Investigating Grade Seven Students’ Critical Thinking Skills through Math Intervention

استقصاء مهارات التفكير النقدي باستخدام برنامج تدخل تدريبي لمادة الرياضيات لطلاب الصف السابع

By

Maha Mahmoud El Ayobi
Student ID number 2013101104

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Supervisor’s Name Dr. Sufian A. Forawi

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Abstract

Teaching critical thinking is crucial in preparing students to face the challenges beyond school. In an unpredictable future, it is the school's role to educate students to be lifelong learners with the skills to creatively encounter the demands of economic growth and the job market. This study explores the effect of enhancing higher order thinking intervention on students' performance in a critical thinking skills test (CTST) and if this effect has a variant result according to gender and level of achievement in math. The participants were 152 students in grade 7 in an American international private school in the UAE. A Critical Thinking Skills Test (CTST) was developed by the researcher using Higher Order Thinking (HOT) practices in the McGraw Hill algebra textbook and revised by experienced math teachers. A pre-test and post-test quasi-experimental design was followed to examine the hypothesis of the study. The data was analyzed by using descriptive and inferential statistics. The study shows that the intervention has a significant effect on above students' performance in CTST. It was also significant that the intervention has a variant effect on students's performance in CTST according to their level of achievement in math. However the correlation between the intervention and gender was not proven significant. Discussion is offered to associate the findings, about enhancing explicit critical thinking skills correlated with embedded CT skills in math for grade 7 using intervention, with the current literature. Further implications and recommendations were given to develop the teaching of critical thinking skills at K-12 schools.

Key Words: Critical thinking, critical thinking skills test, mathematics proficiencies, and Common Core Stare Standards.

ملخص

إن تعليم التفكير النقدي للطلاب هو عامل مؤثر في مرحلة ما بعد المدرسة. اننا نحيا في عالم لا يمكن التنبؤ بالتطورات التي تحدث فيه والتي يحملها لنا المستقبل, لذا كان للمدرسة دورا هاما في اعداد الطلاب ليصبحوا متعلمين دائمين خلال حياتهم وذلك من خلال تزويدهم بالمهارات اللازمة لمواجهة متطلبات التطور الاقتصادي وسوق العمل.

ان هذه الدراسة تهدف الى استكشاف نتائج تعليم برنامج تدخل يحوي التفكير النقدي ومراحل التفكير العليا على اداء الطلاب في امتحان (CTST) واذا ماكلن هذا التريب يؤثر بشكل متفاوت تبعا لجنس الطلاب او مستواهم التعليمي.

لقد بلغ عدد الطلاب الذين شاركوا في هذا البرنامج 152 طالب في الصف السابع في مدرسة دولية خاصة تتبع المنهاج الامريكي في دولة الامارات العربية المتحدة.

لقد تم اعداد امتحان التفكير النقدي (CTST) بواسطة الباحث بالاعتماد على تدريبات التفكير العليا الموجوده في كتاب الجبر" ماكجروهيل" وقد تم مراجعة هذا الامتحان بواسطة مدرسيي رياضيات متخصصين وذوي كفاءه.

أدى الطلاب امتحان (CTST) مرتان, واحده قبل اجراء تدريب التفكير النقدي ومرة أخرى بعد التدريب. تم تحليل النتائج بعد ذلك باستخدام مقاييس التحليل الوصفي والاستدلالي.

إن النتيجه الهامة التي اثبتت من خلال هذا البحث هو التاثير الايجابي للبرنامج التدريبي على مستوى الطلاب في إمتحان التفكير النقدي (CTST) وذلك حسب مستواهم التعليمي ولكن لم يثبت وجود ارتباط بين الجنس من جهة وتعليم التفكير النقدي من جهة أخرى.

لقد نوقشت نتائج هذا البحث مناقشة مستفيضة وتم ربط نتائجه بتدريس التفكير النقدي في مادة الرياضيات لطلاب الصف السابع من خلال برنامج محدد وكذلك تم الخروج بنتائج ومقترحات لتطوير تعليم التفكير النقدي في المدارس.

المصطلحات الرئيسية:

مراحل التفكير العليا, مهارات التفكير النقدي, كفاءات مادة الرياضيات, معايير الرياضيات الاساسية المشتركه.

Dedication

Continuing my postgraduate education and getting a master of education was my passion. This accomplishment would not have happened without:

My mom's inspiration for me to be dedicated and a hard worker.

My daughter Rand for her trust and support.

My sister for her partnership and care.

To all the people who loved me and supported me,

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Table of Content

Chapter 1: Introduction

The teaching of thinking is essential. Studies worldwide emphasize the importance of teaching higher-level skills like creativity, problem solving and being a lifelong learner, as this is crucial for economic growth and the job market (Wegerif & Kaufman 2015). Outside the economic debate, there is also increasing agreement that dispositions necessary for critical thinking, skills like tolerance, creativity and resilience, are also important in our changing globalized world (OECD 2014). The question now is what does 'teaching thinking' mean, or 'higher-order thinking', or more fundamentally, what does 'critical thinking' mean? The practice of teaching thinking is very wide and spans from the study of electrical brain stimulation (Snowball et al 2013) to the study of group drama (Anderson 2004). There is a difference between teaching cognitive skills and teaching thinking; in the latter we have to eliminate non-cognitive approaches like working on emotions, (Newton 2015) or self-image (Burden 2015). Teaching creativity was for a long time separated from teaching critical thinking. In contrast, nowadays teaching creative thinking, which is essential to all kinds of real world problems, is a key for teaching good thinking (Tan & Cropley 2015).

Resnick (1987 in Wegerif 2015) explained his view on teaching thinking, that although we can't explain exactly what it means to think well, we can recognize good thinking when we see it in teaching practices. These words are still valid for all teachers who want to teach beyond memorization and passing exams (Wegerif & Kaufman, 2015). Good teaching happens when teachers build on students' prior knowledge and good teachers employ the teaching time to personalize the content according to each student's interests and needs. The ultimate goal is to embed the critical thinking skills in a meaningful way into the class environment (Forawi 2016).

Creative thinking has been recognized as a main skill for navigating the progressively complex style of life in the 21st century (Partnership for 21st Century Skills 2011). Fast technological, economical and global changes have put more pressure on scholars on what and how to teach, since facts seem less stable (Bonk 2009). Dewey recognized more than a century ago that it is impossible to prepare children for the future since we can't know what the civilization will be like in twenty years from now (Dewey 1897). However, Dewey thought that we have to nurture children's capacities such as creativity, to navigate the uncertain future. Students' creative capacities should be developed to help students take charge of their learning (Greene 1995; Guilford 1950; Vygotsky 1967, 2004; Warnock 1978 in Wegrief 2015). Sawyer (2010) stated that students should go beyond the simple memorization of facts into working creatively to generate new ideas and new knowledge.

There are three main obstacles in teaching creative thinking in K-12 schools. The first challenge is that the lack of clarity about creativity has made it an intractable area of scholarly research and an equally challenging goal for implementing in the mainstream curriculum (Beghetto 2010, 2013). The second challenge is that inertia in the educational system has long been recognized as a main source of resistance to integrating creative thinking into the curriculum (Barron 1969 in Beghetto 2015). Teachers spend the majority of class time transferring factual bits of information, for students to copy and recite while sitting silently in rows. The third challenge is the narrowly focused curriculum (Berliner 2011).

Researchers have identified many approaches to teaching creative thinking through the context of academic subjects (Beghetto & Kaufman 2010). Some studies have focused on identifying instructional strategies that help students develop the ability of creative thinking in many academic subjects and cultural contexts (Niu & Zhou 2010). Many researchers have studied whether and to what extent training programs might enrich creative thinking. The most promising programs are those that focus on realistic and domain specific activities (Nicerson 1999; Scott et al. 2004). Educational reform researchers have emphasized the effectiveness of the experimental research that explores the influence of enhancing intervention (Slavin 2011; Thomas, Anderson & Nashon 2007).

Metacognition has a crucial role in teaching higher order thinking (HOT). This term refers to the cognitive abilities that are beyond memorizing and comprehending. According to Bloom's taxonomy and some of the revised models by Krathwohl (2002) and Leighton (2011), applying, analyzing, evaluating and creating are the main educational objectives at the HOT level. Some of the activities that are classified as HOT are constructing arguments, making comparisons and creating causal relationships (Zohar 2004). These cognitive activities can be used to carry out processes like problem solving, debate and critical thinking.

1.1 Statement of the Problem

One of the most important responsibilities of school is to prepare students to be successful in their higher education and critical thinking has become an important factor in how higher education is conceptualized in societies nowadays (Moore 2015). Beyond the academy, critical thinking skills are included in the lists of skills that employers currently expect of graduates (Hager &Holland 2006). In addition to critical thinking, problem solving and self-directed learning are emphasized in education to prepare students for higher education and a career (Donnelly 2010; Fisher 2011; Lai 2011). Also, critical thinking skills will encourage students to think independently and solve problems in school or in the context of everyday life (Jacob 2012; NCTM 2000).

The correlation between mathematics and critical thinking is positive; many studies have revealed that the development of critical thinking skills can improve mathematics achievement in general (Chukwuyenum 2013; Jacob 2012; NCTM 2000; Silver & Kenney1995; Semerci 2005) and vice versa: teaching mathematics can develop critical thinking skills (Aizikovitsh & Amit, 2010; Rajendran 2010). Critical thinking in mathematics is the process that relates the knowledge of mathematics, mathematical reasoning and mathematical proofs to mathematical problem solving (Krulik & Rudnick, 1995).

Every day, people must make decisions that require understanding, clarifying, interpreting, analyzing, comparing, reasoning and evaluating skills to make reliable and valid decisions. One of the most important issues is how to implement these skills into a real life context and adapt them to a variant environment. Critical thinking is a complex concept, which contains cognitive skills and affective dispositions, which is why it makes the way teachers convey the concept to students challenging. It is crucial for teachers to teach their students how to think critically; mathematics teachers should integrate critical thinking skills to the curriculum to enable students to apply the skills to improve their reasoning ability and math achievement in general. Teaching of critical thinking skills is best controlled not in distinct programs but within the context of each discipline (McPeck 1990). Also, Swartz (2001) and Butera et al. (2014) stated that integrating thinking skills to the context of subjects is very effective in teaching critical thinking skills.

Critical thinking should use logical reasoning to separate facts from opinions, assessing information critically with evidence before rejecting or accepting thoughts about the issues at hand. In another argument, it motivates students to think, question problems, create solutions to problems and take intellectual decisions when confronted with challenges (Semil 2006). Critical thinking also includes deep reasoning and a reflection of the information we receive instead of blindly accepting different ideas (Mansoor & Pezeshki 2012). Students cannot accept an idea if it does not go through the logical and systematic process to find the truth behind the idea. To implement critical thinking in school settings, it is necessary to develop thinking skills. Thus critical thinking students should be able to find the logical connections between ideas, construct and evaluate arguments, identify common mistakes in reasoning and solve problems systematically.

Facione (1990) identified six cognitive skills as central to the concept of critical thinking: interpretation, analysis, explanation, evaluation, self-regulation and inference. Critical thinking skills therefore, are skills that enable one to analyse and synthesize information to solve problems in a broad range of areas (Facione 1990). In ensuring sound arguments, facts will be collected, analyzed, and evaluated, and conclusions will be based on the available facts before them. By giving the students in our school extra time to learn math by constructing the knowledge, working in groups to find solutions to an inquiry and training them to use the critical thinking skills to justify their answers by reasoning, can we make a significant change towards enabling students to gain critical thinking skills, and to what extent can they master these skills?

Low ability among many school students is shown by the studies TIMSS, 2011 and PISA, 2012. The results have shown that many students in the UAE are not as equipped as their peers in other countries to apply math concepts; students could solve arithmetic problems but they are weak in solving questions involving critical thinking skills, like analyzing information, evaluating arguments, identifying common mistakes in reasoning and constructing problem solving solutions. Thus it is crucial to develop critical thinking skills for our students through a process of mathematics learning because mathematics has a structure and a strong and clear connectivity between its concepts; students who learn mathematics have the potential to be rational, logical thinkers (Aizikovitsh & Amit 2011; Rajendran 2010). Moreover, improving critical thinking skills in mathematics is closely related with problem solving, as well as open-ended problems and contextual problems (Henningsen & Stein 1997; NCTM 2000). Higher order-thinking mathematics problems that involve analysis and synthesis can stimulate students' critical thinking skills (Aizikovitsh & Cheng 2015; Krulik & Rudnick 1995).

1.2 Rationale of the Study

UAE' s Vision 2021 includes aspirations for a spirit of entrepreneurship, enhanced educational attainment, and a knowledge-based economy driven by innovation, research, science and technology. The realization of these aspirations requires a world-class education system, responsive to national needs and aligned to international standards (KHDA 2016).

The government of UAE depends heavily on education as one of the main keys that will enable the delivery of the 2021 Vision, and implementing the Smart Learning Program in more than 208 schools has been a very effective tool in technology to push the UAE's educational agenda forward. In today's competitive market, the UAE recognizes the importance of connecting the potential of the human capital and constructing a knowledge-based society in order to be competitive on the global stage. The recent education initiatives are another huge step forward for the UAE towards achieving an integral element of its Vision 2021, and developing a world-class educational infrastructure and workforce. The reform in the sector of education focuses on better planning, greater accountability, higher standards and enriched professionalism. In addition, the instructional approach is being replaced with more interactive approaches of learning (Interacts Emirates 2014). Two of the eight objectives that should lead UAE to being among the most successful countries, which provide world-class education, are to be among the 20 highest performing countries in PISA, and among the 15 highest performing countries in TIMSS. To monitor the progress towards achieving this goal, the School Inspection and Frame Work for KHDA (2015-2016), emphasis was placed on students' attainment of critical thinking, problem solving, reasoning, enquiry, interpretation and how to apply knowledge.

