A Design Study for Multivariable Feedback Control System Regulation for Aircraft Turbo-Jet Engines

دراسة مقارنة في تصميم نظم التحكم ذات المتغيرات المتعددة لمحركات الطائرات النفاثة

By

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Dissertation submitted in partial fulfilment of

MSc Systems Engineering

Faculty of Engineering

Dissertation Supervisor

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January - 2012

Abstract

This research work deals with the design of a multivariable control system for aircraft jet engines. It is motivated by the desire to exploit recent advances in design strategy emphasizing the dissipation of minimum control effort, and the need to investigate suitability and applicability compared with previous applied methodologies. Attention is paid to the energy consumed by the control system itself; in order to minimize wear, cost refits changes and noise generation improving thereby the reliability of aircraft engines.

The first part of the research deals with introduction of gas turbine engines. The research theme problem states the design of a multivariable control system for a twin spool turbo-jet engine model with two inputs. These are the fuel flow rate and nozzle area controlling the high and low pressure spool speeds. Objectives are defined as evaluating the proposed controller in comparison to previous designs.

This study is presented based upon evaluating the performance and power consumption of the proposed controller compared with other controllers designed in accordance with classical and modern control theory. Previous work conducted to design a controller for the theme problem is reviewed. This includes a controller designed by the Inverse Nyquist Array method and the one synthesized by optimal control method.

The research concludes that the new least effort control technique gives superior performance and disturbance recovery. It also generates the simplest controller. This should promote the rapid implementation of this design theory for aircraft engines and for general multivariable applications in near future.

خلاصة البحث

يهدف البحث إلى تصميم نظام تحكم ذو متغيرات متعددة لمحركات الطائرات النفاثة باستخدام نظرية تقنية حديثة تتميز باستراتيجية لتقليل استهلاك الطاقة ، و ذلك لدراسة مدى ملاءمة و عملية هذه النظرية مقارنة بأشهر التقنيات المطبقة حالياً ، حيث يتم التركيز فيها على الطاقة المستهلكة بواسطة نظام التحكم نفسه و ذلك لانعكاسه المباشر على الإنهاك و البلى و الضجيج الذي قد يصيب المحرك على المدى الطويل ، وبالتالي يحسن من كفاءة المحرك و يقلل من صيانته و استهلاكه للوقود.

يتناول القسم الأول من البحث مقدمة عن صناعة محركات الطائرات من الناحية العلمية و الهندسية . و يتم تعريف مشكلة البحث على أن تكون تصميم نظام تحكم ذو متغيرات متعددة لمحرك طائرة نفاث مزدوج بمدخلين مستقلين هما معدل تدفق الوقود و مساحة المنفث المقطعية و مخرجين أساسيين هما سرعة دوران أعمدة الضغط العالي و المنخفض . و يكون هدف البحث تقييم نظام التحكم المقترح مع تصميمات نظم مماثلة لنفس نموذج المحرك .

يتم تقييم نظام التحكم المقترح على أساس دراسة مقارنة مع تصميمين من المدرسة البريطانية الكلاسيكية و المدرسة الأمريكية الحديثة . و المقارنة بين أنظمة التحكم المختلفة ستتناول الصعوبات و التحديات التي تواجه كل تصميم ، و كفاءة النظام خصوصاً في مواجهة الاضطرابات و مدى عمليته و استهلاك طاقته .

نتائج البحث و المقارنة لن تؤكد فقط على أن هذه التقنية هي الأقل استهلاكاً للطاقة ، بل و ستبرز أيضاً أنها عملية و ذات كفاءة عالية في مواجهة الاضطرابات الجوية ، كما أنها تنتج أنظمة تحكم بسيطة و أقل تعقيداً من النظم المستخدمة مما يؤهلها بقوة للتطبيق العملي في صناعة محركات الطائرات و تطبيقات التحكم ذات المتغيرات المتعددة الأخرى على المدى القريب.

