growing concerns about the educational preparation of young American nation, after the publication of the report A Nation at Risk, led to the first educational conference of President "Bush" in 1989, attended by the President and state governors, to set educational goals that will bring America to the lead in international competition. At the same time some professional organizations, like the National Council for Teachers of Mathematics (NCTM) started writing the draft version of the curriculum standards and evaluation of school mathematics, which completed by the year 1989.

After the educational conference of President "George Bush", an official statement has released before the end of his period, and stated some of the principles that should guide education in the states:

1. All children in the United States will start learning when they are ready.

2. High school will graduate not less 90% of the number of students who have completed primary school.

3. An assessment should be steady and regular performances for students in grades fourth, eighth and twelfth, and we need to ensure their competence in basic science: English language, mathematics, science, geography and history, as well as social maturity as citizens in a productive society.

4. American students must be the highest achiever at the global level in the science and mathematics.

5. All Americans will be free from illiteracy, and will provide themselves with knowledge and skills necessary to compete in the context of the global economic system, and increase their competence in the exercise of citizenship rights and duties.

All schools in America provide a disciplined conducive environment for authentic learning liberate America from drugs and violence. (The National Commission on Excellence in Education: 1983

Importance of standards in the education system:

- judging quality in a particular field of knowledge, through: (1) Identifying the quality of what learners know, what they can do, (2) Measuring the Quality of the program, which provide them the opportunity to learn in this field, (3) Knowing the Quality of education in general, (4) The quality system that supports teachers and the program, (5) Quality of the evaluation practices and policies, (6) providing a benchmark to evaluate all dimensions of teaching.

- Standards in the education system Being the base of accountability, which is important approach for the school reform, and therefore the school, will move to education based on performance, using mechanisms, procedures and tests based on performance, thus enhancing the confidence of communities in education.

- Meeting the criteria of Total Quality, reflecting the development of community, service, and meeting with the needs of scientific and technological developments.

- Contributing to building evaluation based on a high degree of stability, and help to follow the development of student' performance and then to judge their progress towards achieving the identified standards.

- Learners learn best in the environment based on the standards, (1) as the teachers can choose the educational activities which enable learners to achieve the standards,(2) Learners know what is the require, and they can use the learning

tools and activities to achieve the learning standards, (3) parents can help their children through doing homework, as they see the close relationship between home activities and standards,(4) the administration provide the all requirements to achieve standards. And thus set standards and clear definitions of all roles and duties of each part in the educational process. (NCTM,2000)

In regard to the importance of standards for student, the education describes the standards of the learners' knowledge and skills through the different stages of learning, and thus helps them to improve their performance and motivation to work toward achieving those standards, and the standards lead them to the challenge to prove their competence and ability.

While the importance of standards for teachers is to provide them an opportunity to develop methods and teaching strategies, determine exact knowledge and skills that learners need, and give the teacher a space of freedom of selection among the different learning resources, rather than relying on the textbook only, in addition, it helps the teachers to select and design educational activities based on the needs of their students.

In an attempt to reform education in Egypt, the Ministry of Education, through collaboration with many experts in the field of scientific research, educational leaders and policy in the community, developed national standards for education in Egypt since 2003.

It is presumed that this research aims to achieve quality in the educational process, by defining a set of normative levels, which based on evaluating all aspects of the educational process to ensure the kind of transparency, accountability and justice. So it has been developed main five domains: effective school, teacher, excellent management, community participation, curriculum and learning outcomes. The document included the curriculum and learning outcomes of ten documents: document the learner, and document the curriculum that deals with all of its elements, and eight documents for subjects that have been selected in the first phase of the project, namely: Islamic religious education, religious education Christian, Arabic, English, French language, social studies, science, mathematics. (Ministry of Education, 2003).

the United Arab Emirates aimed to reform the education system, in collaboration with many foreign experts in the field of educational research, educational policies.

Based on the vision of Abu Dhabi Educational Council (ADEC) "Recognized as a world class education system that supports all learners in reaching their full potential to compete in the global market" ADEC conducted project called public private partnership (PPP) to enhance the education based on the international standards.

Since Applied technology High School (ATHS) has obtained an international accreditation from USA, ATHS curriculum unit should develop the curriculum based on international standards.

Relation between domains, standards and indicators

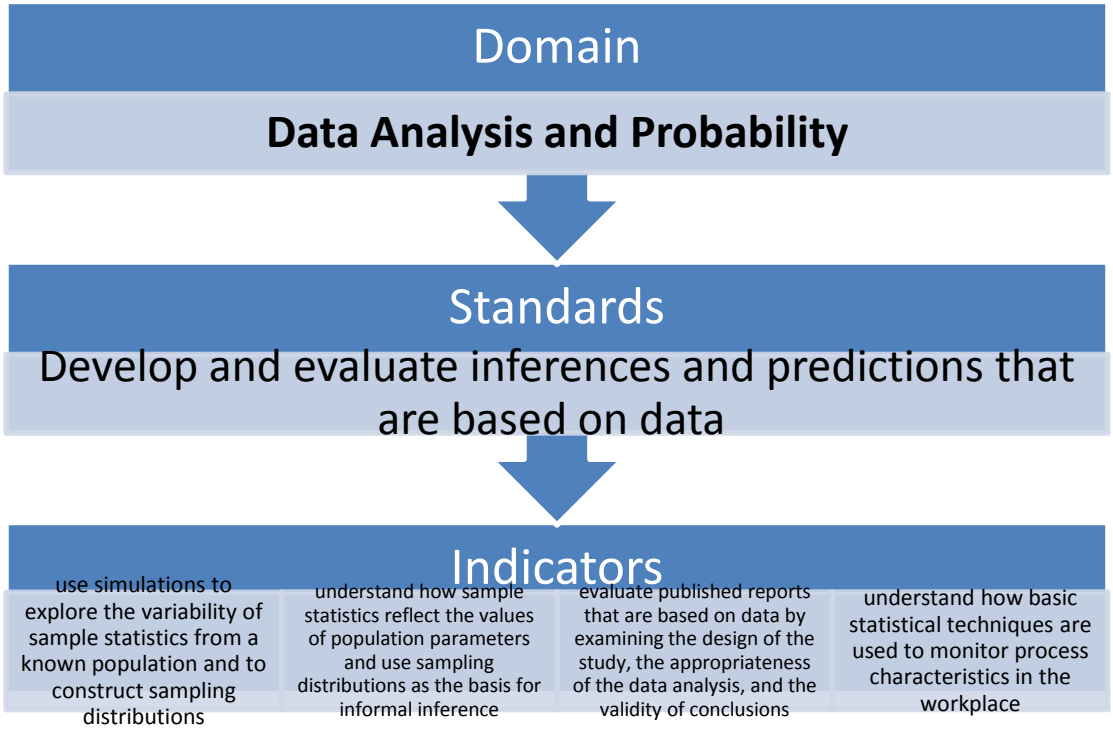


Figure (1): Relation between domains, standards and indicators

Principles of standards:

NCTM document has reviewed the six basic principles which are considered essential to improve the mathematics education of students, these six principles are:

The equity principle:

This principle requires excellence and equality in learning of mathematics, and supporting the capacities of all students.

The curriculum principle:

This principle considers curriculum as more than a gathering of some topics and educational activities that student can achieve, but focuses on the importance of learning mathematics and its nature, that should be logically-structured.

The Teaching principle:

This principle concerns learning mathematics effectively, and to build the teacher's scientific background, and be familiar with what the student knows and wants to learn. Then the teacher will try to provide suitable educational experiences for students to face mathematical challenges through solving mathematical problems

The learning principle:

This principle refers to the importance of the depth of understanding mathematics, so that students can configure the new information through directly organized experience (i.e., building knowledge through direct experience).

The Assessment principle:

This principle must support the importance of mathematics assessment, and provides important and necessary information for both the student and the teacher.

The technology principle:

This principle is essential in teaching and learning of school mathematics which is helpful and influential in developing the learner's ability.

Types of standards:

There are two main types of standards. The first are the Content Standards, and the second are the performance standards;

The Content Standards

Considered a guide for designing any educational subject or course, clarifying the skills and concepts that must be taught in the context. It also offers a description of what is supposed to be taught by teachers, and what students should learn. This is a description of clear knowledge and skills that should be learned by students.

Levels of content Standard for School Mathematics

1. **Numbers and operations Standard:**

   The Number and Operations Standards for educational programs from pre-school through twelfth grade should help the student to perform and to achieve the following learning objectives:

   a. Understanding numbers, ways of representing numbers, relationships among numbers, numbers and nature of the systems
   b. Understanding the meanings of arithmetic operations, and how these processes relate to each other.
   c. Fluency in calculations and ability to make estimates evidentiary.

2. Algebra Standard

a. Understand the patterns and similarities, relationships and functions.

b. Represent and analyze mathematical situations, the use of mathematical structures, and the use of algebraic symbols.

c. Use mathematical models to represent and understand quantitative relationships.

d. Analyze Change in different contexts.

3. Geometry Standard:

a. Analysis of properties of geometric shapes in the plane and space to make the student able to make mathematical discussions and geometric transformation with understanding the relationship between them.
b. Description spatial relationships using geometric coordinates and any other representation systems.
c. Applying transformation geometry and using symmetry concept to analyze mathematical and geometric situations.
d. use Visualization, spatial reasoning, and geometric modeling to solve math problems

4. Measurements Standard:

a. Understanding the characteristics and properties of objects and their measurement units, and measurement systems.
b. Applying different methods and appropriate techniques in the measurement and in the using different measurement tools.

5. Data Analysis and Probability Standard:

a. Formulating questions about a particular set of data, training students to collect data, organize and display appropriate methods to answer these questions.
b. Using statistical methods and appropriate data analysis.

c. Developing and evaluate the interpretations and predictions based on data collected.
d. Understanding and applying basic concepts of probability.

The Process Standards:

Describe the learning tasks and activities which should reflect the content standards, and suggest samples of student work based on benchmarks, specifically it determine use of the knowledge and skills, so it is not a description of the knowledge, but the description of the applications of that knowledge.

The National Council of Teachers of Mathematics (NCTM) defines process standards as statements that describe the methods used or followed by the students to acquire knowledge and skills determined in the mathematics content standards, these processes are problem solving, communication, connections, reasoning and representation. (NCTM, 2000; Fatima Fatouh, 2006)

1. Problem Solving Standard

The student's ability: solving a variety of mathematical problems and non-mathematical problems individually and cooperatively, using appropriate technology, and strategies for non-routine problem-solving; and therefore school mathematics must include regular and multiple opportunities for solving non-routine problems, and the problems related to the life of learners.