An ongoing argument among educators has been on what is the best method to teach critical thinking. The comparison of instructional approaches is between direct instruction and inquiry-based instruction. The focus of discussion is revolving around an either/or comparison, with an understanding that inquiry-based instruction indicates minimally guided learning activities, and direct instruction indicates guided learning (Kuhn2007). The most effective way to compare the two approaches is to explore the grounding assumptions of how learning takes place. The direct instructional approach adopts a top-down tactic, in which learning takes place by knowledge being clearly passed down from the teacher to students. To promote critical thinking skills with this method is to explain and teach independent cognitive skills and procedures that enable students to practice accordingly (Cordingley et al. 2005; Jones et al. 1985; McGuinness 1999). In comparison, the inquiry-based instructional approach adopts a bottom-up method that makes students construct their own understanding of a piece of knowledge. A critical mind is developed through having students involved in interactive investigations of intellectual activities, enabling them to discover and understand cognitive rules for themselves (Brown 1997; Ruggiero 1988; Sigel 1984). Learning takes place through the continuous process of the construction and reconstruction of the learner's own representations of phenomena of interest (Tynjala 1998; Vygotsky 1978) through activities that are directed toward question interpretation, interactive discussion, and reflection. This leads to the investigations of authentic problem-solving in a collaborative learning classroom environment.

There is evidence in some studies of the acquisition of critical thinking skills through direct instruction, in which students used informal reasoning rules in solving real-life problems more frequently and in a sophisticated manner. Also, they made less-biased responses in argument evaluations after receiving explicit reasoning skills' training (Kosonen & Winne, 1995; Neilsen et al. 2009).

Evidence also supports an inquiry-based instructional approach; students who participated in an intervention program that used an inquiry-based teaching approach had employed more critical thinking skills to solve real-life problems than their peers in the traditional approach (Ernst & Monroe 2004; Semerci 2006).

Inquiry-based instruction and direct instruction should not be viewed as absolute opposites (Alfieri et al. 2011). There is a possibility to examine the potential of mixed teaching methods, as an alternative to a closed debate on whether one approach is superior to another.

1.3 Significance of the Study

There has been various research that looked into critical thinking in different perspectives such as, attempting to find a definition of critical thinking (Abrami 2008; Facione 2011), studying the importance of critical thinking skills and its implementation in schools (Paul and Elder 2009; Ennis 2010) and lastly, focusing on critical thinking skills as a habit of mind among school students (Cuco & Goldenberg 1996; Dewey 2009). In the local context, there is a study that explores the focus on critical thinking in public schools in UAE (Thabet 2008). Critical thinking was explored at the British University in Dubai (BUiD); one study focused on the importance of critical thinking in private schools with reference to Abu Dhabi, UAE (Saad 2015) and another one studied the effects of the school curriculum on students' critical thinking skills (Boucif 2014). The current study tries to bridge the gap between the critical thinking skills theories and how to use them in a math context by analyzing the effects of implementing a critical thinking program embedded in the common core practices in math, on students' critical thinking skills in a private school in Sharjah, UAE. This study has used empirical research to examine the effects of enhancing critical thinking skills on students' performance by comparing the results of a math pre-test and post-test.

1.4 Study Purpose and Questions

The purpose of this study is to examine the effects of enhancing the high-frequency thinking skills that students need to master, which are how to model, analyze, evaluate, construct and reason in a richly relevant math content for grade seven. Using the seven keys of proficiency of the Common Core State Standards (CCSS), students are trained to analyze data, evaluate the relevant information, come up with a hypothesis, interpret the solution and reason it, connect the mathematical skills to a real world context, communicate with their peers to solve problems and reason them, check their solutions and determine if the answers 'make sense'. By engaging students in an after-school intervention which is designed to enhance these skills, and comparing their results to their peers in the school who do not have the chance to participate in the intervention, we can recognize if students benefit from such a program. Moreover, to what extent they can implement these skills into math content.

1. 5 Research Questions

The following study is undertaken to address the following main questions:

1. To what extent can a group of grade 7 students, who receive intervention of critical thinking, perform in a higher order thinking of Common Core State Standard Test?

2. Do gender and level of achievement in math have a moderating effects on enhancing the critical thinking skills of students?

The context of the study is a K-12 American curriculum private school in Sharjah, UAE. The participants are 152 students in grade 7, girls and boys from the same cultural and socio-economic background with ages ranging from 13 to 15. The study was conducted after school for eight weeks, two sessions per week for girls and two sessions for boys, the reseaher had to monitor the intervention four days weekly.

1.6 Structure of the Dissertation

Chapter One has presented the topic, objectives, main questions and components of the research itself. The literature review presented in Chapter Two highlights the following major topics: the theoretical review of critical thinking skills, the recent literature, and studies on how to teach critical thinking. This is followed by how this research was translated into educational practice. Chapter Three outlines the research methodology that describes the current study's context, sampling, approach, design, instrumentation and ethical considerations. Chapter Four outlines the data analysis and results. Finally, Chapter Five presents the discussion, conclusion, implications and recommendations.

Chapter 2: Literature Review

Thinking is one of the most basic concepts that has attracted philosophers and researchers since the beginning of humans' existence on earth. This is why it is not easy to recognize when and who started the study. This chapter presents the theoritical framework and pertraining literature review of the study.

2.1 Development of the definition of critical thinking

The roots of critical thinking as an educational approach can be extended to the Ancient Greeks and the teachings of Socrates (Zascavage et al. 2007). Socrates' thought was unlike the Athenians: he knew that he knew nothing. By asking questions he showed that most Athenians thought they knew something, but when asked to clarify the reasons they simply could not (Jones 1970). Many researchers today have shown that students, like the Athenians, cannot explain or rationally justify beliefs (Norris 1985). Aristotle defined thinking as an essential tool which helps us reach the truth, and he thought that thinking is logic combined with reason. Socrates thought that students should be trained to enhance their reasoning skills and eventually gain more rational thinking that is maintained with logic (Copeland 2005).
Scriven and Paul (1987) argue that the concept of critical thinking has been developing for more than 2,500 years and that the term 'critical thinking' first originated in the mid-1900s. John Dewey (1933) was one of the first researchers to distinguish the different levels of thinking and according to Dewey higher level thinking depends on searching as reflective thinking and judging as critical thought. Reflective thinking or inquiry is crucial to both teachers' and students' learning. In the past few years many commissions, boards, and school districts have urged students and teachers to strive towards the acquisition of this standard.

Although the call for achievement in systematic, reflective thinking is strong, it is even difficult to distinguish what systematic, reflective thinking is and what are the effects of reflective teacher education on teachers' practice and students' learning. Going back to the definition of reflection in the work of John Dewey, four distinct criteria characterized Dewey's view, that reflection might be taught, learned, assessed, discussed, and researched, and these offer a starting place for talking about reflection. Dewey claimed that, 'no matter how much an individual knows as a matter of hearsay and information, if he has not attitudes and habits of reflective thinking, he is not intellectually educated' (Dewey, p. 28). One of the main purposes for education according to Dewey (1910, 1991) was to create conditions that nurture the skills of reflective or critical thinking.

Glaser (1941) expands the critical thinking concept and calls for persistent effort to assess beliefs or any supposed form of knowledge, including evidence to support conclusions or to draw inferences. Glaser also includes logical deductions, sequences and order in his explanation of critical thought (Glaser 1941). Just like Glaser, Russell stresses the need for logical judgment of data that is far from emotional bias and fantasies (Russell 1960). In 1962, Ennis also defined critical thinking as logical thinking, but characterized by complex cognitive skills; in 1980 he involved the influence of creative thinking and predispositions to the definition (1991, 1993). He added the correct assessing of statements to critical thinking through reasonable and reflective thinking (Bonk & Maholmes 1996). Ennis included judgment of the credibility of sources, to construct a hypothesis, develop an argument with reasoning, draw conclusion and defend one's point of view to the critical way of thinking. As we discussed before, he defines critical thinking as 'reasonable reflective thinking focused on deciding what to believe or do'. According to this definition, he develops a critical thinking taxonomy that relates to skills that include both an intellectual aspect and behavioral aspect. Moreover, Ennis's taxonomy includes skills, dispositions and abilities (1989).

Dispositions towards critical thinking include a defined search for a thesis, questions and explanations, being adequately informed, using trustworthy sources, adapting it to the situation under study, relating it to the main issue, searching alternatives, putting other point of views into consideration, checking the judgment, striving for accuracy, dealing with the issues in an orderly fashion, and sensitivity. Abilities in critical thinking focus on raising questions, analyzing arguments, evaluating the source's reliability, value judgments, assumption identification, and interacting with others. Ennis argues that critical thinking is a reflective (by critically thinking, one’s own thinking activity is examined) and practical activity leading to a moderate action or belief. According to Ennis, there are five key concepts and characteristics defining critical thinking: practical, reflective, moderate, belief and action.

Paul (1993) criticizes the absorption of society’s prejudices and beliefs and claims that strong-sense critical thinking can be demonstrated when thinkers reflect and integrate the insights discovered to construct their beliefs. Adding emotional and/or moral strategies to the cognitive skills means logical and creative thinking; thinkers should cultivate moral traits such as courage, humility, empathy and integrity. By comparing McPeck’s (1991, 1994) identification of critical thinking with others, he stresses the ability and the tendency to engage in an active and reflective skepticism. The correct use of skepticism serves to find the true reasons on which various beliefs are based; these reasons are associated with the epistemology of each discipline, and he argued that critical thinking should be judged within the framework of each discipline.

According to Siegel (1998, 2003), the concept of critical thinking included a critical spirit along with reason; critical thinking was established through personality, dispositions, habits of the mind, and traits of the character. ‘A critical thinker is a person who acts, takes a stand, works out judgments based on reasons, and who understands and adapts to the principles that govern the evaluation of these reasons’ (Siegel 1988, p.38). Siegel views the direct connection between critical thinking and irrationality; consequently critical thinking should aquire the reasons behind judgements, evaluations and actions (Cuypers & Ishtiyaque 2006).

Critical thinking is a tool for facing imprudent actions and thoughts (Lipman 1988, 1995). He thinks that critical thinking skills develop according to four categories: conceptualization, reasoning, generalization and research. This definition has three fundamental criteria: the use of particular criteria to assess statements; self-correction to engage in an active search for self-improvement, and sensitivity to context when applying rules - rules should adapt with different contexts. Lipman (1988, 1995) emphasizes interaction with peers as an essential condition for critical thinking to occur.

Definitions of critical thinking have developed over thousands of years, from asking questions and looking for their answers through a reasoning process to a pragmatic definition that identifies critical thinking as a complex process that is integrated into a practical design to improve personal and social experience; it is a process versus a product, establishes through peer interactions within a community of inquiry and during this process the foundation of our beliefs should be changed and our behavior should be influenced.

2.1.1 Nature of Critical Thinking

One of the main problems in the critical thinking field is that researchers had not agreed upon a common definition of critical thinking. Facione (1990) used the research methodology known as the Delphi Method to come up with an agreed-upon definition of critical thinking and cognitive skills.

The Delphi Method required the creation of a panel of experts for interactive discussion to work toward achieving common opinion. This group of experts had worked for ten months and participated in six rounds of questions. The Delphi panel constructed a consensus definition of critical thinking and an articulation of the cognitive skills elaborated in critical thinking. Abrami et al.'s (2008) meta-analysis used the Facione Delphi panel definition as the most accepted definition existing in the literature over the past 20 years. Forty-six experts in the fields of psychology, philosophy and education participated in the construction of an agreed-upon definition of critical thinking.

When we talk about critical thinking there are two big branches, the cognitive skills and the intellectual skills. Facione's (2011) study identifies the cognitive skills that include interpretation, analysis, evaluation, inference, and explanation. At the heart of these skills is purposeful reflective judgment. The Delphi panel found a significant agreement on the cognitive skill dimension of critical thinking and that was not far from Facione's identification; he added only self-regulation. Good critical thinking includes the following six cognitive skills: interpretation, analysis, evaluation, inference, explanation, and self-regulation. The six cognitive skills of critical thinking are further broken down into 16 sub-skills (Facione1990), while Kuhn (2005) states that there are two branches of intellectual skills, inquiry and argument, and both work as the basis for developing knowledge. For students to be independent learners they should find answers to their questions by identifying issues, collecting data, looking for evidence, constructing judgments and solving problems. Students should not only inquire their own hypothesis, but also understand others' claims and that collaborative reasoning is ensured while working in groups and with short time periods. Facione not only explained the cognitive skills of critical thinking, but he thought that individuals should have dispositions or attitudes that provide the incentive, prudence, and the motivation to apply them. This vision is related to those individuals wanting to foster critical thinking, since these attitudes can provide areas to target for investigation, assessment, and teaching. Paul (2007) identified intellectual standards, elements of reasoning, and dimensions of critical thought. Employing intellectual standards such as clarity, accuracy and relevance to elements of reasoning such as goals, problems, facts, and assumptions can mature individuals' intellectual traits such as fair-mindedness, intellectual perseverance and confidence in reasoning (Paul and Elder, 2007).