Acknowledgments

I am sincerely grateful to my supervisor, Professor Robert Whalley, for his great assistance and the patience shown throughout this research. It has been a great honour to learn of the theory outlined herein from the mouth of the theorist himself, and to be one of his students. I would like also to show my gratitude and appreciation to Dr. Ala'a Abdul-Ameer, for his continuous support and guidance.

Dedication

To my father, for teaching me the great value of science

To my mother, for believing in me

To my brother and sisters

To my best friends Muhanned, Berair, Fahad , Khalid, Hisham and Rose,
for their continuous support

To my people of Sudan, Arab and Africans and for all developing nations

I dedicate my work to you

“ So 'ere's ~to~ you, Fuzzy-Wuzzy, at your 'ome in the Soudan;

You're a pore benighted 'eathen but a first-class fightin' man;

We gives you your certificate, an' if you want it signed

We'll come an' 'ave a romp with you whenever you're inclined.”

Rudyard Kipling

1885

List of Notations and Abbreviations:

$a_{i,j}(s)$	Element of A(s) , 1$\ll i,j \ll m$
$a_{i,j}, b_{i,j}, \dots, \gamma_{i,j}$	Coefficients of $a_{i,j}(s)$
A	State space matrix
A(s)	Numerator of G(s)
A_J	Area of jet-pipe nozzle , in2
$\Delta A_{J,d}$	Change on demand value of area of jet-pipe nozzle , in2
$b(s)$	Polynomial
$b_o, b_1, \dots, b_{m-1}$	Coefficients of $b(s)$
B	State space matrix
C	State space matrix
$\mathbb{C}$	Controllability test matrix
$C(s)$	Compensator transfer function matrix
d	Demanded value
d_i	Diameter of Gershgorin Circle for column i
$d(s)$	Denominator of G(s)
D	State space matrix
$e(s)$	Error signal between reference input and output
$f, f_1, f_2, \dots, f_m$	Output loop feedback gains
F	Outer loop feedback array
$g_{i,j}$	Element of G(s) , 1$\ll i,j \ll m$
g_c	Gravitational constant = 32.16 ft/s2 lbf/lb

G(s)	Transfer function array (input/output)
h	Feedback path gain
$h(s)$	Feedback path function
H	High-pressure spool
H(s)	Feedback path compensator model
$H(s)$	Closed Loop Transfer Function Matrix
$\widehat{H}(s)$	Inverted closed loop transfer function matrix
$\hat{h}_{ii}$	Diagonal terms of inverted closed loop transfer function matrix
I	Polar moment of inertia , lb ft2
I_H	Polar moment of inertia of High-pressure spool , lb.ft2
I_L	Polar moment of inertia of Low-pressure spool , lb.ft2
I_m	Identity matrix
J	Performance index
k	Forward path gain
$k(s)$	Forward path function
$k >< h$	Outer product of $kandh$
$<k, h >$	Inner product of $kandh$
K(s)	Forward path controller model (pre-compensator)
L	Observer gain matrix
L(s)	Left (row) factors
$\mathcal{L}$	Laplace transformation
$n, n_1, n_2, \ldots., n_{m-1}$	Gain ratios

N	Rotational velocity , rev/min
ΔN_H	High-Pressure Spool Speed , rev/min
ΔN_L	Low-Pressure Spool Speed , rev/min
$\widehat{N}_{q_i}$	Number of origin encirclements by the mapping D-contour for $\hat{q}_{ii}$
$\widehat{N}_{h_i}$	Number of origin encirclements by the mapping D-contour for$\hat{h}_{ii}$
$O(s)$	Observer transfer function matrix
P	Pre-compensator array
P	Solution of Riccati Matrix Equation
P_1	Low-pressure-compressor inlet pressure , lbf/in2
Q	Coefficient array
Q	States weighting matrix
Q_f	Fuel flow rate , lb/sec
ΔQ_f	Change on demand value of fuel flow rate , lb/sec
$Q(s)$	Open loop transfer function matrix
$\hat{Q}(s)$	Inversion of open loop transfer function matrix
$\hat{q}_{ij}$	Off diagonal terms in inverted open-loop transfer function matrix
$\hat{q}_{ii}$	Diagonal terms in inverted open-loop transfer function matrix
r(s)	Transformed reference input
$\bar{r}(s)$	Transformed inner loop reference input
R(s)	Right (column) factors
R	Inputs weighting matrix
S(s)	Sensitivity array