2. Mathematical communication Standard

It is clear that the process enables students to learn the signs , symbols, terminology , mathematical models, and using them in solving problems, reading, writing and discussion of ideas and a variety of issues using correct mathematical language, and so the mathematics school must include opportunities to read and write and discuss a variety of ideas and problems.

3. Mathematical Reasoning Standard

This process shows in the students' ability to:

1. Data collection , observations , interpretation and formation of scientific conjectures

2. Gathering evidences

3. Building discussions to support or reject the mathematical ideas.

So, we really need to include the school mathematics opportunities for the acquisition of mathematical facts and skills, and opportunities to express, interpret mathematical ideas and relationships.

4. Mathematical Connection Standard

This process is demonstrated by the students' ability to make links between mathematical ideas, and between mathematics and other knowledge areas, so it is very important to:

1. Build school mathematical content as an integrated whole with a variety of activities from within mathematics and outside it

2. Including school mathematics applications on a variety of processes, facts, skills and concepts of math to face different problems.

The National Council of Teachers of Mathematics (NCTM) is considered as the first national council concerned with the preparation and documentation of the standards of school mathematics, and the document of "Curriculum & Evaluation standards for school", issued in 1991, was the first document of school mathematics standards at the global level, in response to efforts to reform school mathematics, these document aimed to:

- The announcement of the desired objectives.

- Assistance in the reform of school mathematics.

- Emphasis on achieving total quality in mathematics education.

The document "standards of mathematics curriculum and evaluation" provides a national vision for the high quality mathematics education for all students, regardless of color, religion, social status, language or sex, as it provides a clear view on mathematical content, teaching methods in different levels from kindergarten to 12th grade, and explains how to assess learning of mathematics.

The document "standards of mathematics curriculum and evaluation" is considered as prominent vehicle for reform, because it includes suggestions about how to teach mathematics, taking in consideration the changing nature of school mathematics.

The same document in 1995 also provided a set of standards for judging the quality of assessment practices and learning of mathematics education, which determined in six evaluation standards as follow: (NCTM, 1995)

1. The Mathematics Assessment Standards.
2. The Learning Standards.
3. The Quality Standards.
4. Openness Standards.
5. Inference Standards.
6. Coherence Standards.

Related Studies:

Many studies have addressed curriculum analysis in order to evaluate or develop the curriculum. The researcher will divide the studies into two levels:

1. **Studies related to curriculum development based on standards:**
2. **Studies related to curriculum evaluation based on standards:**

And will discuss these studies in terms of the subject of the study, the results of the study, and applicability of the result in ATHS.

1. Studies related to curriculum development based on standards:

Article Title	Subject of Article	Relevant Results	Applicability to ATHS
Glenda Lappan: "Conclusion: Moving forward with the standards" (Lappan, 2001)	*This paper discusses how the curriculum and evaluation standards for school were first issued and considered as the big picture by the NCTM at a time when such ideas of national guidelines were not a popular thing. *The paper then discusses the dilemma of interpreting the meaning of standards and how these standards according to the NCTM were intended to serve as a guide not as the main recipe. *Another point was discussed in this paper, it is the point which the NCTM focused on in the mid- 90s It is revising and updating the work on the standards where they focused on narrowing the gap between the words and their interpretations	*The paper at the end suggests that in order for the field of mathematics education to achieve significant progress in making the process of teaching and learning any better, we need to undertake more carefully structured types of researches.	Applicable

Article Title	Subject of Article	Relevant Results	Applicability to ATHS
Karen J. Graham, Francis Fennell: "Principles and standards for School Mathematics and Teacher Education: Preparing and Empowering Teachers". (Graham, 2001)	*This paper aims to discuss recommendations for the preparation of teachers, continuous professional development and the certification policy of the mathematics teachers.	*To improve the mathematical education we need the two categories that influence it the most: teachers and teacher educators. *It emphasizes the role teachers' play in improving the learning of mathematics and the role teacher educators have in supporting teachers through the process of change. *This paper states that the structure of the educational organizations needs to change in order to suit the new changes in the learning of teachers	Applicable except the third point because hard to change the structure of the organization.
Barbara Reys , Eric Robinson , Sheila Sconiers , June Mark: "Mathematics Curricula Based on Rigorous National standards What, Why, and How?" (Reys, 1999)	*This article states that it is a must that teachers ,parents ,administrations know about and be acquainted to the special characteristics of the mathematics curricula that are based on the NCTM standards and also the support structures that are designed to ease the adaptation of these standards in the schools	*There are also recommendations included in the article that were made by the reformers to update these standards *It is stated in the article that despite the difference in each of these curriculums they all represent specific interpretations of	Applicable but with conducting training sessions for teachers

Article Title	Subject of Article	Relevant Results	Applicability to ATHS
	*The article also includes a brief history of the formation of the standards and their development phases. *Mention of the programs funded by the national science foundation for the development of the curriculum based on regious mathematical standards is also found in the article.	the NCTM standards' vision as they all introduce significant mathematical concepts, emphasize mathematical reasoning and problem solving, and give students the chance to learn essential mathematics skills.	
Lucy Carpenter Snead: "Professional development for middle school mathematics teachers to help them respond to NCTM standards". (Snead, 1998)	*The evaluation and curriculum standards were made for school mathematics to promote change, according to a result of a city wide survey a professional development program was created to help teachers cope with the change that came over the curriculum as a result of using the new mathematics standards and principles for school.	*Results from this study and survey indicate that teachers who did finish the two courses provided in the program had significant increase in their (Perry level).this study also gave a prove that the higher the Perry level of teacher gets the more their believes match the set standards –a thing that supports the idea of these professional development programs.	It is difficult to apply. Because it needs a decision from the directorate to teach these two courses for teachers

2. Studies related to curriculum evaluation based on standards:

Article Title	Subject of Article	Relevant Results	Applicability to ATHS
Michael J. Bosse , Tammy D. Lee : "The NCTM Process standards and the Five Es of Science: Connecting Math and Science". (Bosse & Lee, 2010)	*This is a study that examines the characteristics that are used to show the difference and similarities between the learning of science and the learning of Mathematics. *These characteristics are the ones found in the standards of both subjects (the standards of mathematics and the 5es of the national science education standards)	*The results of the study reveal noticeable similarities in both standards. The article also displays how hard it is for teachers to face the change in the curriculum stating that yes, it is hard but the results are really worth it. It also provides information about the show me website which includes all the information needed about NSF which help teachers adopt to change through workshops, sample lesson plans,…etc	Applicable because ATHS provide laptops and internet connection to all teachers and students.
Carla J. Thompson: "Preparation, Practice, and Performance: an Empirical Examination of the Impact of standard-based Instruction on Secondary Students' Math and Science Achievement". (Thompson, 2009)	*This paper discusses a study of the impact the standards –based instruction have over the secondary school students achievement in mathematics and in science *These standards according to this paper	*The findings of this study provided support for the views of the reform organizations; the findings also suggest the use of the p3 model preparation and practice as a frame work to validate the effectiveness of the standards –based	Applicable but after conducting training for teachers.

Article Title	Subject of Article	Relevant Results	Applicability to ATHS
	consist of a range of activities that mainly encourage student centered classes and prefer them over teacher centered classes. *This study seeks to prove the existence of empirical connections between standards based instructions (SBI) and student achievement through using quantitative analysis.	instruction as an effort for a systemic change in the education of Mathematics and science. *There is also a regious support given for the specific SBI practices making it the main key for the students' achievements in mathematics and science.	
Mary Ann Huntley, Jon D. Davis: "High –school Students' Approaches to Solving Algebra Problems That Are Posed Symbolically: Results from an Interview Study". (Huntley & Davis, 2008)	*This paper displays a small-based interview study that was held to investigate the approach of 44 pairs of high achieving grade 12 mathematics students to a number of algebra problems. *The three problems offered in the study were all posed symbolically. Two of these problems were a linear inequality and an equation with square roots while the third problem only involved square roots.	*The study findings revealed that most of the students (pairs) preferred to use symbol manipulation in solving problems as they used it in their first attempt to solve the problems before using other strategies like graphical and tabular ones. *The study also revealed that the pairs of students used more than one strategy always. *Another finding of the study which was a	Applicable

Article Title	Subject of Article	Relevant Results	Applicability to ATHS
		surprising one is the small number of students who used graphing calculators. *The findings in this paper claims that students use of graphing calculators while studying in classrooms does not necessary mean that they will be using them in solving unfamiliar symbolic problems. *The study at the end suggests further research examining the effects of technology on high school students of mathematics	
Deborah Graves Camp: "Where Do standards come from? A Phenomenological Study of the Development of National Board Early Childhood/Generalist standards". (Camp, 2007)	*This is a study that aims to create both an explanation and an understanding of the early childhood standards specifically the progress of the development of these standards.	*The results of this study did stand against the criticism that faces them regarding being culturally biased as it proves that the standards have solid foundation based on theory and research. *The results also support the progressive constructivist approach as part of the findings.	Not Applicable

Article Title	Subject of Article	Relevant Results	Applicability to ATHS
Asha K. Jitendra , Cynthia Griffin: "Adherence to Mathematics Professional standards and Instructional Design Criteria for Problem-Solving in Mathematics" (Asha K. Jitendra, 2005)	*This paper states that in the last decade and due to dissatisfaction with the performance of school students in America, calls for national reform were taken into serious consideration and many efforts were made to improve the curriculums and their contents were per sued. Some of these efforts were the standards which call for challenging learning standards, these standards which are the standards –based reforms. *The paper also discusses textbook materials which continue to be an ongoing challenge in the education of Mathematics stating that results from a study about textbooks in America compared to other countries showed that the US textbooks are less in depth, vaster and not successful in creating links between the covered topics.	*The study covered different mathematics books and suggested at the end that teachers should really focus on the instructional design principles to help change or modify math instruction in textbooks in order to match the needs of each individual student.	Applicable

Article Title	Subject of Article	Relevant Results	Applicability to ATHS
Curtis L. Pyke , Sharon Lynch: "Mathematics and Science Teachers' Preparation for National Board of Professional Teaching standards Certification". (Pyke & Lynch, 2005)	*This paper displays the results of a survey of the candidates of both mathematics and science for the NBPTS certification. *This survey aims to investigate how efficient group supports are in preparing for the national board for professional teaching standards (NBPTS)	*The results displayed in the paper reveals the similarity in composition between the subsamples of early adolescent mathematics and science, adolescent and young adult mathematics and science. *Still there is a result of variability in the nature of the effects across subsamples can be noticed in the results. *The paper also discusses the homogeneous nature of preparation that shows in the data available as well as the likelihood that candidates have different levels of being ready for the process of the NBPTS assessment. *Regarding this point the paper suggests that there is a need for more attention to be given to candidates who prepare for the assessment without any provided help or support from an organized group	Applicable