As we discussed earlier the definition of critical thinking according to Ennis (2010) is 'reasonable reflective thinking focused on deciding what to believe or do'. This statement captures the core of the way the term is generally used by researchers of critical thinking.

Students should raise questions and look for sufficient explanations for them using reliable sources, look for alternatives by seriously considering other peoples' points of view and through discussion they should take a stand; strive for accuracy. Critical thinking focuses on raising questiions, analyzing arguments, making value judgments, taking actions, and interacting with others. Ennis claims that critical thinking is a reflective: by critically thinking, one's own thinking activity is examined and is a practical activity aiming for a moderate action or belief.

While Paul and Elder (2009) identified critical thought and reflective thinking, critical thinking is a tool in analyzing and assessing thinking and emphasizing the importance of reflective thinking. Paul and Elder also recognized the importance of critical thinking and stated that many people believe that it is the quality of thinking that determines the quality of lives.

Although many people in the education field know the importance of teaching critical thinking skills, they find it hard to articulate a well-defined conception of it and to demonstrate how teachers can foster these skills. (Gardner 1995; Paul, Elder & Bartell1997).

It is essential to teach critical thinking skills through focused instructions and clear standards of critical thinking skills competency (Paul & Elder 2007); these competency skills will serve as a resource for curriculum designers, adminstrators and teachers. For many students to learn critical thinking skills, they should foster the competency skills in each subject or in most of them, but students are not expected to obtain these skills in one or a few semesters. Many students can aquire the basic critical thinking competencies and that's why these competencies should be taught throughout the curriculcum, while other competencies need different ways. For a teacher to create a class of thinkers, educators should be broadly reflective, systematic, dedicated and visionary (Elder & Paul 2010).

Elder and Paul articulated the critical thinking competency standards that enable educators to decide to what extent students can reason critically through any discipline. The standards include outcome measures that help in assessing students and identifying the extents to which they can use critical thinking as a main tool for learning (Elder and Paul 2010). By applying these competency skills students are expected to become 'more self-directed, self-disciplined, self-monitored thinkers'. It is important that teachers have a solid base in teaching problem solving to enhance the problem solving skills for students. Elder and Paul (2010) realized the importance of teachers having the foundations of critical thinking so they can teach it effectively. ‘The simple truth is that teachers are able to foster critical thinking only to the extent that they themselves think critically’ (Elder and Paul 2010).

This is one of the main barriers to student performance in critical thinking competencies; teachers should demonstrate reflective deep thinking, intellectual humility and multi-logical worldview (Elder & Paul 2010). Many private schools in the region try to hire teachers with low salaries to minimize the expenses. In the absence of highly qualified teachers and the lack of professional development workshops, it is really hard if not impossible to enhance the critical thinking skills in these schools.

2.2 From Dewey to Partnership 21stCentury Skills Framework

One of the 21st century skills as outlined in Partner 21’s Framework for 21st Century Learning is using information, communication and technology literacy,however from the beginning of the last century, Dewey continually argued that learning is a social and interactive process, and consequently schools are social institutions through which social reform should take place. He believed that students thrive in an interactive environment where they are allowed to experience and have the opportunity to take part in their own learning. Means of communication in Dewey’s time was very limited and that’s why school as a place should play a main role in the reform process. What we can see in the 21st century skills is the unlimited ways of communication through a huge network of electronic interactions that means the process of reform takes place in an unlimited area (Partnership for 21st Century Skills, 2011).

Dewey emphasized the importance of education not only as a place where students gain content knowledge, but also as a place to learn how to live. What Dewey wrote has been expanded upon by the Partnership for 21st Century Skills that talks about today’s life and work environments that demand far more than content knowledge and thinking skills. The ability to steer the complex life and work environments in the globally competitive information age requires students to develop more life and career skills such as flexibility, adaptability, ability to manage time, set and reach goals despite hindrances.

Dewey argued that the aim of education should not revolve around the possession of a pre-determined set of skills, but rather the realization of one's full potential and the ability to use those skills to best meet our goals (Dewey 1897 in Beghetto 2015).

Dewey's ideas supported learning through active inquiry. He not only described the way that the learning process should take place, but also the role that the teacher should play within that process. The works of John Dewey criticized the American public education system that uses a limited set of instructional skills and fast training programs for teachers to meet the demands of the workforce.

If we compare this with private schools in UAE we find that the main obstacle in improving the way we teach math in private schools is the lack of trained teachers; although the schools have the newest books, they do not have skillful teachers capable of implementing this curriculum and adapting it to the local context in UAE. In some cases, math teachers have a bachelor degree in engineering, computer programming, accounting or Bachelor of Science but few have an education degree in math, and this undoubtedly affects the way math is taught in schools.

In addition to the previous skills that Partner 21st century stress are communication and collaboration skills. Students should articulate mathematical thoughts and ideas using oral and written communication skills; they have to work either with their peers, effectively reasoning problems and rephrasing another student’s explanation, or work respectfully in diverse teams articulating mathematical thoughts and ideas (Partnership for 21st Century Skills 2011). Dewey argues that it is the school’s responsibility to nurture students to be autonomous, reflective and ethical which qualifies them to participate effectively in society as active citizens, instead of nurturing passive students by emphasizing mastery of facts, and stressing compliance with authority (Dewey 1976, 1980).

It is the teacher's responsibility to explore the potential of each student in a subject and maximize the task that students should complete in order to improve their skills and meet their potential. School should work as a training place for students to gain all the skills that enable them to compete in the world of jobs later; through interactive relationships students can empower their communication skills which qualify them to listen to each other, understand others' needs, respect their opinions and ultimately come up with a common solution for a problem; skills we should strive for our students to gain to face the challenges in the region. It is the school's responsibility to start finding a common base among different students' cultures to enable them to live together in future, as students who learn together, learn how to live together.

Critical thinking and communication are related in many significant ways. On a basic level, the ability to think critically, develop a coherent argument and logical reasoning are important for daily communication. People who have the capability to think critically about an issue can see it from different viewpoints and will be better communicators, and less likely to respond quickly in anger. On another level, critical thinkers can evaluate the way in which other people are thinking and making their arguments before they make a decision. Problems with communication occur when there is an inability to think critically about a situation, and see it from other perspectives. Communication and critical thinking are connected in this way because people who have the ability to problem-solve and consider different perspectives tend to be better communicators.

2.3 Content Instruction Versus Skills Instruction

Research of critical thinking has waxed and waned. In the 1980s, a huge debate grew between the emphases of content instruction of critical thinking versus a general skill instruction of critical thinking (Ennis 1989, 1990). Proponents of a general nature of critical thinking argued that the skills of critical thinking could be learned as a set of dispositions and abilities distinctly from the content of a subject (Abrami et al. 2008). Generalists showed that fallacies of reasoning, for example, can be taught across varying contexts and the failures in achieving correct reasoning depend on the argument design, not on the content (Siegel 1988). If critical thinking is a general skill, then it should be taught as a separate course specializing in critical thinking skills (Royalty 1991; Sá, Stanovich & West 1999).

The content-specific position argues against general critical thinking skills or capacities on logical grounds. It is irrational to think without thinking of a subject specific to a domain (McPeck 1981). In his vision, critical thinking 'in general' is an unconceivable concept, as people always think, but to determine the quality of this thinking one should relate it to the criteria of each specific discipline.

This position maintains that critical thinking is dependent upon background knowledge and varies from domain to domain because the nature of knowledge claims varies across domains as well (Glaser 1984). The content-specific position suggests that the methods of teaching logic or other such general skills are unlikely to transfer, and the best way to teach critical thinking is through an immersion into complex content within a domain of knowledge. Opponents of content-specific argue that if critical thinking skills cannot be taught without a subject, then these skills should be taught exclusively with that subject (Groarke & Tindale 1986; Paul 1985; Siegel 1988).

Swartz (1992) extended the debate between the two main approaches to fostering critical thinking, the general skills approach that considered designing special courses for instructing critical thinking skills, and the infusion approach that considered implementing these skills through teaching the set learning material. According to Swartz, the infusion approach plans to provide specific instruction of critical thinking skills within the domain of different subjects, and therefore there is always a need to process the set material so as to combine it with thinking skills.

It is really hard to imagine that one can teach critical thinking skills without relating them to a specific subject; critical thinking skills should be related to the content and practices of different subjects.

Critical thinking encompasses important skills such as planning, reasoning, constructing judgments and critiquing. These skills enable students to understand and evaluate sources of information and are dominant in students constructing new ideas and knowledge. Developing critical thinking skills covers questioning and reflecting on information based on its accuracy and arguments. As digital technologies facilitate access to a growing amount of information, supporting students to think critically is crucial to their ability to make sense of the world around them and participate effectively in making it a better place to live in.

On one hand, it is superficial education to teach math without implementing critical thinking skills extensively to the content and practices of math. On the other hand, it is easier to think critically within a domain means that we learn to think critically through domain-knowledge. Relating critical thinking to math will shape proficient students in math who can think critically and find solutions for problems and reasoning for those solutions.

2.4 Cognitive Development of Problem Solving

Problem solving is a method used widely in teaching math today. The relationship between problem solving as a component of critical thinking has been defined; critical thinking skills can be identified during problem solving, decision making, and strategizing (Kuhn & Dean 2004).

In agreement with Paul and Elder (2010), Kuhn (2005) argued that individuals develop values, or a disposition, towards learning; they employ their intellectual skills to value the learning process. Glaser (1985) defined critical thinking as employing knowledge and skills in the application of thinking strategies, attached to the disposition or the attitude toward the thoughtful reflection of problems. Facione (2011) valued cognitive skills and dispositions but with more specific descriptions; he included interpretation, analysis, evaluation, inference, and explanation to the definition of cognitive skills. He emphasized the skill of reflective judgment. However, Facione (2011) affirmed the importance of critical thinking over the cognitive skills. Individuals must have attitudes or dispositions that require the incentive and the willingness to apply them. This vision is related to the individuals' considering fostering critical thinking in different areas to target for investigation, teaching and assessment.

Students should read the given information carefully and analyze it to decide which data is relevant, and use their prior knowledge to try to come up with strategies to solve the problem. The solving process forces students to recognize their existing ideas and to produce new ones, with the help of the teacher who acts as an agent and relates the existing knowledge of students to construct new connections. The result is an enrichment of the network of ideas through understanding.

2.5 Critical Thinking and Learning

Educators generally agree that critical thinking capabilities are crucial to one's success in modern life, where making rational decisions is increasingly becoming an important part of daily life. Students should learn how to ask questions, test reliability, investigate situations, study alternatives and check solutions both in school and in daily life.

To connect learning to critical thinking we should use human thinking. 'If we think well while learning, we learn well. If we think poorly while learning, we learn poorly' (Elder & Paul 2010). It is essential that to learn the content of a discipline, a student must learn how to think within that discipline. Thus to learn mathematics, one has to learn to think mathematically; in studying a subject there are foundational concepts that define the core of that subject, and students need to internalize an understanding of the basic concepts of it. To do this they have to express their understanding with their own words and relate them to real-life contexts by giving examples. Shifting from a memorization approach to a critical thinking approach enables students to have knowledge understanding for the body content of the subject and to evaluate and reflect on the content. Critical thinking delivers tools for internalizing content within one's mind to use it later in reasoning and for assessing the quality of that internalization (Elder & Paul 2010).

Critical thinking ensures that educated people think in a different way, they think fair mindedly which helps educators not only to teach mastery of a subject's content and interrelate it with other concepts in the subject or integrate it with other subjects, but also to help educated people to be active citizens who can reason ethically, have alternative ways of thinking and work for the public good.

Harold Fawcett (1938) initiated the idea that students should learn mathematics through experiences of critical thinking. He listed the following ways through which students could demonstrate that they were thinking critically as they engaged in the practices of the classroom: by looking for evidence to support the conclusions they are required to prove, analyzing evidences and differentiating fact from assumption, evaluating these assumptions by accepting what is true and rejecting what is false, and regularly examining the assumptions behind their beliefs.

After another fifty years of research, critical thinking was related to more general notions of what and how to teach mathematics. Students would be able to combine mathematical thinking through communication, articulate mathematical thinking by collaborating with peers, analyze data and evaluate other students' solutions, and use mathematical language to express mathematical ideas precisely (Partnership for 21st Century Learning 2007).

The main addition in this notion is the stress on communication, particularly the importance of peer interaction and training students to represent math ideas though the development of math literacy. Critical thinking should be defined in the context of mathematics education. Students not only have to find an answer to a problem but also communicate their own ideas with other students and collaborate towards finding a solution. Critical thinking skills should be promoted through the group work activities students practice in their classrooms. Cooperative learning is a trigger of critical thinking. Many researchers connect cooperative learning to the development of critical thinking. Critical thinking includes the ability to respond constructively to other members in the group, which implies interaction among the group members by respecting the contribution of each student and enhancing the credibility of claims' reasoning (Bailin et al. 1999; Bonk & Smith 1998; Heyman 2008; Nelson 1994; Paul 1992; Thayer-Bacon 2000). Also Glasser (2009) emphasized the importance of working in pairs, as peers' education leads students to memorize more than 95% of what they have learned. Slavin (1989) concludes that cooperative learning is effective in increasing students' achievement only if group goals and individual accountability are incorporated. Further, the properties of groups impact the experience of students in the groups (Espelage, Holt & Henkel 2003).

One goal is for students to become independent learners and able to find answers to their own questions and foster the development of inquiry skills; they are asked probing questions to promote reflection. Argument can be thought of as a form of inquiry because individuals seek to justify their own claims and at the same time, to question and understand the claims of others. More progress is made in a shorter period of time compared to thinking alone, demonstrating the advantage of collaborative reasoning and problem solving (Kuhn 2005).