T	Torque in lbft
T_H	Torque applied on High-Pressure Spool in lbft
T_L	Torque applied on Low-Pressure Spool in lbft
T_f	Terminal operating time
$u(s)$	Transformed input
$u(t)$	Inputs vector
$x(t)$	States vector
$\bar{x}(t)$	Estimated states vector
$y(s)$	Transformed output
$y(t)$	Outputs vector
z_c	Number of right-half complex plane zeroes for closed loop system
z_o	Number of right-half complex plane zeroes for open loop system
$\Gamma(s)$	Finite time array
$\delta(s)$	Transformed disturbance signal
ג	Eigen value
λ_j	Singular values $1 \ll j \ll m$
$\underline{\lambda}(S(s))$	smallest singular value of S($i\omega$)
$\overline{\lambda}(S(s))$	Largest singular value of S($i\omega$)
$\|\blacksquare\|_2$	Euclidean norm
$\|\blacksquare\|_\infty$	H_∞ norm
τ_A	Time constant of jet nozzle's area
τ_Q	Time constant of fuel flow rate

$\emptyset_i(s)$	Ostrawski shrinking factor of Gershgorin circles for column i
$\Phi(t)$	State transition matrix
Θ	Observability test matrix
CARDIAD	Complex Acceptability Region for Diagonal Dominance
IFAC	International Federation of Automatic Control
INA	Inverse Nyquist Array
L.E.C	Least Effort Control
LQE	Linear Quadratic Estimator
LQG	Linear Quadratic Gaussian
LQR	Linear Quadratic Regulator
LMI	Linear Matrix Inequalities
MVCS	Multivariable Control Synthesis
NACA	National Advisory Committee for Aeronautics
NASA	National Aeronautics and Space Administration
NDRC	National Defense Research Committee
O.C	Optimal Control
PID	Proportional, Integral and Derivative Controller
V/STOL	Vertical and/or Short Take Off and Landing

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Chapter I

Introduction

1.1 Research Background

In 1930, gas turbine was first used as general prime mover; following the award of the patent to Sir Frank Whittle. Eleven years later; Whittle's engine had successfully been tested, and completed its first flight. His patent revolutionized the aircraft and aerospace industry. Not only aviation took advantage from the new engine, but also gas turbine applications expanded to include power plants for electricity generation, ship propulsion, tanks and even nowadays automobiles (Chiras, 2002).

It is commonly called a jet engine, turbofan or turbojet for aircraft applications, while in ships and power plants applications; it is often called a gas turbine or gas generator.

It compromises a compressor, combustion chamber and turbine, connected in series order according to the airflow dynamics. The idea is similar to the internal combustion engine where the power is generated from the combustion of a compressed air and fuel mixture. As shown in figure 1.1, its cycle is combined from four thermodynamic processes. First; the air intake process due to negative pressure which is a result of compressor rotation. Then compression process of air followed by injection with fuel. Then followed by the combustion process, where the air-fuel mixture is burnt under almost a constant pressure leading the resulted gases to expand and rotate a turbine before being expelled as exhaust from the system. This cycle of four thermodynamic processes (air intake, compression, combustion and expansion) are based upon the Brayton ideal cycle, as shown in figure 1.2 (Langston and Opdyke, 2000).

From a thermodynamic point of view, the gas turbine is a heat engine, that converts the chemical energy stored in fuel to thermal energy by combustion, and then, this thermal energy is converted to mechanical work driving the power turbine (Kulikov and Thompson, 2005).