Article Title	Subject of Article	Relevant Results	Applicability to ATHS
Paula Maccini , Joseph Calvin Gagnon: "Perceptions and Application of NCTM standards by Special and General Education Teachers". (Maccini & Calvin, 2002)	*This paper displays a study that determined the perceptions of teachers related to the standards implementation with the students labeled LD and ED in the secondary level of education. *The paper includes some research questions that were designed for a survey mint for general and special educators in the high school of Maryland.	*The findings or the results of the survey report a significant difference across general and special educators and the number of years of experience in teaching students with LD and ED. *The findings suggest that many issues have to be taken into consideration to ensure the successful implementation of the activities that are based on the NCTM standards goals limiting them into five main issues and discussing them in details.	Applicable but after conducting training for teachers.
Jamar Pickreign , Lelon R. Capps: "Alignment of Elementary Geometry Curriculum with Current standards". (Pickreign & Capps, 2000)	Geometry in the curriculum of Mathematics is a subject of concern. The paper states that the gap between the books of geometry and the standards	*The paper states that the gap between the books of geometry and the standards. *This study implies that there are steps and things to be done in order to close this gap without really overcrowding the curriculum of geometry *One critical piece in the argument in this paper is the modeling of suitable vocabulary of geometry in developing concepts of geometry.	Applicable

Chapter III: Methodology

Content Page

in this chapter of the research, the researcher present the procedures followed in the design of list of content analysis standards based on international standards of high school mathematics, and then use this list in the analysis of the content of high school mathematics, as well as the suggested approach to treat mathematics based on emphasis on processing of mathematical process math in the classroom, in addition presenting an observation tool to follow up the teacher's performance within exercising these processes through teaching problem solving in high school mathematics.

List of international standards of mathematics:

The study conducted based on the List of international content and process standards of mathematics (NCTM Standards, 2000), and SAT Reasoning Test Standards.

- Re-viewing the international related studies to determine and analysis more than list of standards of mathematics, specially lists of NCTM standards.

- Taking the specialists' opinions in this list of mathematical content standards by sending the list of standards to two of math curriculum specialists[1].

- Re-viewing the specialists' opinions and their notes to rewrite the list of mathematical content standards.

- Finally using this list in analyzing the content of high school mathematics in ATHS.

- Calculating the percentage of agreement between the ATHS outcomes and NCTM indicators for each level (number and operation, algebra, geometry, measurements, and data analysis and probability) and for all standards together. Also the researcher will calculate the Reliability of the analysis by calculating

[1] Dr. Mahmoud Ahmed Al Ibyari. Prof. in mathematics education, curriculum and methodology dep., Alexandria University - Egypt

Dr. Ekramy Mohamed. Prof. assistant in mathematics education, curriculum and methodology dep., Alexandria University - Egypt

Scott's pi between the researcher analysis and another analysis for the same curriculum that has been made by another coder who named (Dr. Ekramy Mersal, mathematics education lecturer, Alexandria Univ.).

A guide for teacher to develop main process standards in high school

After using the list of standards of NCTM in analyzing the content of high school mathematics, the researcher designed a guide for the teacher to improve the process standard inside the classroom.

Depending on the analysis of NCTM document at 2000, the study will focus on the process standards that required for improving the students' achievement, which are:

(1) Mathematical Problem solving standard.

(2) Mathematical Reasoning standards.

(3) Mathematical communication standards.

Through studying these standards and lit reviewing various knowledge about reasoning, it was found out that all the three standards can be mixed together and developed in the form of reasoning habits.

After that, the researcher presented a simple guide to develop teacher's skills in practicing mathematical process through teaching the content in high school classrooms followed by suggested observation form to evaluate the teachers' skills in practicing these processes.

In addition, the researcher presented with these example an observation tool based on mathematical process standards that cited in NCTM document at 1989,1995,and 2000.

The observation tool:

To design the observation tool, the researcher follows the next steps:

- Re-viewing the international related studies to determine and analyze its observation tool which based on standards of mathematics.

- Determining the domains, standards and indicators that combatable with high school mathematics in AHTS.

- Designing initial form for observation tool based on mathematical process standards which combatable with high school mathematics in ATHS.

- The observation tool- in its final form- included (20) statements is distributed over the three dimensions of the tool: problem solving dimension , reasoning dimension and communication dimension, the first dimension included (8 statements), the second dimension included (5statements) the third dimension included (7 statements).

- These statements require the observer to record the observed degree appropriate to the teacher's performance by putting any mark under the one of the following choices (Unsatisfactory- Needs improvement-Good-Excellent).

- The researcher in assessing the level of teacher's performance while using the observation tool based on mathematical process standards, so that class is allocated to each of them in the range (1-4), so the maximum points becomes (80 points) and the minimum points becomes (20 points).

- The teacher needs professional development if his score less than 75% or (60 points)

- Taking the curriculum specialists' opinions in this observation tool[2].

[2] Dr. Mahmoud Ahmed Al Ibyari. Prof. in mathematics education, curriculum and methodology dep., Alexandria University - Egypt

Dr. Ekramy Mohamed. Prof. assistant in mathematics education, curriculum and methodology dep., Alexandria University - Egypt

- Re-viewing the specialists' opinions and their notes to rewrite the observation tool in the final form.

- Finally presenting the observation tool to administration in Applied Technology High School for using.

- Present observation sheet based on process standards to follow up Teacher's performance in the class room.

Chapter IV: Results and Discussion

Content Page

To answer the first question in this study "***Does the ATHS math curriculum meet the international standards of mathematics content (NCTM standards)?***" the researcher did the following procedures:

Content analysis based on NCTM standards:

Number and Operations		
National Council of Teachers of Mathematics (NCTM)		**Applied Technology High School (ATHS)**
Standards	**Key Performance Indicators (KPI)**	**Learning Outcomes**
Understand numbers, ways of representing numbers, relationships among numbers, and number systems.	Develop a deeper understanding of very large and very small numbers and various representations of them.	Demonstrate an understanding of powers with integral and rational exponents.
	compare and contrast the properties of numbers and number systems, including the rational and real numbers, and understand complex numbers as solutions to quadratic equations that do not have real solutions;	1-Demonstrate an understanding of the absolute value of real numbers. 2-Demonstrate an understanding of performing different operations on complex numbers.
	understand vectors and matrices as	

	systems that have some of the properties of the real-number system	
	use number-theory arguments to justify relationships involving whole numbers	Demonstrate an understanding of powers with integral and rational exponents.
Understand meanings of operations and how they relate to one another	judge the effects of such operations as multiplication, division, and computing powers and roots on the magnitudes of quantities	1-Evaluate numerical and algebraic expressions by using the order of operations 2-Demonstrate an understanding of multiplying and dividing rational expressions. 3-Demonstrate an understanding of adding and subtracting rational expressions.
	develop an understanding of properties of, and representations for, the addition and multiplication of vectors and matrices	Demonstrate an understanding of using the distributive property.
	Develop an understanding of permutations and combinations as counting techniques.	1-Solve problems involving permutations. 2-Solve problems involving combinations. 3-Use combinations and permutations to find probability.

Compute fluently and make reasonable estimates	Develop fluency in operations with real numbers, vectors, and matrices, using mental computation or paper-and-pencil calculations for simple cases and technology for more-complicated cases.	1-Solve problems involving absolute value. 2-Find the probability of inclusive events. 3-Find the probability of mutually exclusive events. 4-Find the probability of two independent events. 5-Find the probability of two dependent events.
	Judge the reasonableness of numerical computations and their results.	Solve problems involving absolute value.

Algebra		
National Council of Teachers of Mathematics (NCTM)		**Applied Technology High School (ATHS)**
Standards	**Key Performance Indicators (KPI)**	**Learning Outcomes**
Understand patterns , relations, and functions	generalize patterns using explicitly defined and recursively defined functions	1-Use arithmetic sequences. 2-Use sigma notation 3-Use geometric sequences 4-Write repeating decimals as fractions.
	understand relations and functions and select, convert flexibly among, and use various representations for them	1-Write linear equations in standard form. 2-Demonstrate an understanding of how to perform different operations on functions and how to write expressions with rational exponents in radical form and vice versa.
	analyze functions of one variable by investigating rates of change, intercepts, zeros, asymptotes, and local and global behavior	1- Analyze relations and functions. 2- Identify linear relations and functions 3- Write linear equations in standard form.
	understand and perform transformations	1-Demonstrate an understanding of graphing radical

	such as arithmetically combining, composing, and inverting commonly used functions, using technology to perform such operations on more-complicated symbolic expressions	functions with different transformations. 2-Demonstrate ability in performing operations with radical expressions.
	understand and compare the properties of classes of functions, including exponential, polynomial, rational, logarithmic, and periodic functions	1-Demonstrate an understanding of factoring monomials. 2-Demonstrate an understanding of how to graph quadratic functions and how to find and interpret the maximum and minimum values. 3-Demonstrate an understanding of graphing both the growth and decay exponential functions. 4-Demonstrate an understanding of logarithms and the graph of logarithmic functions. 5-Demonstrate an understanding of properties of logarithms. 6-Demonstrate an understanding of special logarithms; the common logarithm and the natural logarithm.

	interpret representations of functions of two variables	1-Interpret graphs of relations 2-Demonstrate an understanding of how to graph quadratic functions and how to find and interpret the maximum and minimum values. 3-Find the midpoint of a segment on the coordinate plane. 4-Find the distance between two points on the coordinate plane
Represent and analyze mathematical situations and structures using algebraic symbols	understand the meaning of equivalent forms of expressions, equations, inequalities, and relations	1-Translate sentences into equations, and equations into sentences 2-Factor and solve equations of the form $x^2+bx+c=0$ 3-Factor and solve equations of the form $ax^2+bx+c=0$ 4-Factor and solve equations of the form $a^2-b^2=0$ 5-Factor and solve perfect square trinomials. 6-Demonstrate an understanding of solving quadratic equations by factoring.

		7-Demonstrate an understanding of the multiplication of polynomial expressions (limited to monomials, binomials, and trinomials). 8-Demonstrate an understanding of multiplying and dividing rational expressions. 9-Demonstrate an understanding of adding and subtracting rational expressions.
	write equivalent forms of equations, inequalities, and systems of equations and solve them with fluency—mentally or with paper and pencil in simple cases and using technology in all cases	1-Represent a linear function, using function notation. 2-Solve linear equations in different methods. 3-Demonstrate an understanding of solving absolute value equations. 4-Demonstrate an understanding of simplifying radical expressions. 5-Explain and illustrate strategies to solve single variable linear inequalities, Absolute value inequalities, and Compound inequalities. 6-Demonstrate an understanding of solving and estimating solutions of quadratic equations by graphing.