Critical thinking skills can be developed in a math classroom and should provide practical experience; mathematical critical thinking skills should work as a bridge to the real world of jobs and adult responsibilities. This requires the math teacher to go beyond memorization into a world of reasoning and problem solving. Students should interact with each other, as well as work independently, just as adults do, using textbooks as one of many resources, and know how and when to use technology as a problem-solving tool, apply math to real-life problems and not just practice a collection of isolated skills. Students should seek a best solution among many solutions to a problem, explain the different ways they reach these solutions and defend the choice of one over another. Also, students should communicate mathematical ideas to one another through examples, demonstrations, models, drawings, and logical arguments. By working in teams, students challenge and defend possible solutions and help each other to develop the critical thinking skills.

2.6 Teaching Thinking Skills within the Common Core

There is a need to unpack the Common Core State Standards (CCSS) in a balanced way, using rich content and rigorous thinking processes. The CCSS thread the skills of literacy and reading, writing, speaking and listening through narrative and informative text (CCSS initiative 2016). The key to implementing the CCSS with relevancy is to address them with explicit teaching of the higher order thinking skills that are embedded in rich subject matter content (Cheuk 2012).

The focus in CCSS is on the 'high frequency' thinking skills that students need to master. These skills teach students how to process, analyze, evaluate, produce and present with rich content and relevant thinking. Further, they teach students how to process, analyze, evaluate, produce and present with rich content and relevant thinking. There is the Syllabus of Seven Thinking Skill Sets and 21 Explicit Thinking Skills that thread across all content areas for student proficiency, explained in Appendix 3.

Students thinking critically in mathematics means they make judgments or reasoned decisions about an inquiry or a problem; students should support their decisions and not simply guess or apply a formula without evaluating its relevance. It is the teachers' role to guide their students to choose the best strategy and defend this particular strategy among various strategies, to solve an assigned problem. The time teachers invest in improving critical thinking skills pays off, since students 'learn to think and think to learn'. Students who are critical thinkers in mathematics develop deeper understanding and engagement. Common sense and research both prove that, no matter how hard teachers try, they cannot think or understand for their students. Teachers can, however, create the conditions that encourage students to spark their brains and motivate them to actively engage in learning mathematics through critical problem solving. Also students will have more independence and self-regulation. By supporting students to develop a repertoire of thinking skills that enable them to learn independently we can raise their confidence in thinking for themselves and assessing their own learning, which will enable them to have greater competence with mathematical practices. Current standards in math education have a great emphasis on problem solving, modeling, reasoning, representing, collaborating and communicating. Each of these practices is reinforced when students think critically about mathematics.

If a thoughtful critical skill benefits students they comprehend better what they are learning. A teacher should invite students to use reasoning to justify their decisions about almost every aspect of mathematics, including selecting strategies for building number sense and mastery of basic facts, analyzing how to approach a problem for which they do not have a ready-made solution, choosing the most appropriate way to represent a mathematical situation, monitoring the problem solving progress and adjusting as needed, analyzing their own answers and asking, 'Does this make sense?', communicating with other students their mathematical ideas effectively, and connecting mathematics with real life situations.

The role of the teacher is essential in enhancing critical thinking in a class, by deliberately nurturing communities of thinkers through emphasizing mathematical discussions as part of the daily routine in math classes. They could start with a brief 'question of the day' after which students could share mental math strategies to solve the question, drawing the class into a whole discussion about the strategies they have used to resolve the problem and set the expectation for appropriate discussion rules among the students. By framing critical challenges through using a variety of critical challenges to engage and provoke math tasks, presenting problems for which students do not have predetermined solutions strategies, open questions that have more than one solution, they relate math to real-life contexts and encourage different ways to find solutions. They should teach the intellectual tools to solve math, develop background knowledge, teach flexible strategies to solve problems like analyzing through asking what I know, what I need to find, and how can I use this data to solve the problem, and foster mind habits of critical thinking like perseverance in problem solving. Also, assess thinking through using self-assessment and peers' feedback, offer different ways for students to show evidence of their thinking, and make sure that the assessment strengthens the value of explaining their thinking and showing their work.

2.7 Theoretical Framework

This research will use the model of critical thinking which was developed by Kuhn (2005) to explain the intellectual development and epistemological understandings.

Using empirical research on the process of intellectual development of students, Kuhn (1999) delivered an understanding of the knowledge and skills that provide the foundations for critical thinking. Meta-knowing, which is essential to critical thinking, has three categories. First is the meta-strategic form that deals with procedural knowledge and controls one's own thinking and choices of strategies to reach goals. Second, metacognition deals with one's declarative knowledge which coordinates ideas and gives evidence in justifying information assertions (Kuhn 1999), and involves understanding of both thinking and knowing in general (Kuhn and Dean 2004). Finally, epistemological knowing has both a philosophical and personal aspect that influences the other two workings; in other words, these three categories of cognition function as means to manage and develop 'knowing what one knows and how one knows it' (Kuhn 1999, p.20).

The most important goal for students is to become independent learners: be able to find answers to their own questions. To foster the development of inquiry skills students should construct their argument as a form of inquiry and seek reasoning to justify their own claims, to question and understand the claims of other students. More progress is made in a shorter period of time compared to thinking alone, demonstrating the advantage of collaborative reasoning and problem solving (Kuhn 2005).

This theory is supported in this research because it emphasizes the importance of two families of intellectual skills, inquiry and argument, that function as the foundation to acquiring knowledge as students collect evidence, make judgments, solve problems, to learn (Kuhn 2005). These basic critical thinking skills are also employed within the common core standards which were implemented during the workshops of this research, such as: problems perseverance, reason abstractly and quantitatively, construct arguments and critique the reasoning of others, as a requirement for students to be proficient critical thinkers.

Equally important as intellectual skills are intellectual values such as persistence. As students recognize and practice using their intellectual skills, they recognize their values. Skills without values have little use. Together, knowledge, skills, and values are nurtured by one another and the individual is potentially equipped with the tools to become a critical thinker.

The literature review in the current study is aligned with the recent literature review about teaching critical thinking and how it is related to enhance HOT in math education. Authors (Crenshaw, Hale and Harper, 2011; Forawi 2016; Geeertsen 2003; Ness 2015; Walace, Berry & Cave 2009) address some of the gaps, such as while the focus on developing students' critical thinking skills through explicit instructions is taking more of the research, the knowledge about the conditions that enable critical thought within the math domain still needs more study, and for students to develop critical thinking skills they need both the correct instructions and the right conditions. Also, there is a lack of empirical research that examines the measures of critical thinking skills and assesses their validity and reliability. There is a scarcity of local studies on teaching critical thinking in the UAE schools. Based on this framework, the next chapter will discuss the research design, methodological approach, site, samples and measures of the study so it can add to the growing body of literature in the domain of teaching critical thinking in math.

Chapter 3: Methodology

The previous chapters have shown that there are few studies that have investigated the effects of implementing a critical thinking intervention in math classes, followed by testing students' achievement in a critical thinking test. Also, no studies in UAE have used the pre-test and post-test to examine the impact of the intervention on students' achievements in HOT skills embedded in the common core standards mathematics curriculum. The current study is a quasi-experimental design that uses a control group pre-and post-test. It was carried out over a period of three months in a private school in Sharjah, where the math teachers were explaining mathematics using mainly the teacher-centered approach, or the direct approach, during the regular math classes. The intervention of critical thinking skills was conducted after the school day, four days a week and continued for eight weeks. As girls and boys are segregated in the middle school, the researcher had to teach each group separately: two days, Sunday and Tuesday, were given to the participant girls in grade 7, and two days, Monday and Wednesday, were given to the participant boys in grade 7. The intervention had the same instruction methods, content, practices and assessment in both groups.

This chapter describes the site and population, the specific study approach employed for sampling, data collection, analysis and explanation.

3.1 Research Design

Empirical evidence is needed to validate the literature's discussion on the potential of a mixed instructional approach for teaching critical thinking. The current study is designed to search for the most effective mode that integrates the direct and the inquiry-based instructional approaches in teaching critical thinking. The study was designed to compare two modes of instruction, with each mode combining the direct instruction and the inquiry-based instruction. The researcher provides intervention to grade 7 in 16 x 2 sessions of the intervention program. The program included the generic cognitive skills underlying informal reasoning and debate; this design was in line with the general approach framework, which proposed teaching critical thinking embedded in a particular subject domain.

A pre-test and post-test quasi-experimental design was implemented to find empirical evidence on the effectiveness of instructional interventions in fostering grade 7 students' critical thinking. Particularly, structures of instructional interventions of individual studies in relation to a critical thinking instructional approach and teaching strategy. This study aimed to find a proof of the effect of mathematics critical thinking intervention on students' achievement in the mathematics critical thinking test. Therefore, the objective was to compare the effect of the critical thinking intervention as intellectual treatment (independent variable) on the learner' critical thinking skills (dependent variable) for the experimental group.

The quantatative data is considered the dominant data that guides the study by using a quasi-experimental design which is appropriate, as the researcher aims to evaluate the critical thinking intervention.

This research can be considered as a quantitative approach, the experimental group is not chosen randomly and the control group is chosen as a comparison group. Carrying out pre-test on both the experimental and the control groups allow us to measure the initial comparability of the groups. If the experimental group and the control group have similar results at the pre-test, the less likelihood there is of confounding variables differing between the two groups (Harris 2006).

Empirical evidence is required to prove that teaching explicit critical thinking skills correlated with HOT skills embedded in rich math content through a well-planned program would affect students' critical thinking skills, which can be measured by checking their performance in a critical thinking skills test before and after the intervention.

3.2 Site

The study took place in a K-12 American international private school in Sharjah, UAE. The school is one of the oldest accredited American schools in the country and it has three branches within UAE, but the study was conducted on one campus. This campus has around 2000 multicultural students: around 60% UAE citizens, 35% Arabs, 3% Asians and 2% other nationalities. The school segregates boys and girls in the middle and high school and grade 7 students are divided into six classes, three boys' classes and three girls' classes. Teachers can teach in both sections. At the end of each academic year grade 7 math teachers and the math head of department sit together to write the year plan for grade 7. A grade 7 math coordinator is responsible for breaking down the yearly plan into weekly plans and submitting them for all parallel teachers at the beginning of each week, to ensure that all students are studying the same material.

3.3 Participants and Sample

The total population of the study was 152 students, 80 males and 72 females. A letter of consent to participate in the math critical thinking skills program was sent to the parents of each of the grade 7 students. There was no obligation and no choices of participants were made - it was a voluntary option to every student in grade 7. All students who had their parents' consent were accepted to attend the critical thinking skills' intervention.

The experimental group in this study comprised the students participating in the math intervention program - 50 students who were exposed to the mixed approach, the direct method of teaching critical thinking through the regular math classes and the inquiry-based instruction through attending the critical thinking intervention work shops. The control group was 98 of the students who only studied using the direct method.

3.3.1 Samples

The sample that represented the experimental group had a condition, to attend at least 12 sessions out of the 16 sessions of the program. All students who failed to satisfy this condition were excluded from the sample, as students who attended less than 12 sessions had missed some of the targeted critical thinking skills, which would affect the result of the study. The experimental group sample had 26 females and 10 males. All of them complied with the attendance requirement.

The experimental group and the control group had different characteristics, like gender and achievement levels in mathematics, that could influence the independent variables (Fraenkel, Wallen & Hyun 2012, p.267). Therefore, to increase the internal validity of the study, the control group sample was chosen to best fit the experimental group by choosing similar samples in gender and level of achievement according to students' final mark in math in term one from all of the six sections in the school, to ensure the same starting points for all groups at the beginning of the study. The sampling methodI was a 'cluster random sampling (Fraenkel, Wallen & Hyun 2012, p.96) where the researcher selected cluster groups that already existed from different classes. The boys' participant group was from grade 7B, 7C and 7D and the girls' participant group was from grades 7A, 7AA and 7AG. These were all grade classes in the school under the study, to be assigned to experimental and control groups. The classes were divided into two groups: control groups that did not participate in the intervention and were studying the normal math classes using mainly the direct teacher-centered approach, and the experimental groups that had the inquiry-based instructional approach in addition to the normal math.

3.4 Grade 7 Math Curriculum and Intervention

The school had adapted the Common Core State Standards to teach math in grade 7. These standards should be implemented in a balanced way by using math content and comprehensive thinking processes. The main point to implementing the CCSS with relevancy is to address them, with clear teaching of the HOT skills that are implanted in rich math subject matter content. The CCSS has seven key student proficiencies: first is the critical thinking, second is the creative thinking, third is the complex thinking, fourth is the comprehensive thinking, fifth is the collaborative thinking, sixth is the communicative thinking and seventh is the cognitive transfer of thinking. The researcher taught all the intervention's workshops in both sections, boys and girls. The intervention took place after school for eight weeks. An attendance sheet was designed and the attendance was taken every session.

In each week of the intervention, there was emphasis on three or four of the proficiencies of the CCSS as exlained in appendix 3. Students were set in groups to construct a solution to mathematical problems using the proficiencies of group thinking.

In week one, the objective was to create equations and inequalities in one variable and use them to solve problems. The emphasis was on proficiency #2 - creative thinking. Student first hypothesized a variable then generalized an equation and synthesized the solution by using the four element models: understanding the problem, creating a plan, carrying out this plan and looking back to check the solution. Students may need to revisit the elements and reformat the problem as they work towards a common solution. As an example, students construct an equation to find the price of two brands, then proficiency #4 - comprehensive thinking, which is to compare between the two prices, and finally proficiency #5, collaborative thinking to infer and decide which one is better to buy.