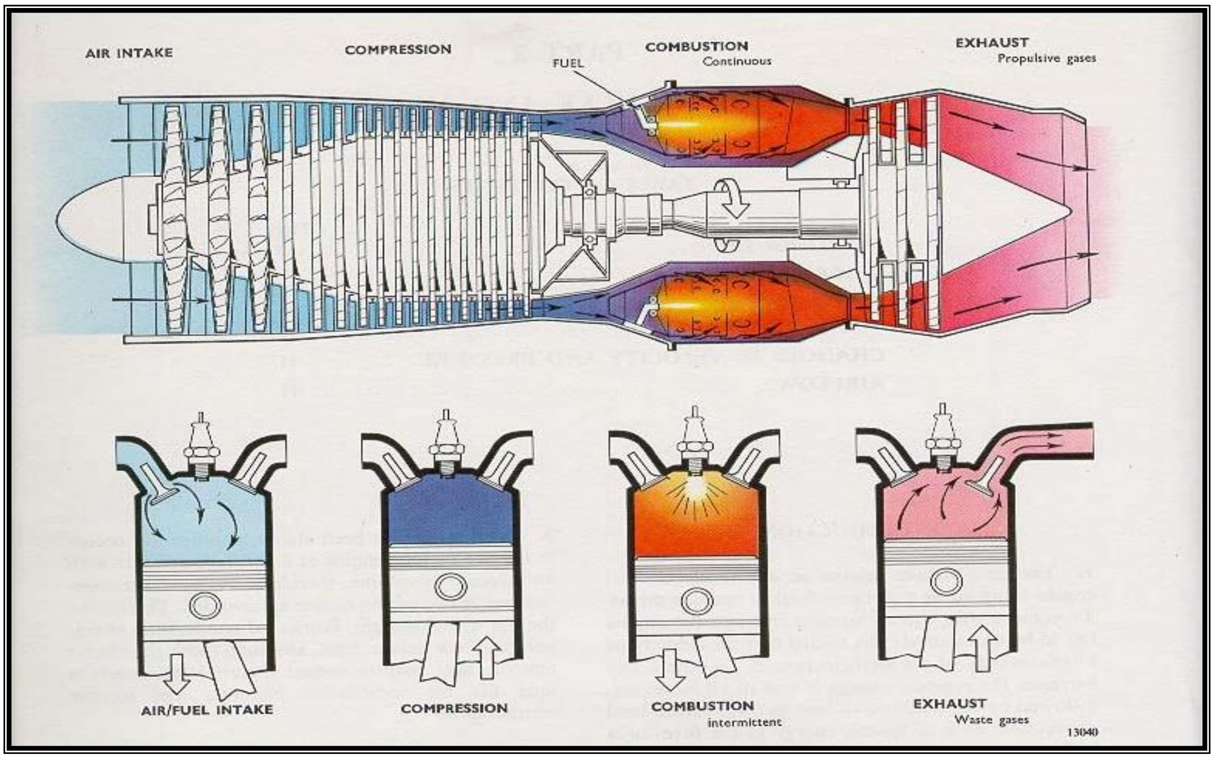


Figure 1.1, Processes Occur in Gas Turbine Engine Analogues to Internal Combustion Engine. (Rolls Royse,1996)

The power turbine is designed to extract sufficient energy from the expanded gases to keep the compressor rotating; so that the engine is self-sustaining (Chiras, 2002). The remaining energy of the exhaust gases is exploited either to generate thrust power, as in aircraft application, or to rotate another separate turbine, as in electrical power generation. These arrangements are shown in figures 1.2a and 1.2b respectively.