		7-Demonstrate an understanding of solving quadratic equations by completing the square. (Only case $a=1 \text{ in } ax^2+bx+c=0$). 8-Demonstrate an understanding of solving quadratic equations by the quadratic formula. 9-Demonstrate an understanding of common factors and trinomial factoring. 10-Solve radical equations and inequalities. 11-Solve exponential equations and inequalities. 12-Solve logarithmic equations and inequalities. 13-Demonstrate an understanding of solving rational equations and inequalities. 14-Find arithmetic means. 15-Find sums of arithmetic series. 16-Find geometric means. 17-Find sums of geometric series. 18-Find specific terms of geometric series. 19-Find the sum of an infinite geometric series. 20- Write equations of parabolas in standard form

		21- Write equations of circles. 22- Write equations of ellipses. 23- Write equations of hyperbolas. 24- Write repeating decimals as fractions.
	use symbolic algebra to represent and explain mathematical relationships	Describe and represent linear relations, using: words, ordered pairs, tables of values, graphs, equations
	use a variety of symbolic representations, including recursive and parametric equations, for functions and relations	Demonstrate an understanding of literal equation and dimensional analysis.
	judge the meaning, utility, and reasonableness of the results of symbol manipulations, including those carried out by technology	1-Evaluate numerical and algebraic expressions by using the order of operations 2-Demonstrate an understanding of weighted averages. 3-Evaluate numerical and algebraic expressions by using the order of operations
Use mathematical models to represent and understand quantitative relationships	identify essential quantitative relationships in a situation and determine the class or classes of functions that	Demonstrate an understanding of ratios and proportions.

	might model the relationships	
	use symbolic expressions, including iterative and recursive forms, to represent relationships arising from various contexts	1-Demonstrate an understanding of variation functions. 2-Use Pascal's triangle to expand powers of binomials. 3-Use the Binomial Theorem to expand powers of binomials.
	draw reasonable conclusions about a situation being modeled	1-Demonstrate an understanding of literal equation and dimensional analysis. 2-Create and use graphs of probability distributions. 3-Graph parabolas. 4- Graph circles. 5- Graph ellipses. 6- Graph hyperbolas.
Analyze change in various contexts	Approximate and interpret rates of change from graphical and numerical data.	1-Demonstrate an understanding of percent of change. 2-Demonstrate an understanding of slope with respect to: • rise and run • line segments and lines

		• rate of change • parallel lines • Perpendicular lines. 3-Write the equation of a linear relation, given: • a point and the slope • two points • a point and the equation of a parallel or perpendicular line

Geometry		
National Council of Teachers of Mathematics (NCTM)		**Applied Technology High School (ATHS)**
Standards	**Key Performance Indicators (KPI)**	**Learning Outcomes**
Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships	analyze properties and determine attributes of two- and three-dimensional objects	1-Demonstrate an understanding of the properties of parallelograms and its connections to Algebra. 2-Demonstrate an understanding of properties of rectangles and determine whether parallelograms are rectangles. 3-Demonstrate an understanding of properties of rhombi and squares and determine whether quadrilaterals are rectangles, rhombi, or squares. 4-Demonstrate an understanding of Circles and Circumference. 5-Demonstrate an understanding of the relationship between arcs and chords. 6-Demonstrate an understanding of inscribed angles. 7-Demonstrate an understanding of tangents.

	explore relationships (including congruence and similarity) among classes of two- and three-dimensional geometric objects, make and test conjectures about them, and solve problems involving them	1-Demonstrate an understanding of the interior angles sum and the exterior angles sum theorems. 2-Demonstrate an understanding of similar polygons. 3-Demonstrate an understanding of similar triangles. 4-Demonstrate an understanding of parallel lines and proportional parts. 5-Demonstrate an understanding of parts of similar triangles.
	establish the validity of geometric conjectures using deduction, prove theorems, and critique arguments made by others	Demonstrate an understanding of the Sum and Difference of angles Identities.
	use trigonometric relationships to determine lengths and angle measures	1-Develop and apply the primary trigonometric ratios (sine, cosine, tangent) to solve problems that involve right triangles. 2-Demonstrate an understanding of angles in standard position, expressed in degrees and radians. 3-Demonstrate ability in finding values of

		trigonometric functions for general angles. 4-solve problems using the sines and cosines laws. 5-Demonstrate an understanding of the six trigonometric functions based on the unit circle. 6-Demonstrate an understanding of inverse trigonometric functions 7-Students will establish trigonometric identities and use them to simplify trigonometric expressions and verify equivalence statements. 8-Demonstrate an understanding of Double-angle Identities. 9-Solve Trigonometric Equations.
Specify locations and describe spatial relationships using coordinate geometry and other representational systems	use Cartesian coordinates and other coordinate systems, such as navigational, polar, or spherical systems, to analyze geometric situations	1-Demonstrate an understanding of graphing radical functions with different transformations. 2-Demonstrate an understanding of how to graph quadratic functions and how to find and interpret the maximum and minimum values. 3-Represent data, using polynomial functions (of degree ≤ 3), to solve problems.

		4-Graph and analyze the six trigonometric functions with special emphasis on the graphs of sine, cosine and tangent to solve problems. 5-Graph parabolas. 6- Graph circles. 7- Graph ellipses. 8- Graph hyperbolas.
	investigate conjectures and solve problems involving two- and three-dimensional objects represented with Cartesian coordinates	1-Represent data, using polynomial functions (of degree ≤ 3), to solve problems. 2-Analyze graphs of polynomial functions (limited to polynomial functions of degree ≤ 5).
Apply transformations and use symmetry to analyze mathematical situations	understand and represent translations, reflections, rotations, and dilations of objects in the plane by using sketches, coordinates, vectors, function notation, and matrices	Demonstrate an understanding of graphing radical functions with different transformations.
	use various representations to help understand the effects of simple transformations and their compositions	

Use visualization, spatial reasoning, and geometric modeling to solve problems	draw and construct representations of two- and three-dimensional geometric objects using a variety of tools	1-Demonstrate an understanding of solving and estimating solutions of quadratic equations by graphing. 2-Analyze graphs of polynomial functions (limited to polynomial functions of degree ≤ 5).
	visualize three-dimensional objects and spaces from different perspectives and analyze their cross sections	
	use vertex-edge graphs to model and solve problems	Use grid paper and string to investigate the shape of an ellipse.
	use geometric models to gain insights into, and answer questions in, other areas of mathematics	
	use geometric ideas to solve problems in, and gain insights into, other disciplines and other areas of interest such as art and architecture	Use trigonometric functions with special emphasis on the graphs of sine, cosine and tangent to solve problems.

Measurement		
National Council of Teachers of Mathematics (NCTM)		**Applied Technology High School (ATHS)**
Standards	**Key Performance Indicators (KPI)**	**Learning Outcomes**
Understand measurable attributes of objects and the units, systems, and processes of measurement	make decisions about units and scales that are appropriate for problem situations involving measurement	Demonstrate an understanding of how to measure angles and arcs in a circle.
Apply appropriate techniques , tools, and formulas to determine measurements	analyze precision, accuracy, and approximate error in measurement situations	1-Demonstrate an understanding of how to measure angles and arcs in a circle. 2-Demonstrate an understanding of secants, tangents, and angle measures.
	understand and use formulas for the area, surface area, and volume of geometric figures, including cones, spheres, and cylinders	1- Find the surface area and volume of geometric figures.
	apply informal concepts of successive approximation, upper and lower bounds, and limit in measurement situations	Find class boundaries in grouped frequency distribution table
	use unit analysis to check measurement computations	

Data Analysis and Probability		
National Council of Teachers of Mathematics (NCTM)		**Applied Technology High School (ATHS)**
Standards	**Key Performance Indicators (KPI)**	**Learning Outcomes**
Formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them	understand the differences among various kinds of studies and which types of inferences can legitimately be drawn from each	1- Differentiate between the two branches of statistics. 2- Identify types of data. 3- Identify the measurement level for each variable. 4- Identify the four basic sampling techniques. 4- Explain the difference between an observational and an experimental study.
	know the characteristics of well-designed studies, including the role of randomization in surveys and experiments	Students should be able to create a study to apply the knowledge.
	understand the meaning of measurement data and categorical data, of univariate and bivariate data, and of the term variable	1- Organize data using frequency distributions. 2- Represent data in frequency distributions graphically using histograms, frequency polygons, and ogives. 3- Represent data using Pareto charts, time series graphs, and pie graphs.

	understand histograms, parallel box plots, and scatter plots and use them to display data	Represent data in frequency distributions graphically using histograms, frequency polygons, and ogives.
	compute basic statistics and understand the distinction between a statistic and a parameter	Summarize data using measures of central tendency, such as the mean, median, mode, and midrange.
Select and use appropriate statistical methods to analyze data	for univariate measurement data, be able to display the distribution, describe its shape, and select and calculate summary statistics	Students should be able to display the distribution and describe its shape
	for bivariate measurement data, be able to display a scatter plot, describe its shape, and determine regression coefficients, regression equations, and correlation coefficients using technological tools	
	display and discuss bivariate data where at least one variable is categorical	
	recognize how linear transformations of	

	univariate data affect shape, center, and spread	
	identify trends in bivariate data and find functions that model the data or transform the data so that they can be modeled	
Develop and evaluate inferences and predictions that are based on data	use simulations to explore the variability of sample statistics from a known population and to construct sampling distributions	Identify the four basic sampling techniques.
	understand how sample statistics reflect the values of population parameters and use sampling distributions as the basis for informal inference	
	evaluate published reports that are based on data by examining the design of the study, the appropriateness of the data analysis, and the validity of conclusions	
	understand how basic statistical	

	techniques are used to monitor process characteristics in the workplace	
Understand and apply basic concepts of probability	understand the concepts of sample space and probability distribution and construct sample spaces and distributions in simple cases	Identify the four basic sampling techniques.
	use simulations to construct empirical probability distributions	
	compute and interpret the expected value of random variables in simple cases	1-Find the probability of inclusive events. 2-Find the probability of mutually exclusive events. 3-Find the probability of two independent events. 4-Find the probability of two dependent events.
	understand the concepts of conditional probability and independent events	1-Understand the probability of two independent events. 2-Understand the probability of two dependent events.
	understand how to compute the probability of a compound event	1-Understand how to Find the probability of inclusive events. 2- Understand how to Find the probability of mutually exclusive events.