In week two, the objective was to translate sentences into equations and translate equations into sentences. The emphasis was on proficiency #2 - creative thinking, to hypothesize the age of one sister and relate the given information to find the ages of all three sisters, then proficiency # 1- critical thinking, to analyze the information and evaluate the ages of the three and solve the problem, and proficiency # 6 - communicative thinking, to connect the information, synthesize the solution and write the reasoning of the solution.

In week three, the objective was using addition/subtraction/multiplication/division to solve equations with one operation. The emphasis was on proficiency # 3 - complex thinking to clarify the sequence of the solution, interpret the properties of equality, addition/subtraction/multiplication or division, to solve an equation and determine which solution is correct. Also, proficiency # 7- cognitive transfer of thinking, to apply the properties of equality to everyday practices, and finally proficiency # 5 - collaborative thinking, as a shift from competitive to collaborative thinking as students train and develop explanations and decide the erroneous and the correct solution.

In week four, the objective was to solve equations involving more than one operation. The emphasis was on proficiency # 5 - collaborative thinking. As a student-centered approach was implemented the most, students explained and developped the maximum and minimum amounts that could be dispensed from a machine to satisfy an equation. Employing proficiency # 6 - communicative thinking, to present the solution of an equation. Also, apply proficiency # 3- complex thinking to clarify and determine which minimum and maximum values can be allotted.

In week five, the objective was to solve for an indicated variable. The emphasis was on proficiency # 7 - cognitive transfer of thinking, to generalize a formula to find the base of a parallelogram using its area and height. Also using proficiency # 5 - collaborative thinking to explain the steps to isolate a variable, which are, locate the variable, identify the operations and use inverse operation. Use proficiency #4 - comprehensive thinking to understand and infer the applications of solving for a variable by deriving formulae for a variable from complex equations.

In week six, the objective was to solve equations with variables on both sides. The emphasis was on proficiency # 7 - cognitive transfer of thinking, to generalize a formula to synthesize the solution of an equation using the equality properties. Also, use proficiency # 3 - complex thinking to clarify if the equation has one solution, identity or contradiction and interpret which case is applicable to determine if a statement is sometimes, always or never true and use proficiency # 6 – communicative thinking to find the reasoning for the solution.

In week seven, the objective was to compare ratios and solve proportions. The students implemented proficiency # 4 – comprehensive thinking to compare between two ratios and used proficiency # 3 – complex thinking to decide if two ratios are equivalent by implementing the cross-product property. Also, they used proficiency # 5 - collaborative thinking to explain the error in a student's response, and develop and decide the height of the actual space shuttle if we know the size of a model and its scale.

In week eight, the objective was to solve problems involving percentages. The emphasis was on proficiency # 5 - collaborative thinking to describe the conditions in which adding a 40% solution to a 60% solution would produce a 50 % solution, and use proficiency # 4 – comprehensive thinking to compare the different percentages. Also, they used proficiency # 7 – cognitive transfer to synthesize the solution of some percentage applications like interest, finance, tax and tips.

It should be noted that the researcher has more than 25 years of experience in teaching math in different countries including 15 years of experience in the UAE.

Teacher	Gender	Academic Qualifications	Professional Qualification	Years of Experience
Researcher & grade 7 teacher	Female	Bachelor of science/ math	Bachelor of Education	25
Grade 7 Math teacher	Female	Bachelor of Actuarial Sciences (Math/ Finance)	N.A	6
Grade 7 Math teacher	Male	Bachelor of Math	N.A.	3

Table 1 : Teachers' experience

The researcher taught all the after-school intervention workshops that used the mixed approach in teaching critical thinking skills, in addition to teaching two classes of grade 7 sections using the direct approach. The other two math teachers taught the other classes of grade 7 using the direct approach. Grade 7 math teachers follow the weekly plan that is prepared by the grade 7 coordinator and their weekly meeting discussed the curriculum and the objectives' achievement. The two teachers helped the researcher in handing in and collecting the consent letters and in conducting the pre-tests and post-tests.

3.5 Instruments

The researcher had developed the Critical Thinking Skills Test (CTST) by collecting the HOT questions in the McGraw Hill math book for grade 7 in the relevant lessons. Grade 7 math teachers and the principal of the school, an experienced math teacher in Lebanon and UAE, revised the test. Also, the researcher's professor revised it.

The test measured seven main criteria in critical thinking skills that are embedded in the CCSS in math: precision, critique, perseverance, argument, regularity, structure, reasoning and challenge.

The test used a mixed item format, as Ku (2009) claims that teachers should implement variant methods in assessing students' critical thinking skills; the exercises should direct students to 'self-construct answers, assignments that facilitate the practice of strategic use of thinking skills in everyday contexts' (Ku 2009 p.75). He also suggested that multiple-choice exercises should be followed by questions that investigate students' underlying reasoning.

The test has three parts; the first part includes eight multiple-choice questions that cover the math content and the practices that had been taught during the intervention. The second part has eight questions developed from the HOT questions that follow each lesson that was taught for students in the regular math classes; one of them is an open-ended question. Each question measures one of the above HOT skills of common core standards in math. The third part includes six short-response questions and one extended-response question. However, only three students solved this part, so the researcher decided not to include it the data analysis in either pre-test and post-test.

The pre-test was done in the last week of September. All the participating students were tested at the same time. It took 45 minutes, during the second period, for all boys' classes, and the third period for all girls' classes. The post-test was done after the intervention finished, in the first week of December during the exam week.

3.6 Data Analysis

The quantitative data was collected from the pre-test and post-test for 16 questions that represent the HOT skills. The data was collected from the test papers of 36 students representing the experimental group and the sample of 40 students that represented the control group. Four excel sheets included the results for the pre-test control group, pre-test experimental group, post-test control group and post-test experimental group. The scores of the mean and standard deviation of all questions in the pre-test were collected and analyzed to ensure that all groups had the same starting point.

To answer the first question of the study on whether the intervention (independent variable) has an effect on students' performance on CTST (dependent variable), the scores of the mean and standard deviation of all questions in the post-test and pair of t-test were implemented to compare the results between the experimental group and the control group. To answer the second question of the research on whether the gender or level of achievement in math (independent variables) have a moderating effect on students' performance in CTST (dependent variable), the data was analyzed using descriptive statistics and t-test to study the demographic factor for both genders and level of achievement on students' performance in the CTST.

Additional variables may have an influence on the outcome, analyses of covariance would help us identify the covariates and assess their impact on our results.

Correlation analysis is a statistical procedure to estimate the relationship among the different questions in the test. The Pearson factor was calculated to study the correlation among the CTST questions.

3.7 Pilot Study

The quasi-experiment has to be tested against internal and external validity. For a quasi-experiment there should be an evidence 'for the absence of physical control of the experimental situation'. The internal validity was examined for the following factors. For history, whether there was some current event other than the intervention that could effect the change in the dependent variable (PPA 696 Research Methods 2001). No, because both groups experienced the same current events, they both studied the math classes normally in school, and only the experimental group had the after-school intervention. Regarding statistical regression, if the subjects come from low or high performing groups. Both groups have different math levels and the chance was equal for all students in variant levels of achievement groups to participate in the study. For selection, if the subjects self-selected into experimental and control groups which could affect the dependent variable. No, both groups had applied to the intervention program, and had a similar chance to attend the intervention and no restrictions were made for students. For experimental mortality, if some students dropped out the intervention, only the students who attended at least 12 sessions out of 16 were included in the experimental group in the data analysis. Other students had been dropped from the experimental group results as they missed 25% of the intervention (independent variable) and the performance in the CTST (dependent variable) would change as well, and consequently the results of the post-test of the experimental group (the group that was exposed to the intervention). For testing, if the pre-test had affected the results of the post-test. No, because the students had done the pre-test in September and did not have any access to it and had done the post-test on December, thus there was no possible effect. For the instrument the CTST, it had not changed throughout the study. For the design contamination, if the control group found out about their treatment in the experiment, or did either group have a reason to want to make the research succeed or fail, none noted. Regarding the external validity, and effects of the setting, the study was made in one school in UAE. For the effects of history, the study was conducted in term one which is very convenient for students, as behavior issues do not usually progress during this term. For the reactive effects of experimental arrangements, the study would need to replicate the findings in other grades, schools and time periods.

The design of the research as quasi-experimental includes pre-test and a control group to maximize the validity of this design more than other quasi-experimental designs.

CTST was tested for reliability as shown in Appendix 4. The reliability result states that if Cronbach’s Alpha test is > 0.6 then the level of internal consistency is good for the items of the test. Cronbach's Alpha = 0.761 > 0.7, which indicates consistency for the 20 questions of the CTST in the present study.

3.8 Ethical Considerations

There are a number of key phases that describe the system of ethical protections that the contemporary social research establishment has created to protect better the rights of their research participants (PPA 696 Resarch Methods 2001). These were followed in this study. The first principle of voluntary participation requires that students have not been forced into participating in the research. The second principle is the requirement of informed consent. Essentially, the participants were fully informed about the procedures and risks involved in research and gave their parents' consent to participate. Another ethical standard requires that researchers do not put participants in a situation where they might be at risk of harm as a result of their participation. Harm can be defined as both physical and psychological and that was complied with in this study by giving the students 15 minutes' break after the end of the school day to eat or drink, also transportation was provided for all students who live in the city and the researcher did not leave the school before the assurance that all participants had left the school safely.

There are two standards that were applied in order to protect the privacy of research participants. The research guarantees the participants' confidentiality; students are assured that identifying information will not be made available to anyone, also the school name will not be published. The other principle is the right to service, because all grade 7 students had the right to participate in the study no matter what their age, gender, socioeconomic situations or achievement level in math. Since the intervention would have beneficial effects, persons assigned to the non-experimental group may feel their right to equal access to the math support is being curtailed.

Chapter 4: Data Analysis and Results

The goal of this study was to examine the results of implementing a critical thinking intervention program through math instruction in order to enhance critical thinking skills for students in grade 7. This chapter presents the analysis of the collected data from the pre-test and post-test scores of a Critical Thinking Skills Test for mathematics (CTST). A pre-test and post-test quasi-experimental design was applied to meet the study's need to determine the effect of higher order thinking intervention (HOT) on students' critical thinking skills in mathematics classes. The drive is to compare the effects of teaching HOT proficiencies as a treatment (independent variable) on the learner's critical thinking skills (dependent variable) and whether this treatment had variant results on gender (dependent variable) or level of achievement in math (dependent variable) for the experimental group versus the control group that had not received the intervention. The participants were 152 grade 7 students in mathematics classes in a K-12 American curriculum school in UAE. Two hypotheses were articulated to control the study's investigation to answer the study questions: To what extent can a group of grade 7 students, who receive intervention of higher order thinking skills perform in a critical thinking skills test? And does gender or level of achievement in math have a moderating effect on enhancing the critical thinking skills of students?

To ensure that the experimental group and the control group were equivalent and the sampling was adequate, an independent t-test was conducted and descriptive statistics were implemented to compare the results of the control group and experimental group in pre-test. To answer the first question of the study, the data was analyzed by comparing the scores of the means and the standard deviations of the control group and the experimental group in the post-test. Also t-tests were conducted to compare the results of the control group and the experimental group in the post-test. To answer the second question, if gender or level of achievement in math has a variant effect on students'performance in CTST, the mean scores of each question were calculated to compare the results between pre-test and post-test for male and female students. Also, the mean scores of each question in different levels of achievement were calculated to study the effect of level of achievement in math as a variant factor on students' performance in CTST. To measure the reliability statistics test Cronbach's Alpha was conducted.

4.1 Experimental and Control Groups' Equivalency and Adequacy

The participants were 152 grade 7 students in 6 classes, 3 boys' classes and 3 girls' classes. They were selected to form four groups. Hence, the results are obtained from two experimental groups: one male group of 10 students and one female group of 26 students. The number of students in the experimental groups at the beginning of the study was 25 males and 27 females. However all the students who did not attend at least 12 workshops of the critical thinking skills intervention were eliminated from the data analysis of the experimental group. For the control group 40 students (25 female and 15 male) were selected randomly from different levels of achievement in math. To study the equivalence of the four groups, students were pre-tested and the results were compared. Independent t-test has been applied and the value of statistical significance was calculated. Table 2 below represents the results.

Group	Time	Gender	Number
Control	Pre-test	Male	15
		Female	25
	Post-test	Male	15
		Female	25
Experimental	Pre-test	Male	10
		Female	26
	Post-test	Male	10
		Female	26

Table 2: The distribution of sample size in the study by gender

As shown in Table 2, there is no significant difference in the pre-test scores between males in the experimental group (M = 2.6, SD = 0.82) and males in the control group (M = 2.23, SD = 0.58). There is also no significant difference between female students in the experimental group (M = 2.74, SD = 0.54) and female students in the control groups (M = 2.91, SD = 0.58). There is also no significant difference in the pre-test scores between the control group (M = 2.65, SD = 0.42) and the experimental group (M = 2.67, SD = 0.45), indicating that both groups had a similar starting level. When analyzing each gender group separately, it was found that there is no significant difference in the pretest scores for gender in control and experimental groups.