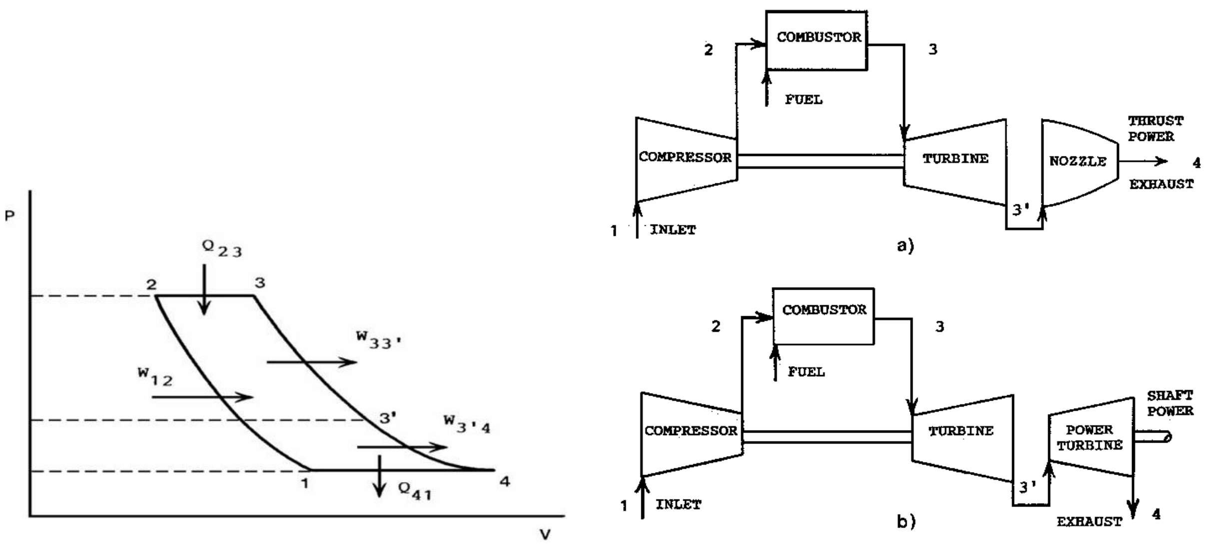


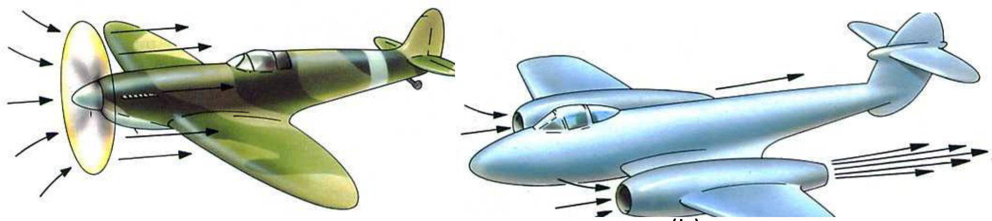
Figure 1.2,Brayton Cycle for Ideal Gas (e.g. air) Pressure (P) Versus Volume (v) Diagram

(a), Jet Turbine Brayton Cycle (**b)**, Power Turbine Brayton Cycle

(Langston and Opdyke, 2000)

The thrust power generation is based upon Newton's third law. Newton’s law states that "for every action there is always an equal and opposite reaction or the forces of two bodies on each other are always equal and are in opposite directions". A jet engine applies force on the air, in order to accelerate it and expel it through a nozzle. This would result in a reaction force of the same magnitude but opposite in direction applied by the air on the engine's body (therefore equally upon the aircraft body), known as thrust. The thrust results in the motion of the aircraft in the opposite direction of the airflow through the engine. The total thrust power by all engines should accelerate the airplane to its final speed; this being sufficient to overcome aerodynamic drag.

Thrust power depends on the multiplication of mass flow and airspeed. Classical aircraft use propeller fans mounted in front of the body to generate thrust by accelerating large mass of air at low speed backwards, as shown in figure 1.3a. Consequently, jet engines generate thrust by accelerating small mass of air and expelling it backwards at high speeds, as shown in figure 1.3b.



(a) (b)

Figure 1.3,(a) Thrust by Propeller Fan , (b) Thrust by Jet Engine .