Calculating the percentage of agreement and the Reliability:

In the following the researcher will calculate the percentage of agreement between the Applied Technology High School (ATHS) math curriculum and the National council of teachers of math standards (NCTM).

1- Agreement percent in content analysis Number and Operation

Table (1): Number and Operations

Standards	NCTM Indicators	ATHS Outcomes	ATHS Outcomes (Second Coder)	joint marginal's proportions
Understand numbers, ways of representing numbers, relationships among numbers, and number systems.	4	3	4	0.44
Understand meanings of operations and how they relate to one another	3	3	2	0.31
Compute fluently and make reasonable estimates	2	2	2	0.25
Sum	9	8	8	

a) Percent agreement:

$$\frac{3+3+2}{9} = 0.89$$

b) Reliability in content analysis

Percent agreement:

$$PA0 = \text{Total A's} = \frac{3+2+2}{8} = 0.88$$

$$\text{Scott's } pi = \frac{PA_0 - PA_E}{1 - PA_E}$$

Where $PA_E = \sum Pi^2$ and Pi = each joint marginal proportion

So $PA_E = (0.44)^2 + (0.31)^2 + (0.25)^2$

$= 0.19 + 0.096 + 0.063$

$= 0.35$

And, Scott's pi = $\frac{0.88 - 0.35}{1 - 0.35} = 0.81$

So, Reliability of this content analysis equals to 0.81 which is high

From the analysis of the data in the previous table:

Number and operation standards consist of three standards, the first standard contains (4) indicators while ATHS curriculum contains (3) outcomes, the second standard contains (3) indicators which is the same with ATHS curriculum, and the third standard contains (2) indicators which is also the same with ATHS curriculum, and the total number of NCTM indicators are (9) indicators, on the other hand the total number of ATHS curriculum outcomes are (8).

From the calculated percentage of agreement (0.89) and it is considered to be high percentage, it shows that the content of ATHS mathematics curriculum is matching international standards on the level of numbers and operations.

2- Agreement percent in content analysis Algebra

Table (2): Algebra

Standards	NCTM Indicators	ATHS Outcomes	ATHS Outcomes (Second Coder)	joint marginal's proportions
Understand patterns , relations, and functions	6	6	6	0.4
Represent and analyze mathematical situations and structures using algebraic symbols	5	5	5	0.33
Use mathematical models to represent and understand quantitative relationships	3	3	3	0.3
Analyze change in various contexts	1	1	1	0.06
Sum	15	15	15	

a) Percent agreement:

$$= \frac{6+5+3+1}{15} = 1.0$$

b) Reliability in content analysis

Percent agreement:

$$PA0 = \text{Total A's} = \frac{6+5+3+1}{15} = 1.0$$

Scott's $pi = \dfrac{PA_0 - PA_E}{1 - PA_E}$

Where $PA_E = \sum Pi^2$ and Pi = each joint marginal proportion

So $PA_E = (0.4)^2 + (0.33)^2 + (0.2)^2 + (0.06)^2$

$= 0.16 + 0.11 + 0.04 + 0.0036$

$= 0.3136$

And, Scott's pi = $\dfrac{1.0 - 0.3136}{1 - 0.3136} = 1.0$

So, Reliability of this content analysis equals to 1.0 which is perfect

From the analysis of the data in the previous table:

Algebra standards consist of four standards, the first standard contains (6) indicators, the second standard contains (5) indicators, the third standard contains (3) indicators, and the fourth standard contains (1) indicator which is the same with ATHS curriculum, and the total number of NCTM indicators are (15) indicators, which is the same total number of ATHS curriculum outcomes.

From the calculated percentage of agreement (1) and it is perfect percentage, it shows that the content of ATHS mathematics curriculum is matching international standards on the level of algebra.

3- Agreement percent in content analysis Geometry

Table (3): Geometry

Standards	NCTM Indicators	ATHS Outcomes	ATHS Outcomes (Second Coder)	joint marginal's proportions
Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships	4	4	4	0.38
Specify locations and describe spatial relationships using coordinate geometry and other representational systems	2	2	2	0.19
Apply transformations and use symmetry to analyze mathematical situations	2	1	1	0.09
Use visualization, spatial reasoning, and geometric modeling to solve problems	5	3	4	0.33
Sum	13	10	11	

a) Percent agreement:

$$PA0 = \text{Total A's} = \frac{4+2+1+3}{13} = 0.77$$

b) Reliability in content analysis

Percent agreement:

$$PA0 = \text{Total A's} = \frac{4+2+1+3}{11} = 0.91$$

$$\text{Scott's } pi = \frac{PA_0 - PA_E}{1 - PA_E}$$

Where $PA_E = \sum Pi^2$ and Pi = each joint marginal proportion

So $PA_E = (0.38)^2 + (0.19)^2 + (0.09)^2 + (0.33)^2$

$= 0.14 + 0.04 + 0.008 + 0.11$

$= 0.298$

And, Scott's pi = $\frac{0.91 - 0.298}{1 - 0.298} = 0.87$

So, Reliability of this content analysis equals to 0.87 which is high

From the analysis of the data in the previous table:

Geometry standards consist of four standards, the first standard contains (4) indicators which is also the same with ATHS curriculum, the second standard contains (2) indicators which is the same with ATHS curriculum, and the third standard contains (2) indicators while ATHS curriculum contains (1) outcomes, and the total number of NCTM indicators are (13) indicators, on the other hand the total number of ATHS curriculum outcomes are (10).

From the calculated percentage of agreement (0.77) and it is considered to be high percentage, it shows that the content of ATHS mathematics curriculum is matching international standards on the level of geometry.

4- Agreement percent in content analysis Measurement

Table (4): Measurement

Standards	NCTM Indicators	ATHS Outcomes	ATHS Outcomes (Second Coder)	joint marginal's proportions
Understand measurable attributes of objects and the units, systems, and processes of measurement	1	1	1	0.25
Apply appropriate techniques , tools, and formulas to determine measurements	4	3	3	0.75
Sum	5	4	4	

a) Percent agreement:

$$= \quad \frac{1+3}{5} \quad = 0.8$$

b) Reliability in content analysis

Percent agreement:

$$\text{PA0} = \text{Total A's} \quad = \quad \frac{1+3}{4} \quad = 1.0$$

$$\text{Scott's } pi = \quad \frac{PA_0 - PA_E}{1 - PA_E}$$

Where $PA_E = \sum Pi^2$ and Pi = each joint marginal proportion

So $PA_E = (0.25)^2 + (0.75)^2$

$= 0.06 + 0.56$

$= 0.62$

And, Scott's pi = $\frac{1.0 - 0.62}{1 - 0.62} = 1.0$

So, Reliability of this content analysis equals to 1.0 which is perfect

From the analysis of the data in the previous table:

Measurement standards consist of only two standards, the first standard contains (1) indicators which is also the same with ATHS curriculum, the second standard contains (4) indicators while ATHS curriculum contains (3) outcomes, and the total number of NCTM indicators are (5) indicators, on the other hand the total number of ATHS curriculum outcomes are (4).

From the calculated percentage of agreement (0.8) and although the lowest number of standards in this domain, it is considered to be high percentage; it shows that the content of ATHS mathematics curriculum is matching international standards on the level of measurement.

5- Agreement percent in content analysis Data analysis and probability

Table (5): Data analysis and probability

Standards	NCTM Indicators	ATHS Outcomes	ATHS Outcomes (Second Coder)	joint marginal's proportions
Formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them	5	5	5	0.43
Select and use appropriate statistical methods to analyze data	5	1	2	0.13
Develop and evaluate inferences and predictions that are based on data	4	1	1	0.09
Understand and apply basic concepts of probability	5	4	4	0.35
Sum	19	11	12	

a) Percent agreement:

$$= \frac{5+1+1+4}{19} = 0.58$$

b) Reliability in content analysis

Percent agreement:

$$PA0 = \text{Total A's} = \frac{5+1+1+4}{12} = 0.92$$

Scott's $pi = \dfrac{PA_0 - PA_E}{1 - PA_E}$

Where $PA_E = \sum Pi^2$ and Pi = each joint marginal proportion

So $PA_E = (0.43)^2 + (0.13)^2 + (0.09)^2 + (0.35)^2$

$= 0.18 + 0.017 + 0.008 + 0.123$

$= 0.328$

And, Scott's pi = $\dfrac{0.92 - 0.328}{1 - 0.328} = 0.88$

So, Reliability of this content analysis equals to 0.88 which is high

From the analysis of the data in the previous table:

Data analysis and probability standards consist of four standards, the first standard contains (5) indicators which is also the same with ATHS curriculum, the second standard contains (5) indicators while ATHS curriculum contains (1), and the third standard contains (4) indicators while ATHS curriculum contains (1) outcomes, and the fourth standard contains (5) indicators while ATHS curriculum contains (4) outcomes and the total number of NCTM indicators are (19) indicators, on the other hand the total number of ATHS curriculum outcomes are (11).

From the calculated percentage of agreement (0.58) and it isn't considered to be high percentage, it shows that the content of ATHS mathematics curriculum doesn't match international standards on the level of Data analysis and probability.

This data show that there is a problem in ATHS math curriculum on the level of Data analysis and probability, so math curriculum developer should review the math content related to data analysis and probability.

6- Agreement percent in content analysis overall math curriculum

Table (6): overall math curriculum

Standards	NCTM Indicators	ATHS Outcomes	ATHS Outcomes (Second Coder)	joint marginal's proportions
Number and Operation	9	8	8	0.16
Algebra	15	15	15	0.31
Geometry	13	10	11	0.21
Measurement	5	4	4	0.08
Data Analysis and Probability	19	11	12	0.23
Sum	61	48	50	

a) Percent agreement:

$$= \frac{8+15+11+4+12}{61} = 0.78$$

b) Reliability in content analysis

Percent agreement:

$$PA0 = \text{Total A's} = \frac{8+15+10+4+11}{50} = 0.96$$

$$\text{Scott's } pi = \frac{PA_0 - PA_E}{1 - PA_E}$$

Where $PA_E = \sum Pi^2$ and Pi = each joint marginal proportion

So $PA_E = (0.16)^2 + (0.31)^2 + (0.21)^2 + (0.08)^2 + (0.23)^2$

$= 0.026 + 0.096 + 0.044 + 0.006 + 0.052$

$= 0.224$

And, Scott's pi = $\frac{0.96 - 0.224}{1 - 0.224} = 0.95$

So, Reliability of the whole content analysis equals to 0.95 which is high

From the analysis of the data in the previous table:

Overall NCTM standards consist of five domains, the first domain contains (9) indicators while ATHS curriculum contains (8)outcomes, the second domain contains (15) indicators which is the same with ATHS curriculum, the third domain contains (13) indicators while ATHS curriculum contains (10) outcomes, the fourth domain contains (5) indicators while ATHS curriculum contains (4) outcomes, and the fifth domain contains(19) indicators while ATHS curriculum contains (11) outcomes and the total number of NCTM indicators are (61) indicators, on the other hand the total number of ATHS curriculum outcomes are (48).