Question #	Groups	Mean	Standard Deviation	T-Test	Alpha	Result
All Questions (Male X Pre-test)	Control	2.23	0.58	0.651	0.522	No significant difference
	Experimental	2.60	0.82			
All Questions	Control	2.91	0.58	0.471	0.640	No

(Female X Pre-test)	Experimental	2.74	0.54			significant difference
Control (Pre-test)	Male	2.23	2.23	-0.167	0.868	No significant difference
	Female	2.91	2.91			
Experimental (Pre-test)	Male	2.60	2.60	-0.405	0.688	No significant difference
	Female	2.74	2.74			
Overall all questions (Pre-test)	Control	2.65	0.42	0.705	0.483	No significant difference
	Experimental	2.67	0.45			

Table 3: Descriptive statistics and paired t-test results of CTST pre-test

Therefore, it can be interpreted that there is no significant difference between the groups in terms of pre-test scores either within the same gender or across genders, and experimental and control groups had the same starting point before the intervention, so this study can be considered appropriate for the current groups.

4.2 The Effect of Critical Thinking Skills Intervention

This section includes the statistics of the CTST to explore the effect of the intervention on students' performance in the CTST.

The results show that there is a signiicant difference for the intervention on students' performance in the CTST.

The mean scores and the standard deviation of all the questions in the control group were compared versus the experimental group in the post-test. It can be shown clearly from Table 4 the effect of the intervention by comparing the experimental group (M = 12.08, SD = 5.34) and the control group (M = 7.93, SD = 3.63). By conducting the independent t- test the results show a highly significant difference, t = 4.005, $p < 0.000$ (see also Table 3), with alpha = 0.000 between the control group and the experimental group.

Question #	Groups	Mean	Standard Deviation	T-Test	Alpha	Result
Over all questions	Control	7.93	3.63	4.005	0.000	Highly significant difference
	Experimental	12.08	5.34			

Table 4: Descriptive statistics and paired t-test results of CTST post-test

These findings answer the first study question, that there is significant improvement in the students' critical thinking skills in the experimental group who participated in the critical thinking skills intervention, while the students who did not participate in the intervention did not show any improvement.

4.3 Variant Effect of the Intervention on Gender and Level of Achievement

The second question is to study the effect of the independent variables (gender and level of achievement) on students' performance in HOT skills as dependent variables, to find out if there is statistical significant difference between post-test scores for both control and experimental groups.

Table 5 indicates that there was a significant difference in post-test between male students in the control group and male students in the experimental group, and the result was also valid for female students between the control group and the experimental group. By conducting the independent t-test the results show a highly significant difference. A t-test was conducted to study the effect of the intervention on males and females and the result is as shown in the table, that there is a significant difference in both cases, with F= -2.887, $p < 0.000$ (see also Table 5), with alpha = 0.000 between the control group and the experimental group for the male students; F= -2.996, $p < 0.000$ (see also Table 4), with alpha = 0.000 between the control group and the experimental group for the female students.

	Groups	Mean	Standard Deviation	T-Test	Alpha	Result
Question #	Groups	Mean	Standard	T-Test	Alpha	Result

			Deviation			
All Questions (Male)	Control	8.07	3.84	-2.887	0.008	Highly significant difference
	Experimental	12.50	3.63			
All Questions (Female)	Control	7.84	3.57	-2.966	0.005	Highly significant difference
	Experimental	11.92	5.92			

Table 5: Descriptive statistics and paired t-test results of CTST post-test according to gender

To study the effect of the intervention across genders, the table shows that there is no significance difference between males and females in the post-test, either in the control group or in the experimental group.

	Groups	Mean	Standard Deviation	T-Test	Alpha	Result
Control (Post-test)	Male	8.07	3.84	0.189	0.851	No significant difference
	Female	7.84	3.58			
Experimental(Post-test)	Male	12.5	3.63	0.287	0.776	No significant difference
	Female	11.92	5.92			

Table 6: Descriptive statistics and paired t-test results of CTST post-test across gender

The finding can partially answer the second question of the study, if there is a variant effect of the intervention on gender. It can be concluded from the findings that the intervention has not had any variant effect on gender, meaning the intervention had affected the participants in the study, either males or females, in the same way. To study the variant effect of the level of achievement in math in enhncing the critical thinking skills, a t-test was conducted to compare the results of the experimental group and control group in the posttest among the different level of achievement. Our hypothsis that the intervention has a moderating effect among level of achievement was proven right. Students who scored above 90 in their level of achievement in math had highly significant performed better than the control group.

Findings show that t = -2.887, F < 0.000 (see also Table 7), with alpha = 0.000 between the control group and the experimental group for the students who scored 90-100. Finding also show highly significant difference for the experimntal group than the control group for 80-90 level in math. Results show that t = -3.104, F < 0.000 (see also Table 7), with alpha = 0.000 between the control group and the experimental group for the students who scored 80-90 in their math test, that indicates that the experimental group students had outpereformed the control group studetns in this level but with less differnce than the level of 90-100. The same thing can be concluded for the 70-80 level but with less significance than the 80-90 level of achievement. It can be concluded that level of achievement in math had a variant effect on students critical thinking skills. However there was no significant difference for the intervention on students'critical thinking skills for students who had an average that is less than 70 in math.

Level of Achievement	Groups	Mean	Standard Deviation	T-Test	Alpha	Result
90 – 100	Control	10.82	3.60	-4.453	0.000	Highly significant difference
	Experimental	16.75	2.77			
80 – 89	Control	7.67	2.74	-3.104	0.007	Highly significant

	Experimental	12.88	4.12			difference
70 – 79	Control	6.68	2.54	-3.297	0.005	Highly significant difference
	Experimental	11.13	2.90			
60 – 69	Control	6.57	3.64	-1.007	0.343	No significant difference
	Experimental	9.00	3.00			
Less than 60	Control	5.50	4.20	1.330	0.225	No significant difference
	Experimental	3.00	0.71			

Table 7: Descriptive statistics and paired t-test results of CTST post-test according to level of achievement

These findings partially answer the second question of the study. Results show that there is a highly significant improvement in the students' HOT skills among level of achievement in the experimental group. In other words the intervention was a variant factor among different levels of achievement and the high-level achievers in math benefitted the most from the intervention, while this benefit decreased for other levels of achievement and there was no significance difference among the below-level students in math.

4.4 Analysis of Covariance

For more advanced analysis of what and how each of the CTST questions were solved by the students in the intervention group and their attributes to the HOT skills, the twenty questions were ranked in descending order. According to the t-test for the control group and experimental group among the HOT questions, the findings show that the 'structure' question, 'Determine whether each statement is sometimes true, always or never true. Explain your reasoning:

a. x+ x = x and b. x + 0 = x has scored the highest ranking in the t-test among questions, with alpha = 0.000 (M=1.22 , SD = 0.76) for the experimental group and (M= 0.58, SD = 0.5) for the control group. The second ranking was the 'critique' question: 'Determine whether each solution is correct. If the solution is not correct, describe the error and give the correct solution' with alpha = 0.000 (M=1.22, SD = 0.83) for the experimental group and (M= 0.52, SD = 0.68) for the control group.

The third ranking was the ‘regularity’ question: ‘ Determine whether the following statement is sometimes true, always or never true. Explain your reasoning. The sum of three consecutive odd integers equals an even integer.’ with alpha= 0.018 (M = 1.22, SD = 0.8) for the experimental group and (M= 0.68, SD = 0.83) for the control group.

It can be shown clearly that the mean of the experimental group in ‘critique’ and ‘regularity’ is greater than the mean of the control group in both questions. Also, the standard deviation is greater than the mean for the control group, which shows a higher variation of students’ scores from the mean in the control group than the experimental group in these questions.

Another noteworthy resultwas that the third lowest ranking question was'challenge’:‘Given the perimeter P and width W of a rectangle, write a formula to find the length L’ scored (M = 0.5, SD = 0.74) for the experimental group and (M = 0.35, SD = 0.66) for the control group. Then ‘argument’: ‘Describe the conditions so that adding a 50% solution to a 100% solution would produce a 75% solution’. This scored (M = 0.47, SD = 0.74) for the experimental group and (M = 0.38, SD = 0.59) for the control group. The lowest ranking question was ‘perseverance', which is ‘Solve each equation or formula for the variable indicated.’ It scored (M = 0.28, SD = 0.45) for the experimental group and (M = 0.32, SD = 0.53) for the control group. Results show no significant difference between the control group and the experimental group in the post-test in these questions.

4.5 Summary of Results

The collective information obtained from the tables, in conformity with previous results, answered the study questions by concluding that the critical thinking skills intervention on the treatment groups have led to a significant difference between the experimental and control groups in most of the math HOT skills in the post-test results.

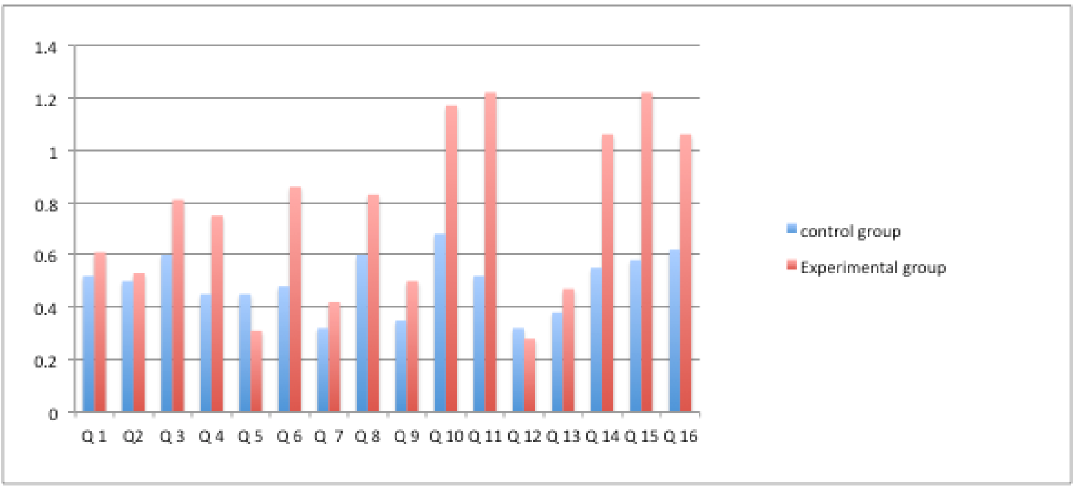

Figure 1: The mean scores of all questions in the post-test

It can be concluded that there was a rise in the critical thinking skills of students in the experimental group, while the students in the control group did not make such a change in their critical thinking skills. The first question of the study was answered, as there is a significant difference between the higher order thinking test scores of the students who participated in the intervention and those who did not. The results also indicate that the intervention had a variant significant improvement in the students' critical thinking skills according to their level of achievement in mathematics. Thus the highest achievers in math have shown the most significant change in the CTST and vice versa: the below-level achievers in mathematics did not show any significant change in their critical thinking skills in the post-test. However the hypothesis about the variant effect of the intervention on gender was not proven and there was no significance between male and female in the experimental group in the post-test results.

Based on these results, the next chapter will discuss these findings and make appropriate recommendations in the area of improving critical thinking skills in mathematics among grade 7 students. It will also discuss the limitations of this study.

Chapter 5: Discussion and Conclusions

Critical thinking skills can be enhanced in a math classroom and should be used as a bridge to practical life beyond school. Discussion of the findings of the study, conclusion, limitations of the study and recommendations for improving the teaching of critical thinking skills for teachers will be presented in this chapter.

5.1 Discussion

The goal of this study was to explore the effects of enhancing critical thinking skills by integrating the direct approach in a regular math class and the inquiry-based instructional approach, which was implemented in an afterschool intervention, on students' performance in HOT skills in math. Accordingly, a quasi-experimental design pre-test control group was adopted, in order to answer the study questions.

To answer the first question of the study, to what extent a group of grade 7 students who receive the intervention of critical thinking can perform in a higher order thinking of common core standards test, analyzing the results using t-test shows a statistically significant difference between the experimental group and the control group when looking at the post-test. Students who attended the intervention had got a higher mean in their answers than the control group. It can be safely concluded that the intervention group had significantly scored higher than the control group in the HOT test.

This result agrees with many empirical studies that examine the influence of explicit instructional intervention on students'critical thinking skills. Abrami et al. (2008) found these interventions to have a positive impact, however the result depends on the type of the intervention and the characteristics of the sample. In addition, Kennedy (1991) believed that instructional interventions that work at improving students' critical thinking skills have shown positive results in general. It is essential in enhancing critical thinking skills to teach the objectives of critical thinking explicitly in a track and embedded in subject content and it is important that the critical thinking instructors receive professional training in teaching critical thinking skills (Abrami, Bernard, Borokhovsky & Wade 2008). Teaching critical thinking skills and their dispositions is influenced by the intervention of critical thinking instruction (Bernard et al. 2004). Also, studies by Renaud and Murray (2008) and Williams et al. (2004) stated that the group that received instructions about solving higher-order questions significantly outperformed the group that received instructions about lower-order questions in domain-specific critical thnking measures. In another study Yeh (2009) emphasized teacher modeling and scaffolding of critical thinking skills, reporting a significant improvement of critcal thinking in the treatment group compared to students who did not receive direct instruction in critical thinking. Other studies that examined critical thinking skills like argument analysis using teacher's scaffolding and modeling showed significant results in pre-test and post-test comparisons (Hitchcock 2004; Mazer, Hunt & Kuznekoff 2007). However some studies that had other variations in implementing direct instructions reported no significant critical thinking improvement (McLean & Miller 2010; Nieto & Saiz 2008).