From the calculated percentage of agreement (0.78) and it is considered to be high percentage, it shows that the overall content of ATHS mathematics curriculum match international standards of mathematics (NCTM).

Through the analysis of ATHS math curriculum, the researcher finds the following:

1- There is a part of ATHS math curriculum doesn't exist in the high school international standards (NCTM) which is the calculus.

2- Some of the NCTM indicators in the high school included in the engineering math not core math like vectors and matrices.

Thus ATHS math curriculum meets the international standards of mathematics content (NCTM standards)

Suggested procedure

To answer the second question in this study ***"Based on the result of the content analysis, what is the suggested procedure?"*** the researcher – based on his teaching Responsibilities at ATHS – did the following:

- Analyze grade 11 students' answers (question by question) in term one exam for the academic year 2010 – 2011

Marks	Question I (28)							Question II (26)					Question III (12)					Question IV (11)			Question V (23)				
	1 (4)	2 (4)	3 (4)	4 (4)	5 (4)	6 (4)	7 (4)	A (7)	B (7)	C (4)		D (8)	1 (1.5)	2 (1.5)	3 (1.5)	4 (1.5)	5 (6)	A (5)	B (6)		A (6)	B (3)	C (4)	D (5)	E (5)
										C1 (2)	C2 (2)								B1 (3)	B2 (3)					
0	26.3%	27.0%	17.0%	54.4%	54.8%	30.5%	47.9%	22.4%	49.4%	19.3%	36.7%	35.9%	16.2%	8.1%	18.9%	19.7%	12.0%	34.4%	56.4%	32.4%	37.5%	8.1%	38.6%	84.9%	34.7%
0.5								1.2%	1.2%	6.9%	18.5%	1.9%	1.2%	0.0%	1.5%	0.8%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.8%	0.8%
1								5.4%	6.2%	18.9%	13.9%	10.8%	0.4%	0.0%	0.8%	12.4%	4.2%	19.3%	30.1%	14.3%	5.4%	2.3%	8.1%	6.2%	36.3%
1.5								1.9%	1.9%	4.2%	14.3%	0.0%	81.5%	91.1%	78.0%	66.4%	2.7%	0.0%	1.2%	1.5%	0.4%	0.4%	0.0%	4.6%	0.0%
2								5.8%	3.1%	48.6%	14.7%	2.3%					10.0%	28.2%	9.3%	6.2%	9.7%	0.8%	22.0%	1.2%	2.3%
2.5								1.5%	1.2%			0.0%					0.0%	0.0%	0.0%	0.4%	0.4%	0.0%	0.4%	0.4%	0.0%
3								13.5%	6.6%			3.9%					8.1%	6.6%	1.9%	44.0%	4.6%	87.3%	10.0%	0.8%	1.2%
3.5								0.4%	0.4%			0.4%					0.8%	0.0%			0.8%		0.0%	0.0%	0.4%
4	73.0%	72.2%	82.2%	44.8%	44.4%	68.7%	51.4%	20.1%	1.9%			4.2%					5.8%	0.8%			16.6%		19.7%	0.0%	10.4%
4.5								0.0%	0.4%			0.0%					10.4%	1.2%			0.0%			0.0%	0.8%
5								0.4%	4.2%			1.9%					5.0%	8.5%			4.2%			0.0%	12.0%
5.5								0.0%	0.8%			0.0%					2.3%				0.4%				
6								2.3%	2.7%			2.3%					37.8%				18.9%				
6.5								0.8%	0.4%			0.4%													
7								23.2%	18.9%			4.2%													
7.5												11.2%													
8												19.7%													

	Average Question
	above 60% fail
	above 60% pass

By comparing these results with the exam paper I found that the main problem in the exam was with the word problems

Figure (2): Grade 11 Exam Analysis

By comparing these results with the exam paper I found that the main problem in the exam was with the word problem.

The required skills to solve word problem questions are as follow:

1. Mathematical reasoning
2. Mathematical Problem solving
3. Mathematical Communications

- Analyze the response of the same students' answers (question by question) in SAT reasoning Mock test for the academic year 2011 – 2012

Overall SAT Mock 2 Analysis																																							
DIFFICULTY LEVEL																																								
Level	Level 1							Level 2								Level 3								Level 4								Level 5									
Section	Sec 2				Sec 6			Sec 2					Sec 6				Sec 2				Sec 6				Sec 2				Sec 6					Sec 2				Sec 6		
Question No.	1	2	3	4	1	2	9	5	6	7	10	12	3	4	10	11	8	9	11	13	5	12	13	14	14	15	16	18	7	8	15	16	18	17	19	20	6	17		
Right	92	66	74	81	80	46	34	51	34	18	43	51	31	44	26	8.8	44	40	27	40	23	15	5.8	1.8	17	29	11	19	13	4.4	2.2	0	1.3	8.4	4	9.3	3.1	0.4		
Wrong	8	34	26	19	20	54	66	49	66	82	57	49	69	56	74	91	56	60	73	60	77	85	94	98	83	71	89	81	87	96	98	100	99	92	96	91	97	100		
	Average							Average									Average								Average									Average							
Right	67.51							34.12									24.69								10.72									5.04							
Wrong	32.49							65.88									75.31								89.28									94.96							

Figure (3): Overall SAT Mock 2 Analysis

Core Students SAT Mock 2 Analysis																																							
DIFFICULTY LEVEL																																							
Level	Level 1							Level 2								Level 3								Level 4								Level 5									
Section	Sec 2				Sec 6			Sec 2					Sec 6				Sec 2				Sec 6				Sec 2				Sec 6					Sec 2				Sec 6		
Question No.	1	2	3	4	1	2	9	5	6	7	10	12	3	4	10	11	8	9	11	13	5	12	13	14	14	15	16	18	7	8	15	16	18	17	19	20	6	17		
Right	90	59	68	74	76	34	33	43	26	10	43	42	20	30	23	3.8	43	33	19	31	17	11	3.8	1.9	16	26	8.8	18	16	3.8	1.3	0	0	6.9	5.6	6.3	2.5	0		
Wrong	10	41	33	26	24	66	67	57	74	90	58	58	80	70	77	96	57	67	81	69	83	89	96	98	84	74	91	83	84	96	99	100	100	93	94	94	98	100		
	Average							Average									Average								Average									Average							
Right	61.96							26.74									20.00								9.86									4.25							
Wrong	38.04							73.26									80.00								90.14									95.75							

Figure (4): Core Students SAT Mock 2 Analysis

ES Students SAT Mock 2 Analysis																																							
DIFFICULTY LEVEL																																							
Level	Level 1							Level 2								Level 3								Level 4								Level 5									
Section	Sec 2				Sec 6			Sec 2					Sec 6				Sec 2				Sec 6				Sec 2				Sec 6					Sec 2				Sec 6		
Question No.	1	2	3	4	1	2	9	5	6	7	10	12	3	4	10	11	8	9	11	13	5	12	13	14	14	15	16	18	7	8	15	16	18	17	19	20	6	17		
Right	97	85	89	97	89	73	36	71	53	36	45	73	58	77	33	21	47	58	47	65	36	24	11	1.5	21	36	15	21	6.1	6.1	4.5	0	4.5	12	0	17	4.5	1.5		
Wrong	3	15	11	3	11	27	64	29	47	64	55	27	42	23	67	79	53	42	53	35	64	76	89	98	79	64	85	79	94	94	95	100	95	88	100	83	95	98		
	Average							Average									Average								Average									Average							
Right	80.95							52.02									36.11								12.79									6.97							
Wrong	19.05							47.98									63.89								87.21									93.03							

Figure (5): ES Students SAT Mock 2 Analysis

SAT reasoning questions divided into 5 levels of difficulties based on the required level of reasoning skills.

According to the above analysis it is clear that the core students achieve only level one which required one direct step problem solving. On the other hand ES students achieve level one and level two by 48% which required two or more direct substitution.

Both core students and ES students didn't achieve levels 3, 4, and 5 which required logic thinking and reasoning skills.

The educational process consists of three dimensions the curriculum, the learner and the teacher. Since Applied Technology High School follows a specific system in the selection of students as enrolled students must pass the entrance exam, so the enrolled students have the minimum requirement to study at ATHS. And since the analysis of the curriculum content showed that the ATHS curriculum is compatible with the international standards, since the ATHS students' performance in SAT reasoning test – which is depends on mathematical process - is very low, and also the result of question by question analysis of term one grade 11 math exam on the academic year 2010- 2011 (created by the researcher) shows that about 83% of students did not solve the word problems which required the skills of communication, problem solving, and reasoning, so the reason behind the students' low performance is the teaching process (problem solving, mathematical reasoning, and mathematical communication).

Based on the above the researcher suggested an approach to develop the teacher's skills in teaching with mathematical process to enhance the students' achievement in the ATHS exams and international exams. Also the researcher suggested an observation tool to follow up the teachers' performance in the classroom.

(A guide for teacher to develop main process standards in high school)

Introduction:

According to NCTM document at 2000, the process standards are determined in four main standards, which are: (1) Mathematical Problem solving standard, (2) Mathematical Reasoning standards, (3) Mathematical communication standards and (4) Mathematical Connection standards.

This document refers to some aspects that must be taken in consideration by the teacher within the classroom environment for developing the mathematical processes, these aspects as follow:

Mathematical Problem solving:

Since it is the center of attention in the curriculum, so it must be also the basis for the teaching method, because we do not expect that student be able to solve mathematical problems unless we help him to learn how to think mathematically or do mathematics through problem-solving more than doing a series of actions to solve problems.

Mathematical Reasoning standards:

Students can inference in mathematics if they have the opportunity to do; it is through the classroom environment where the goal- which both the teacher and students seeks to achieve- is to know why, not just knowing how.

Mathematical Communication standards:

Classroom environment that encourages communication is a classroom environment where students speak and listen to each other, in addition read and write mathematical ideas in their own language.

In the following step, the researcher will present a simple guide an example of a professional development program to develop teacher's skills in practicing mathematical process through teaching the content in high school classrooms followed by suggested observation form to evaluate the teachers' skills in practicing these processes.

Through studying these standards and lit reviewing various knowledge about reasoning, it was found out that all the three standards can be mixed together and developed in the form of reasoning habits.

The Reasoning Habit:

It is simply a common way of thinking productively. It should be common in the process of sense making and reasoning when studying mathematics.

Below is a list of reasoning habits that points up the expected routine types of thinking that should show up in classrooms of mathematics.

This habit shows in the students' ability to:

- Data collection , observations , interpretation and formation of scientific conjectures
- Gathering evidences
- Building discussions to support or reject the mathematical ideas.

The Importance of reasoning skills

Students can inference in mathematics if they have the opportunity to do; it is through the classroom environment where the goal- which both the teacher and students seeks to achieve- is to know why, not just knowing how.

Reasoning and sense making of things are both skills that should be in every mathematics classroom and on daily basis. In Such classes you are much likely to come across questions like “What’s going on here?” and “Why do you think that?”

Some teachers argue that developing reasoning habits could be an extra burden, since they are already suffering with some students just to teach them the procedures. Using the structure that reasoning brings to the process of earning mathematics deeply supports the understanding and continued learning. Reasoning adds meaningfulness to the learning of math, a thing that due to the lack of it many students have found it

difficult to learn mathematics as for teachers the lack of it has taken them through an endless cycle of reteaching as well.

Therefore with the presence of good planning and attention from the teachers' side can very likely help make students responsible for their own learning and hold them accountable for engaging in reasoning and exploring reasoning themselves.

Hence and according to what we have explored, we can state that reasoning and sense making are the cornerstones of the subject of Mathematics and rebuilding the math curriculum to include both skills will allow the development the process knowledge and content required for a successful learning of mathematics, as well as the use of the subject of mathematics in their daily lives.

The reasoning habit components:

1. Analyzing a problem

- Identifying relevant mathematical concepts, procedures, or representations

Example 1:

If $f(x) = x^2 + 2$, which of the following

Could be a value of f(x)?

A) -2
B) -1
C) 0
D) 1
E) 2

Teacher: what does it mean $f(x) = x^2 + 2$

Student1: this is the definition of the function

Student2: this function represented by parabola

Student3: the value of the function usually positive

Teacher: Great, can the value of the function be negative?

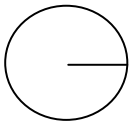
Student1: No, because x2 usually positive and it added to 2.

- Reveal important information about the problem and contribute to its solution

Example 2:

If the circle with center O has a diameter of 9, then what is the area of the circle with center O?

Teacher: this problem about circle so we have to sketch a circle
Student:



Teacher: Now what are the formulas related to the circle?
Student1: area of the circle = πr^2
Student2: circumference of the circle = $2\pi r$

- Defining relevant variables and conditions carefully, including units if appropriate;

Example 3:

Tickets for a play cost $6 each for adults and $3 each for children. If 160 of these tickets were bought for a total of $816. How many adult's tickets were bought?

Teacher: ask student to start read the problem and when he find (,) or (.) he must stop and ask himself what does this statement mean.
Student1: read the first statement "Tickets for a play cost $6 each for adults and $3 each for children"
It means we have two types of tickets for adult and children, and the cost for adult is $6 and for children is $3.

Student 2: read the second statement "160 of these tickets were bought for a total of $816"
(If the student can't explain the meaning) teacher will help him
Teacher: what is the total number of tickets?
Student 3: 160 tickets
Teacher: all tickets for adults?
Student 4: No, for both adults and children
Teacher: ask another student do we have the number of adults
Students3: no, so let number of adults = x, then the number of children= 160-x

- Seeking patterns and relationships

Example 4:

In the previous example 3

Teacher: now, what is the total price of tickets?

Student1: $816

Teacher: is this the price for adults' tickets only

Student 2: no, it is the sum of adults' and children's tickets

So, 6x+3(160-x) =816

- Looking for hidden structure (for example, drawing auxiliary lines in geometric figures or finding equivalent forms of expressions that reveal different aspects of a problem);

Example 5:

If 2x+5=7

It means we can find the value of x (so, if we have any mathematics expression, we have to simplify it directly)

- Making preliminary deductions and conjectures, including predicting what a solution to a problem might involve or putting constraints on solutions;

Example 6:
A construction company will be fined for each day it is late completing a bridge. The daily fine will be $4000 for the first day and will increase by $1000 each day. Based on its budget, the company can only afford $60,000 in total fines. What is the maximum number of days it can be late?
Teacher: will ask the students to write all possible formulas that may be used to solve the problem?

Student:

a) $a_n = a_1 + (n-1)d$

b) $s_n = \frac{n}{2}(a_1 + a_n)$

c) $s_n = \frac{n}{2}(2a_1 + (n-1)d)$

Teacher: What are the best formulas to solve the problem? Why?

Student:

The best formulas are the second and the third because the total sum is given so we must use the sum formulas

2. Implementing a strategy

- Making good use of procedure
- Ordering the solution, including calculations, algebraic manipulations, and data displays;
- Making logical deductions according to the current progress level, defining inferences, extending the primary findings and results, and monitoring the attempts done to find the correct solution, including going through the strategy used and other possible ones.

Example 7:

In the previous example (example 8)

Teacher: Ask students to try to us the formula $s_n = \frac{n}{2}(a_1 + a_n)$

Student:

In this step students will compare the givens with the required elements in the formula

The givens are: a_1 = 4000,d = 1000, S_n = 60000

But the required to use this formula are: S_n, a_1, a_n.

Since a_n is not given, so teacher will ask students to try to find it by using the formula $a_n = a_1 + (n-1)d$

Students will try to use this formula to find a_n, but they will discover that they can't use it because (n) is not given.

Teacher: Since the first formula does not work, so try the second formula $s_n = \frac{n}{2}(2a_1 + (n-1)d)$

Student:

In this step students will compare the givens with the required elements in the formula

The givens are: a_1 = 4000,d = 1000, S_n = 60000

Example 7 - Continued:

On the other hand the required elements to use this formula are: S_n, a_1, d.

Since (n) is the only unknown, so this formula can be used to solve the problem.

Teacher: ask the students to solve the problem

Student:

Givens: a_1= $ 4000, d = $ 1000, S_n = $ 60000

Since $s_n = \frac{n}{2}(2a_1 + (n-1)d)$

Then $60000 = \frac{n}{2}(2\times 4000 + (n-1)\times 1000)$

$60000 = \frac{n}{2}(8000 + 1000n - 1000)$

$60000 = \frac{n}{2}(7000 + 1000n)$

$60000 = 3500n + 500n^2$

$600 = 35n + 5n^2$

$120 = 7n + n^2$

$n^2 + 7n - 120 = 0$

(n – 8) (n + 15) = 0

n – 8 = 0

n= 8 days

Or n + 15 = 0, n = -15 (rejected)

3. Seeking and using connections across different mathematical domains, different contexts, and different representations.

4. Reflecting on a solution to a problem

- Infer a solution and concluding it, in other words, finding how it answers the problem.
- Take into consideration the level of accuracy of numbers included in the solution of the problem, in other words, mind the amount or reasonableness found in a solution.
- Review primary assumptions regarding the solution's nature while minding special cases.
- Validate a problem solution (proof or inferential reasoning)|
- Recognize inference range in the case of a statistical solution
- Consider other suggested approaches to solve a problem
- Refine arguments so as to make them effectively communicated
- Generalize a solution to a wider range of problems (find the common thing or the connection between them)

A practical example for teacher to develop the three mathematical processes together in high school:

EX: A construction company will be fined for each day it is late completing a bridge. The daily fine will be $4000 for the first day and will increase by $1000 each day. Based on its budget, the company can only afford $60,000 in total fines. What is the maximum number of days it can be late?

First: Analyze the problem: Teacher will do the following:

1. Who can read the problem? ***(Oral communication)***

2. Who can explain the problem in his own words? ***(Oral communication)***

3. Assign two students to discuss the givens and the required. ***(Oral communication)***

Students' responds:

The givens are:

a) The first daily fine $4000, it considered to be the first term of the series

b) The increase of the fine each day $ 1000, it considered to be the common difference of the series

c) The total amount of fines $ 60000, it considered to be the sum of arithmetic sequence.

The required:

The number of days it can be late or the number of terms (n)

4. Now who can tell me which type of sequences or series does this problem reflect? Why? ***(Oral communication & reasoning)***

Student's responds:

This problem reflects arithmetic series because:

a) Each time we add a constant amount, so it is arithmetic

b) The total amount is given, so it is series. So the problem reflects arithmetic series.

Second: Implementing Strategy: Teacher will do the following:

1. Asking the students to try to solve the problem without formula. ***(Induction reasoning)***

Student's responds:

Students will have different responds and the teacher will discuss each respond to find the solution if it available for this problem.

2. Now the teacher will ask the students to write all possible formulas that may be used to solve the problem? ***(Written communication)***

Student's responds:

a) $a_n = a_1 + (n-1)d$

b) $s_n = \frac{n}{2}(a_1 + a_n)$

c) $s_n = \frac{n}{2}(2a_1 + (n-1)d)$

3. What are the best formulas to solve the problem? Why? **(*Reasoning)***

Student's responds:

The best formulas are the second and the third because the total sum is given so we must use the sum formulas.

Third: Solving the problem: Teacher will do the following:

1. Asking the students to try to us the formula $s_n = \frac{n}{2}(a_1 + a_n)$

(Written communication& deduction reasoning)

Student's responds:

In this step students will compare the givens with the required elements in the formula

The givens are: $a_1 = 4000, d = 1000, S_n = 60000$

But the required to use this formula are: S_n, a_1, a_n.

Since a_n *is not given, so teacher will ask students to try to find it by using the formula* $a_n = a_1 + (n-1)d$

Students will try to use this formula to find a_n*, but they will discover that they can't use it because (n) is not given.*

2. Since the first formula does not work, so try the second formula $s_n = \frac{n}{2}(2a_1 + (n-1)d)$

(Written communication& deduction reasoning)

Student's responds:

In this step students will compare the givens with the required elements in the formula

The givens are: $a_1 = 4000, d = 1000, S_n = 60000$

On the other hand the required elements to use this formula are: S_n, a_1, d.

Since (n) is the only unknown, so this formula can be used to solve the problem.

3. The teacher will ask the students to solve the problem

Student's responds:

Givens: a_1= \$ 4000, d = \$ 1000, S_n = \$ 60000

Since $s_n = \frac{n}{2}(2a_1 + (n-1)d)$

Then $60000 = \frac{n}{2}(2\times 4000 + (n-1)\times 1000)$

$$60000 = \frac{n}{2}(8000 + 1000n - 1000)$$
$$60000 = \frac{n}{2}(7000 + 1000n)$$
$$60000 = 3500n + 500n^2$$
$$600 = 35n + 5n^2$$
$$120 = 7n + n^2$$

$$n^2 + 7n - 120 = 0$$

(n – 8) (n + 15) = 0

n – 8 = 0

n= 8 days

Or n + 15 = 0, n = -15 (rejected)

Fourth: Reflecting the solution steps: Teacher will do the following:

Asking students to look over their solution to check

1. Does it seem probable?
2. Does the mathematical operations are accurate?
3. Did you answer the question? Are you sure?
4. Did you answer using the language in the question? Same units?