To answer the second question of whether gender or level of achievement in math have a moderating effect on enhancing critical thinking skills of students: for the first part of the question, scores did not vary either between males and females in the control group result at the post-test, or between males and females in the experimental group result at the post-test. It can be concluded that the intervention had no significant difference between males and females at the post-test results. However, the results show a significant difference for the same gender, between males in the experimental group compared to the control group and also females' results show a significant difference between control group and experimental group at the post-test.

This results agree with some other quasi-experimental designs' findings that there is no significance difference in the post-test according to gender. This can be due to the awareness of the importance of critical thinking intervention by both sexes, and that one can hardly survive without it (Chukwuyenum 2013). Other research in the area of critical thinking interventions found significant differences according to gender, where boys performed better than girls in grade 9 in Bangalore (Harrish 2013).

For the second part of the question, if the level of achievements has a moderating effect on enhancing the critical thinking skills of students at the post-test, the scores were significantly higher for students who scored above 70% in the term one exam in math in the experimental group compared to the results in the control group. Therefore, it can be concluded that there is a significant difference for students who are above average in math between the two groups at the post-test.

However, the scores were not significantly higher for students in the experimental group who scored less than 70% in the term one exam in math compared to the control group. Consequently, results had not shown a significant difference among below-average students between the experimental group and the control group. This result supports other findings in the field of critical thinking where Williams et al. (2004) studied the influence of an intervention in accordance with the exam results of students on the course. They claimed that high exam performers showed a significant improvement over the low exam performers in the critical thinking post-test.

From an educational perspective of critical thinking, according to the revised Bloom's Taxonomy (Karthwohl 2002), it is expected that students who are above average in math will improve their skills, to analyze by differentiating, attributing and organizing; to evaluate by checking and critiquing; and to create by generating, planning and producing, which will enable them to perform better in CTST. However, average students are expected to understand by interpreting, summarizing, classifying or comparing; and to apply by executing and implementing. Although some studies argue that much evidence advocates that ' average' people struggle to think critically, researchers like Edler (2003) believed that the basics of critical thinking can be taught to most students and they can implement critical thinking skills as efficiently as gifted students if they are given the time and sufficient conditions to learn. In our case, below-average students did not have extra time to practice and they were not given differentiated instructions during the critical thinking intervention.

From all the critical thinking skills that were under the study, three of them were ranking the highest: 'structure' as students use reasoning to judge a sentence, 'critique' as students evaluate the solution of a question, and 'regularity' as students analyze arguments. According to Gelder (2005) the critical thinker uses a systematic way to present a reason for an assumption.

This result comes consistent with Forawi's (2016) findings about one of the most checked critical thinking attributes skills CTAS instruments 'analyze arguments, interpretations, beliefs or theories and their implications' (Forawi 2016, p.18). These main features of critical thinking skills match with Lai's (2011) findings that despite the differences among approaches to define critical thinking, researchers agree on specific abilities, including analyzing arguments and evidence or claims (Ennis 1985; Facione 1990; Halpern 1998; Paul 1992).

Another feature of the finding is the 'perseverance' question, as students in the intervention group had not shown improvement compared to the control group. Perseverance is a mind habit that students need more time and extensive effort to gain. In addition to skills, critical thinking also includes dispositions (Facione 1990). Since 1985, researchers working in critical thinking have recognized that the ability to think critically is separate from the disposition to do so (Ennis 1985). Empirical evidence shows the notion that critical thinking abilities and dispositions are distinct entities (Facione 2000). These dispositions have been recognized as attitudes or habits of mind. Facione (2000) defines critical thinking dispositions as 'consistent internal motivations to act toward or respond to persons, events, or circumstances in habitual, yet potentially malleable ways' (Facione 2000, p. 64).

Instructional time in math in grade 7 should focus on four main areas and relate them to critical thinking skills. In the first area, students develop an understanding of proportionality and apply this understanding to solving single-step and multi-step problems. Students should analyze the proportional relationships and apply them to synthesize a solution to a real-world problem such as tax, discount, tips and percentages. In the second area, students should deepen their understanding of operations with rational numbers and write them with different representations as a fraction, decimal and percentage, and attend to precision. Students use the properties of operations on numbers to reason viable arguments and interpret the rules of operations, and extend them to negative numbers and their applications in real life such as temperature below zero and amounts owed. Students should connect the arithmetic of rational numbers to hypothesize variables then use these to formulate expressions and model equations to persevere in solving real-life problems. In the third area, students construct geometrical figures and relate their knowledge about the area and circumference of two-dimensional figures to generalize formulae for the surface area and volume of three-dimensional figures, then use them to create solutions for real-life situations. In the fourth area, students work with data samples to draw inferences about population and evaluate the probability of an event, and use these to predict the number of times an event can occur (CCSS initiative 2016).

5.2 Conclusions and Implications

Educators have always aimed to make critical thinking an educational outcome; Partnership for 21st Century Skills (2007) had emphasized, in its framework, critical thinking as one of the necessary skills in order for students to continue their college education and join the job market later. Also, Common Core State Standards emphasize teaching critical thinking skills in the math curriculum. The CCSS provide explicit teaching of the HOT skills that are embedded in rich subject math content (CCSS initiative 2016).

This study aimed to provide empirical evidence for the effect of enhancing critical thinking skills on the performance of students in a HOT test. A pre-test and post-test quasi-experimental study design was implemented, with 152 seventh grader students in math class who participated in the study from an American curriculum private school in UAE for 8 weeks. The quantitative approach applied in this study was mainly to find the effect of an independent variable, which is the critical thinking intervention on the dependent variable, which is the students' performance in the CTST. The conclusion of the study is that instructional intervention does have an effect on the development of critical thinking skills among grade 7 students.

The study finds that teaching clear critical thinking skills embedded in math class is highly significant for students' performances in a HOT test. These findings are consistent with those of many other researchers who support the infusion approach of critical thinking that combines instructions to be provided in the context of the math curriculum with explicit instructions on general critical thinking principles. Ennis (1989) called this approach 'across the curriculum movement'. The explicit instructions should focus on students' acquisition of these skills as a logical consequence of engaging them with the subject matter. Silva (2008) rebuts this viewpoint, sustaining that knowledge and thinking should be taught simultaneously. Similarly, Case (2005) claims that critical thinking is a lens through which content and skills should be taught, through embedding it in the curriculum.

Many researchers have advocated using particular instructional strategies such as collaborative learning (Abrami et al. 2008; Bailin et al. 1991; Bonk & Smith 1998; Heyman 2008; Nelson 1994; Paul 1992; Thayer-Bacon 2000), modeling, and constructivist techniques to encourage the development of critical thinking skills and abilities. Also, they have noted that these skills and abilities are unlikely to improve in the absence of clear instructions (Abrami et al. 2008; Case 2005; Facione 1990; Halpern 1998; Paul 1992). It is important here to recognize that the cooperative learning appears to be rooted in Piaget and Vygotsky traditions that stress the value of social interactions for promoting cognitive development (Dillenbourg et al. 1996).

By teaching CT skill, students in the intervention group were active, had the chance to sit in groups, think about the problem and try to collaborate towards finding a common solution for challenging questions through applying the strategies of problem solving. Students had the opportunity to critique the solution of a question, find answers to questions with precision, evaluate arguments, analyze the conditions of a solution, make inferences using reasoning, determine the regularity of a statement, and use structure to make judgements about a sentence.

The workshops in the intervention were made up of a teacher's explanation of the CT principles followed by the guidelines of the math content, then use of a constructivist learning method to involve students in extensive discussion and practice of the CT skill. The result of the teacher's modeling and coaching has been examined in many studies that reported significant improvement in CT skills (Hitchcock 2004; Mazer, Hunt, & Kuznekoff 2007; Plath et al. 1999; Reed & Kromrey 2001 cited in Tiruneh, Verburgh & Elen 2014) and the role of the teacher during the intervention was vital to support the students' learning. The teacher has to move around the students and ensure that they are engaged actively in solving questions, guide them to appropriately to use technology, encourage them to ask questions and help them to gain mathematical competence.

Group work as an effective element in improving students' achievement (Slavin 1989) was adopted during the intervention, also peer interaction in groups is another element in enhancing critical thinking skills (Forawi 2016). Conversely, students in the regular math class were mainly passive, listening to the teacher's explanation, watching the teacher solving the example, copying the answers and applying the same procedure to solve other questions. Moreover, students in the control group did not have the chance to develop their critical thinking skills and that explains why their results were below the experimental groups'. As the two groups had experienced the same circumstances during the school day, the intervention had significantly affected the students' performance in the CTST, and it can be concluded that the intervention enhanced the development of the experimental groups' performance in the test.

One of the important points that deserves discussion is whether the critical thinking skills that had been taught in the intervention can be transferred to new contexts or disciplines. For the transfer to happen students should be given a wide range of contexts and situations. Further, instructions should focus on metacognitive skills like planning, setting goals and monitoring the progress towards goals (Kennedy et al.1991). Also, students should dig deep into the structure of the problem and not merely focus on the surface structure and superficial aspects of problems and tasks (Halpern 1998; Willingham 2007). Finally, Halpern (1998) argues that the use of real-world problems helps in transferring critical thinking skills.

5.3 Limitations

The teachers in the study had a general idea about teaching HOT in math. Interventions in which teachers receive special training in teaching critical thinking would have better results compared with studies in which course curricula was aligned to critical thinking standards. Thus, effective interventions require specific professional development for teachers, particularly focused on teaching critical thinking (Abrami et al. 2008).

While intervention includes students from all levels, the instructions were the same for all students and there was no differentiating method implemented to adapt to every students' needs. This would further explain why low-achievement level students did not show development in the CTST post-test.

While marking the CTST, scores were divided into three categories: full marks if the answer was completely correct, half a mark if the answer was partially correct and zero if it was wrong. Accordingly, the process was not checked precisely. Norris (1989) claims that testing validly for critical thinking requires us to view an examinee's process of thinking.

CTSTs should have criteria to help in making decisions and evaluating arguments. From a philosophical perspective, critical thinking includes using criteria to analyze decisions and make judgments (Case 2005; Lipman 1988). Also, criteria are needed to evaluate the positions or arguments of others (Lai 2011). Criteria should communicate to students the quality of thought they should strive to gain (Paul 1992).

One of the limitations of the study is having the researcher as the only one who teaches the intervention. On one hand, it assures that the strategies and skills had been taught equally for every student. On the other hand, it would be very useful for other teachers in the school to be trained to teach critical thinking skills, which would help widen the circle of people who support the teaching of critical thinking skills within the school.

5.4 Recommendation for the Field

The school, like other typical schools, does not support the development of critical thinking skills. Typical schools focus on covering the content, and knowledge is mainly associated with memorization. Thus these schools stress the lower-order thinking (Paul 1992). Schools are urged to change the focus of teaching from instructions that stress memorizing to instructions that urge critical thinking. Teachers should maximize the time that students get involved in intellectual activities that sharpen their CT skills. Students' perceptions of critical thinking skills would happen in the presence of open-ended and guided instructions (Forawi 2016).

Although critical thinking is a demanding topic in our school, there is a huge ambiguity in the science pedagogy about such skills (Forawi 2016). This can be extended to math pedagogy as well and many efforts should be done to facilitate teaching critical thinking skills through the math curriculum.

Many math teachers are not qualified to teach critical thinking skills because of the lack of clarity about creative thinking (Beghetto 2013). Consequently they would tend to avoid teaching them, especially if we add that the majority of students are used to 'spoon feeding' learning. Schools should involve teachers in critical thinking professional development to acquire the required knowledge and pedagogy to teach such a crucial and demanding topic. According to Forawi (2016), teachers, especially new teachers, have to be trained to include critical thinking in their instructions, as well as how to transfer these skills by identifying the nature of critical thinking skills, and the methods to merge them into classroom practices.

5.5 Recommendations for Further Research

The present study examined teaching critcal thinking through explicit instructions and was embedded in the grade 7 math curriculum for eight weeks. Further research should study other instructional approaches such as general, immersion and mixed instructions. Moreover the intervention could be applied to other grades within the school or extended to involve more schools.

Further research should study extensively the ways the math curriculum in K-12 could facilitate teaching critical thinking and specifically correlate math pedagogy to critical thinking through practices and activities.

The author created the measure of the study CTST. It is recommended that other studies employ international measures of critical thinking such as WGCTA, the California Critical Thinking Skills Test and the Cornell Critical Thinking Test, and adapt them to the UAE educational system.

It is the schools' responsibility to prepare students who employ critical thinking and who are able to successfully continue their college education and productively pursue their careers.

Schools in the Middle East region should strive to create a generation who have critical thinking skills in addition to dispositions of critical thinking, such as: being open minded, accepting of others, motivating to be well-informed, flexible, and perseverant in seeking reasoning, to qualify them to be global citizens.

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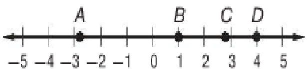
Appendix 1:

Critical Thinking Skills Test (CTST)

Multiple Choice

Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.

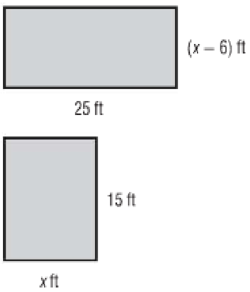
1. Which point on the number line best represents the position of $\sqrt{8}$?



A −2.8 **C** 2.8

B 1 **D** 4

2. Find the value of x so that the figures have the same area.



F 10 **H** 13

G 12 **J** 15

3. The elevation of Black Mountain is 27 feet more than 16 times the lowest point in the state. If the elevation of the lowest point in the state is 257 feet, what is the elevation of Black Mountain?