Tips for developing Reasoning Habits in the Classroom

Teachers can help their students develop their higher level thinking skills through providing them with a well judged and chosen series of tasks as well as through using inquisitive questions.

Through such things students can learn to:

- Analyze their way of solving problems
- Pin point the weaknesses and the strength of their way
- Use more formal reasoning to formulate mathematical findings

The consistency in continuing to use the reasoning and sense making skills inside mathematics classes is a must ,and in order to have this type of consistency and develop such habits ,we have to come up with the following list of tips:

1. Give assignments that make students find out things on their own.
2. Make students restate the problem using their own words along with their assumptions.
3. Give time so that students can primarily analyze a problem using models at first ,then moving forward to choosing a suitable approach to solve it.
4. Do not tell students how to solve a problem, instead, support them through their thinking procedures
5. Give students questions to prompt their thinking
6. Give enough thinking time after asking a question
7. support to make their own probing questions and use them ask themselves
8. Encourage students to communicate their reasoning to both their classmates and teacher, orally and in writing, using math related terminology.
9. Bring to light ideal explanations and elicit what makes them effective.
10. Create a positive learning environment inside the class where students can have their productive mathematical arguments in a friendly atmosphere.

- Reasoning and sense making are the bases of a genuine foundation for true mathematical competence. Therefore they have to be a consistent part of every mathematics class and be a major part in the mathematics curriculums of high schools.

- Integrating isolated experiences with reasoning and sense making will not be sufficient unless teachers co-operate and stress the practices of such skills in each and every mathematics class as well as pushing the use of more sophisticated levels of reasoning

The Observation Tool

ثانوية التكنولوجيا التطبيقية
Applied Technology High School

Class Observation Tool (Based on Mathematical Process)										
Teacher's Name							Campus			
Cluster / Subject		Grade / Section					Date			
Room							Period			
Process	Criteria	Rating					Score	Comment		
		U	N	G	E					
Problem Solving	Teacher asking students to									
	Read the problem									
	Determine what are the givens and the required									
	Think about any formulas they may need to use.									
	Decide if the problem requires you to graph.									
	Define some strategies to solve this problem									
	Try out the different strategies									
	Solve the problem									
	Check the solution									
	Total Problem Solving									
Mathematical Reasoning	Teacher asking students to									
	Solve the problem mentally (without using any formula)									
	Choose the correct formula to solve the problem and why									
	Compare the givens with the formulas									
	Write beside any step the cause (why)									
	Explain wither the solution seems probable or not and why.									
	Total Mathematical Reasoning									
Mathematical Communication	Explain the problem in their own words									
	Discuss the problem with his peers									
	Write the required formulas									
	Discuss the relation between the formulas and both the given and required									
	Discuss solutions with whole class									
	Write the problem in their own words									
	Write similar problems in their own									
	Total Mathematical Communication									
Total										
U: Unsatisfactory = 1	N: Needs Improvement = 2	G: Good = 3						E: Excellent = 4		
Teacher's Signature		Observer Name & Signature					Principal's Signature			
Date:		Date:					Date:			

Important consideration while using the suggested observation tool

1. This form applicable while teachers solve practices in the classroom

2. This form used beside the original form used by the organization.

3. This form helps to figure out the required professional development for mathematics teachers.

4. Teachers' awareness for the content of this form helps them to enhance their performance within practicing these processes with the students.

5. Enhancing teacher's skills in practicing mathematical process will lead to develop students' mathematical power (means practicing all mathematical process together while solving any mathematical problem).

6. Teacher needs professional development if his score less than 75% (60 point)

Recommendations and related studies:

Recommendations

Based on the results of the current study, the following recommendations could be offered:

1. Take the NCTM indicators in to consideration while planning the core math curriculum to include the missing indicators in the data analysis and probability level.
2. Treat the mathematical process a central focus of mathematics in all levels of education in general, and high school in particular, and it requires the need to appear in the list of the objectives of teaching mathematics at various stages.
3. Focus on the teacher's skills in teaching of mathematical process during the selection of teachers to join the Applied Technology High School.
4. The above relates to the need for attention in professional development programs for teachers of mathematics are related to mathematical process:
    - Problem Solving
    - Mathematical Reasoning
    - Mathematical Communication
5. Follow up the continued growth of mathematical processes' levels for students in different stages at the Applied Technology High School.
6. Attention to mathematics teacher to focus on problem solving, mathematical reasoning, and mathematical communication for his students in particular, side by side with the interest of the output of the problem
7. Include the content of mathematics attitudes and problems help to link the content with the life experiences for students, which help to transfer the effect of learning, it means using of mathematical process.
8. Planning exploring activities contribute to increase the students' opportunities to develop the ability to explore relationships and geometric patterns.
9. The need for the teacher to arrange his students in cooperative groups (3-5 students) and ask them to solve problems, and then routes the groups to share these tasks, or to solve problems after making sure it is correct.

10. Planning activities that allow students the opportunity to create a variety of ways to solve, using the mathematical rules and without using the mathematical rules.
11. Changing the culture of learning environment for the involved in the educational process, so the focus is in teaching mathematics operations practiced by the students, and their ability to manage these operations properly, the data and the learning environment that helps to achieve this.

Related Studies:

The current study is an introduction to other future studies, to address other aspects, and be more comprehensive and deeper, and proposed future studies as a field of study, and the center of attention include the following

1. The effectiveness of using the suggested approach in improving the students' achievement in math in applied technology high school.
2. A proposed program to train teachers how to teach SAT reasoning test based on the Standards of SAT reasoning teat.
3. The effectiveness of using "shadow teaching strategy" in developing the ability to solve mathematical problems of applied technology high school's students.
4. The effectiveness of using "mathematical process" in developing the ability to solve mathematical problems of applied technology high school's students.
5. The effect of the interaction between the mathematical process, and some types of education (individual - the cooperative - group) in the development of mathematical thinking of applied technology high school's students.
6. The impact of the use of proposed teaching strategies for the development of mathematical process of applied technology high school's students.
7. A comparative study between the performance of students of both sexes in the Institutes of Applied Technology
8. A proposed program to train teachers how to teach units of the mathematics curriculum using mathematical processes for developing of some aspects of mathematical knowledge for students at the Applied Technology High School.
9. A proposed program to train teachers on how to build and design a variety of tools designed to measure the mathematical processes of students at the Applied Technology High School.

References

- National council of teachers of mathematics. (2000**). *principles and standards for school mathematics,*** Reston, VA.

- Robert,Arthur,D.(1980), "The Roberts checlist-Selecting And Evalution Social Materials", The Social studies ,vol 71 no(3).

- National Couneil for Social studies (1994), Expections of Execellence Curriculum standards for social studies. 3501 Newark ST, Washington, U.S.A.

- El-qa'oud Ibrahim.(1990). Proposed model for the development of curriculum for the first secondary geography class in Jordan, unpublished Ph.D. thesis, Ain Shams University, Egypt.

- De Roch, Edward F. (1981). An Administrators Guide for evaluating Programs and Personal. Ally and Bacon. Inc. Boston.

- Lorantawilt, (1998). Criteria for Evaluating Grade Level Content Objectives. Performance Assessment and Standards-Based Curricula: The Achievement Cycle.

- Ungurait, Michelle Davidson.(2001). Social studies curriculum standards, social studies consultant.

- Abdullah faouzi.(1991). Evaluation geography books prescribed in the secondary school in Jordan, from the viewpoint of teachers and students, Unpublished Master Thesis, University of Jordan, Amman, Jordan.

- Olson, M. & Berk, D. (2001). Two Mathematician's Perspectives on Standards: Interviews with Judith Roitman and Alfred Manasltw. School Science and Mathematics, 101(6), 305-309

- Abed and Al-Khatib,. (2002). Mathematics teachers beliefs about school mathematics curriculum standards and their relationship believes and attitudes

in teaching. A series of psychological studies and educational, Sultan Qaboos University, November, 72 - 122.

- NCTM. (2011). Standards Overview. Retrieved july 20, 2011, from nctm: http://www.nctm.org/standards/content.aspx?id=26798
- AdvancEd. (2007). *AdvancEd standards.*
- Kamal Abdel-Hamid. (2004). Technology education in the era of information and communication, Cairo.
- National Commission on Excellence in Education. (1983). A Nation at Risk: The imperative for Educational Reform. Report on the quality of education in America to you and to the American people
- Ministry of education. (2003). National Standards for Education in Egypt.
- Asha K. Jitendra, C. G. (2005). Adherence to Mathematics Professional standards and Instructional Design Criteria for Problem-Solving in Mathematics. Exceptional Children , 319+.
- Bosse, M. J., & Lee, T. D. (2010). The NCTM Process standards and the Five Es of Science: Connecting Math and Science. School Science and Mathematics , 262.
- Camp, D. G. (2007). Where Do standards come from? A Phenomenological Study of the Development of National Board Early Childhood/Generalist standards. Research in Childhood , 420.
- Graham, K. J. (2001). Principles and Standards for School Mathematics and Teacher Education: Preparing and Empowering Teachers. School Science and Mathematics .
- Huntley, M. A., & Davis, J. D. (2008). High –school Students' Approaches to Solving Algebra Problems That Are Posed Symbolically: Results from an Interview Study. School Science and Mathematics , 380.
- Lappan, G. (2001). Conclusion: Moving forward with the standards. School Science and Mathematics .

- Maccini, P., & Calvin, J. (2002). Perceptions and Application of NCTM standards by Special and General Education Teachers. Exceptional Children , 325+.

- Pickreign, J., & Capps, L. R. (2000). Alignment of Elementary Geometry Curriculum with Current standards. School Science and Mathematics , 243.

- Pyke, C. L., & Lynch, S. (2005). Mathematics and Science Teachers' Preparation for National Board of Professional Teaching standards Certification. School Science and Mathematics , 25+.

- Reys, B. (1999). Mathematics Curricula Based on Rigorous National Standards What, Why, and How? Phi Delta Kappan .

- Snead, L. C. (1998). Professional development for middle school mathematics teachers to help them respond to NCTM standards. Teacher Education .

- Thompson, C. J. (2009). Preparation, Practice, and Performance: an Empirical Examination of the Impact of standard-based Instruction on Secondary Students' Math and Science Achievement. Reseach in Education , 53+.