A 4,085 feet **C** 4,139 feet

B 4,103 feet **D** 4,215 feet

4. The expression $(3x^2 + 5x - 12) - 2(x^2 + 4x + 9)$ is equivalent to which of the following?

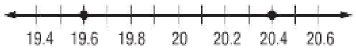
F $x^2 - 3x - 30$

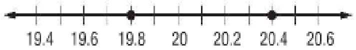
G $x^2 + 13x + 6$

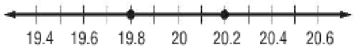
H $5x^2 + x - 18$

J $x^2 + 3x - 21$

5. The amount of soda, in fluid ounces, dispensed from a machine must satisfy the equation $|a - 0.4| = 20$. Which of the following graphs shows the acceptable minimum and maximum amounts that can be dispensed from the machine?

A


B


C


D


6. If a and b represent integers, $ab = ba$ is an example of which property?

F Associative Property

G Commutative Property

H Distributive Property

J Closure Property

7. The sum of one fifth of a number and three is equal to half of the number. What is the number?

A 5 **C** 15

B 10 **D** 20

8. Aaron charges \$15 to mow the lawn and \$10 per hour for other gardening work. Which expression represents his earnings?

F $10h$

G $15h$

H $15h + 10$

J $15 + 10h$

CHALLENGE Given the perimeter P and width w of a rectangle, write a formula to find the length ℓ.

PRECISION Adelina is comparing prices for two brands of health and energy bars at the local grocery store. She wants to get the best price for each bar.

a. Write an equation to find the price for each bar of the Feel Great brand.

b. Write an equation to find the price of each bar for the Super Power brand.

c. Which bar should Adelina buy? Explain.

STRUCTURE Determine whether each sentence is *sometimes*, *always*, or *never true*. Explain your reasoning.

a. $x + x = x$

b. $x + 0 = x$

Write an equation and solve each problem.

REASONING The ages of three brothers are consecutive integers with the sum of 96. How old are the brothers?

REGULARITY Determine whether the following statement is *sometimes*, *always*, or *never* true. Explain your reasoning.

The sum of three consecutive odd integers equals an even integer.

CRITIQUE Determine whether each solution is correct. If the solution is not correct, describe the error and give the correct solution.

a.
$$\begin{aligned} 2(g+5) &= 22 \\ 2g+5 &= 22 \\ 2g+5-5 &= 22 \\ 2g &= 17 \\ g &= 8.5 \end{aligned}$$

b.
$$\begin{aligned} 5d &= 2d-18 \\ 5d-2d &= 2d-18-2d \\ 3d &= -18 \\ d &= -6 \end{aligned}$$

c.
$$\begin{aligned} -6z+13 &= 7z \\ -6z+13-6z &= 7z-6z \\ 13 &= z \end{aligned}$$

PRECISION An artist used interlocking building blocks to build a scale model of Kennedy Space Center, Florida. In the model, 1 inch equals 1.67 feet of an actual space shuttle. The model is 110.3 inches tall. How tall is the actual space shuttle? Round to the nearest tenth.

PERSEVERANCE Solve each equation or formula for the variable indicated.

a. $n = \frac{x + y - 1}{xy}$ for x

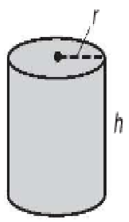
b. $\frac{x + y}{x - y} = \frac{1}{2}$f or y

ARGUMENTS Describe the conditions so that adding a 50% solution to a 100% solution would produce a 75% solution.

Short Response/Gridded Response

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

9. The formula for the lateral area of a cylinder is $A = 2\pi rh$, where r is the radius and h is the height. Solve the equation for h.



10. GRIDDED RESPONSE Solve the proportion $\frac{x}{18} = \frac{7}{21}$.

11. GRIDDED RESPONSE The table shows the cost of renting a moving van. If Miguel budgeted $75, how many miles could he drive the van and maintain his budget?

Moving Van Rentals	
Flat Fee	$50 for up to 300 miles
Variable Fee	$0.20 per mile over 300

12. Find the height of a soup can if the area of the label is 302 square centimeters and the radius of the can is 4 centimeters. Round to the nearest whole number.

13. GRIDDED RESPONSE Lara's car needed a particular part that costs $75. The mechanic charges $50 per hour to install the part. If the total cost was $350, how many hours did it take to install the part?

14. Lucinda is buying a set of patio furniture that is on sale for $\frac{4}{5}$ of the original price. After she uses a $50 gift certificate, the total cost before sales tax is $222. What was the original price of the patio furniture?

Extended Response

Record your answers on a sheet of paper. Show your work.

15. The city zoo offers a yearly membership that costs $120. A yearly membership includes free parking. Members can also purchase a ride pass for an additional $2 per day that allows them unlimited access to the rides in the park. Nonmembers pay $12 for admission to the park, $5 for parking, and $5 for a ride pass.

a. Write an equation that could be solved for the number of visits it would take for the total cost to be the same for a member and a nonmember if they both purchase a ride pass each day. Solve the equation.

b. What would the total cost be for members and nonmembers after this number of visits?

c. Georgena is deciding whether or not to purchase a yearly membership. Explain how she could use the results above to help make her decision.

Appendix 2

Critical Thinking Program Consent Form

A free critical thinking math program will be held for grade 7 students SAIS. This program will be held after school. It will provide support for **ALL** students in math and will reinforce the educational standards at our school. This workshop has been created to increase opportunities to succeed academically and to develop other essential skills such as working with diverse teams, communication, creative thinking and self-direction which are essential to student's success in practical life. This program will provide hands-on experiential learning opportunities, apprenticeships, mentoring, as well as opportunities for exploring new aides. In addition to developing core and new basic skills, after school programs provide students with the opportunity to explore and deepen individual interests in problem solving and critical thinking.

The program will provide a positive learning experience that will foster the interconnections among a student's social and cognitive skills so that students receive the maximum benefits from the academic support system in our after school programs.

The program will be held two days a week starting on the week if October the 4th and ending on the week of November 29th. Sessions for girls will be held on Sunday and Tuesday from 3PM- 4PM.

A school bus will be provided **ONLY** for students living in Sharjah. Other students must have arranged transportation.

The program is a part of Ms. Maha Masters' of Education.

Girls Middle School Coordinator,

Maha El Ayobi

This form must be completed, signed and returned in order for the student to attend the math critical thinking program.

STUDENT NAME (print)

____

Parent/ Guardian Name:

____

If you do not want your son to attend the program, no need to sign the form.

Appendix 3:

Seven Key Student Proficiencies of the New National Standards:

Proficiency #1 - Critical Thinking - Analyze, Evaluate, Problem Solve

Proficiency #2 - Creative Thinking - Generate, Associate, Hypothesize

Proficiency #3 - Complex Thinking - Clarify, Interpret, Determine
Proficiency #4 - Comprehensive Thinking - Understand, Infer, Compare

Proficiency #5 - Collaborative Thinking - Explain, Develop, Decide
Proficiency #6 - Communicative Thinking - Reason, Connect, Represent

Proficiency #7 - Cognitive Transfer of Thinking - Synthesize, Generalize, Apply

Appendix 4

The distribution of sample size in the study by achievement level and gender:

Group	Time	Achievement level	Male	Female	Total
Control	Pre-test	90 – 100	3	8	11
		80 – 89	6	3	9
		70 – 79	3	6	9
		60 – 69	3	4	7
		Less than 60	0	4	4
	Post-test	90 – 100	3	8	11
		80 – 89	6	3	9
		70 – 79	3	6	9
		60 – 69	3	4	7
		Less than 60	0	4	4
Experimental	Pre-test	90 – 100	5	7	12
		80 – 89	3	5	8
		70 – 79	2	6	8
		60 – 69	0	3	3
		Less than 60	0	5	5
	Post-test	90 – 100	5	7	12
		80 – 89	3	5	8
		70 – 79	2	6	8
		60 – 69	0	3	3
		Less than 60	0	5	5

Appendix 5:

Comparison gender achievement in open questions between pre-test and post-test:

Gender	Group	Time	Q9	Q10	Q11	Q12	Q13	Q14	Q15	Q16
			Average	Average	Average	Average	Average	Average	Average	Average
Male	Control	Pre-test	0.07	0.80	0.40	0.07	0.20	0.53	0.73	0.13
		Post-test	0.33	0.80	0.47	0.27	0.27	0.47	0.67	0.60
	Experimental	Pre-test	0.10	0.60	0.50	0.10	0.20	0.50	0.50	0.20
		Post-test	0.70	1.10	1.10	0.20	0.40	1.10	1.20	1.30
Female	Control	Pre-test	0.16	0.56	0.44	0.08	0.32	0.44	0.64	0.28
		Post-test	0.36	0.60	0.56	0.36	0.44	0.60	0.52	0.64
	Experimental	Pre-test	0.08	0.54	0.54	0.12	0.27	0.42	0.62	0.12
		Post-test	0.42	1.15	1.27	0.31	0.50	1.04	1.23	0.96

Appendix 6:

Comparison gender achievement in multiple choice questions between pre-test and post-test:

L. A.	Group	Time	Q1 Average	Q2 Average	Q3 Average	Q4 Average	Q5 Average	Q6 Average	Q7 Average	Q8 Average
100 - 90	Control	Pre-test	0.50	0.50	0.00	0.00	0.50	0.25	0.25	0.50
		Post-test	0.75	0.50	0.25	0.50	0.75	0.00	0.25	0.00
	Experimental	Pre-test	0.20	0.60	0.00	1.00	0.40	0.20	0.00	0.40
		Post-test	0.20	0.60	0.40	0.40	0.00	0.40	0.00	0.40
89 - 80	Control	Pre-test	0.64	0.73	0.82	0.27	0.27	0.73	0.55	0.55
		Post-test	0.82	0.55	0.73	0.45	0.45	0.45	0.36	0.55
	Experimental	Pre-test	0.42	0.25	0.75	0.50	0.33	0.75	0.25	0.83
		Post-test	0.83	0.58	0.83	0.92	0.58	1.00	0.75	1.00
79 - 70	Control	Pre-test	0.56	0.33	0.33	0.22	0.56	0.67	0.22	0.67
		Post-test	0.44	0.33	0.67	0.33	0.44	0.78	0.33	0.78
	Experimental	Pre-test	0.38	0.62	0.62	0.50	0.38	0.50	0.25	0.38
		Post-test	0.62	0.62	0.88	0.62	0.25	0.75	0.63	0.88
69 - 60	Control	Pre-test	0.44	0.33	0.44	0.00	0.22	0.44	0.00	0.44
		Post-test	0.33	0.56	0.67	0.44	0.11	0.44	0.44	0.78
	Experimental	Pre-test	0.75	0.25	0.50	0.25	0.25	0.38	0.12	0.50
		Post-test	0.62	0.38	1.00	0.88	0.25	1.00	0.12	0.75
Less than 60	Control	Pre-test	0.14	0.43	1.00	0.29	0.43	0.43	0.29	0.71
		Post-test	0.29	0.57	0.43	0.57	0.71	0.43	0.14	0.57
	Experimental	Pre-test	0.67	0.33	0.00	0.00	0.33	0.67	0.67	0.67
		Post-test	0.33	0.33	0.67	0.67	0.00	1.00	0.00	1.00

Appendix 7:

Comparison gender achievement in open questions between pre-test and post-test:

L. A.	Group	Time	Q9	Q10	Q11	Q12	Q13	Q14	Q15	Q16
			Average	Average	Average	Average	Average	Average	Average	Average
100 - 90	Control	Pre-test	0.36	1.18	0.64	0.27	0.55	0.55	0.73	0.45
		Post-test	0.82	1.09	1.00	0.73	0.64	0.55	0.64	1.00
	Experimental	Pre-test	0.17	0.92	0.67	0.08	0.50	0.67	0.58	0.25
		Post-test	1.25	1.58	1.75	0.42	1.08	1.42	1.33	1.42
89 - 80	Control	Pre-test	0.00	0.67	0.44	0.00	0.44	0.56	0.67	0.22
		Post-test	0.00	0.56	0.44	0.22	0.44	0.56	0.67	0.67
	Experimental	Pre-test	0.00	0.75	0.50	0.12	0.25	0.50	0.75	0.12
		Post-test	0.25	1.50	1.25	0.25	0.50	1.00	1.25	1.62
79 - 70	Control	Pre-test	0.00	0.56	0.22	0.00	0.11	0.44	0.67	0.11
		Post-test	0.22	0.78	0.33	0.11	0.00	0.44	0.67	0.44
	Experimental	Pre-test	0.12	0.25	0.25	0.12	0.12	0.38	0.38	0.13
		Post-test	0.00	1.25	1.12	0.25	0.00	1.25	1.38	0.88
69 - 60	Control	Pre-test	0.14	0.29	0.57	0.00	0.00	0.43	0.57	0.00
		Post-test	0.29	0.29	0.43	0.14	0.14	0.57	0.57	0.43
	Experimental	Pre-test	0.00	0.33	1.00	0.00	0.00	0.33	1.00	0.00
		Post-test	0.33	0.00	1.33	0.33	0.00	1.00	1.67	0.33
Less than 60	Control	Pre-test	0.00	0.00	0.00	0.00	0.00	0.25	0.75	0.25
		Post-test	0.25	0.25	0.00	0.25	0.75	0.75	0.00	0.25
	Experimental	Pre-test	0.00	0.00	0.40	0.20	0.00	0.00	0.40	0.00
		Post-test	0.00	0.20	0.00	0.00	0.00	0.00	0.40	0.00