(Rolls Royce, 1996)

Moreover, the combination of both types of engines is implemented by so-called turbo-fans, where the gas turbine drives a propeller fan and compressor by one shaft. The fan exhausts large volumes of air and some of the flow bypasses the turbine core as shown in figure 1.4. Thrust is then generated by accelerating bypassed air and exhaust gasses through nozzle at low speed and high mass flow rates. Turbo-fans, in general, improve the fuel consumption compared to turbo-jets, especially at sub-sonic speeds, and so they are ideal for commercial aircraft. However, turbo-fans require larger frontal areas compared to turbo-jets, which consist of small frontal areas. Turbo-jets are clearly suitable for military fighters, especially at supersonic speeds and takeoff from short runways. (Langston and Opdyke, 2000).

Many developments have been introduced to Whittle's original engine, such as the twin spool, triple spool and by-pass ducted fans. Industrial research and development were carried out in order to achieve the maximum thrust from jet engines and to improve the overall efficiency.

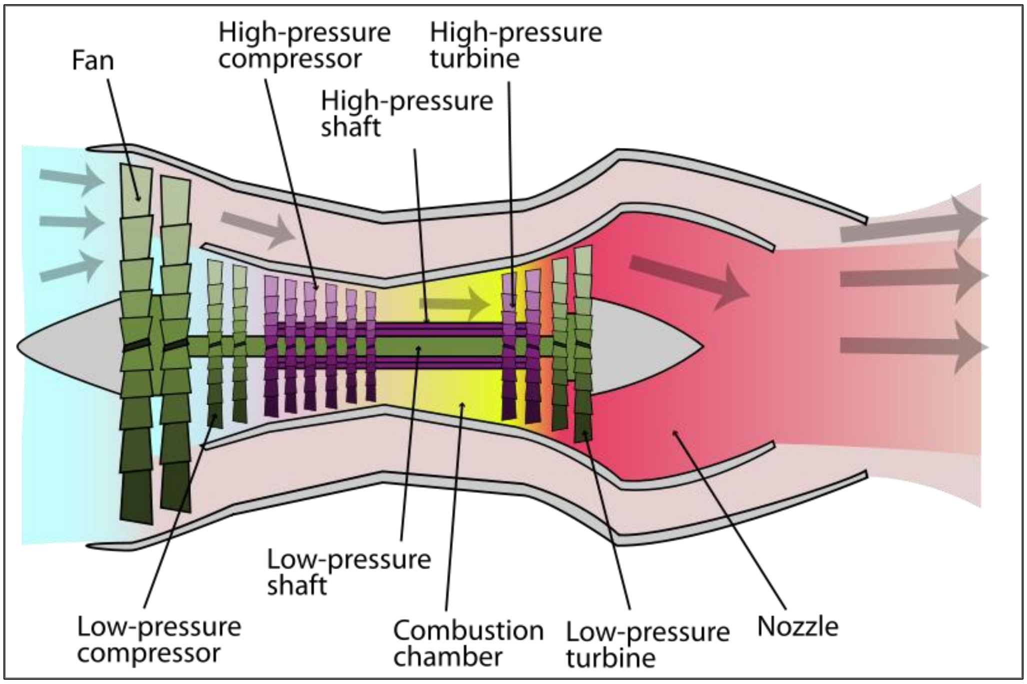


Figure 1.4, Turbofan Engine.

(Rolls Royce, 1996)

This is led to the design of twin spool gas turbines. Two stages of compression are introduced to increase the overall pressure ratio, and so the overall efficiency. The first stage is known as low pressure (LP) or low speed and its shaft is connecting between the first stage compressor and the second stage turbine. This second stage is known as the high pressure (HP) or the high speed stage and its shaft is connecting between the second stage compressor and the first stage turbine as shown in figure 1.5. The two shafts are independent, concentric and are rotating at high and low speeds (Chiras, 2002).

In general, the control objective of the jet engine is dependent on the flight mode. Usually at takeoff, the maximum thrust is required. In cruise flight, maintaining the power efficiency as high as possible and keeping specific fuel consumption to a minimum are important (Kulikov& Thompson, 2005).

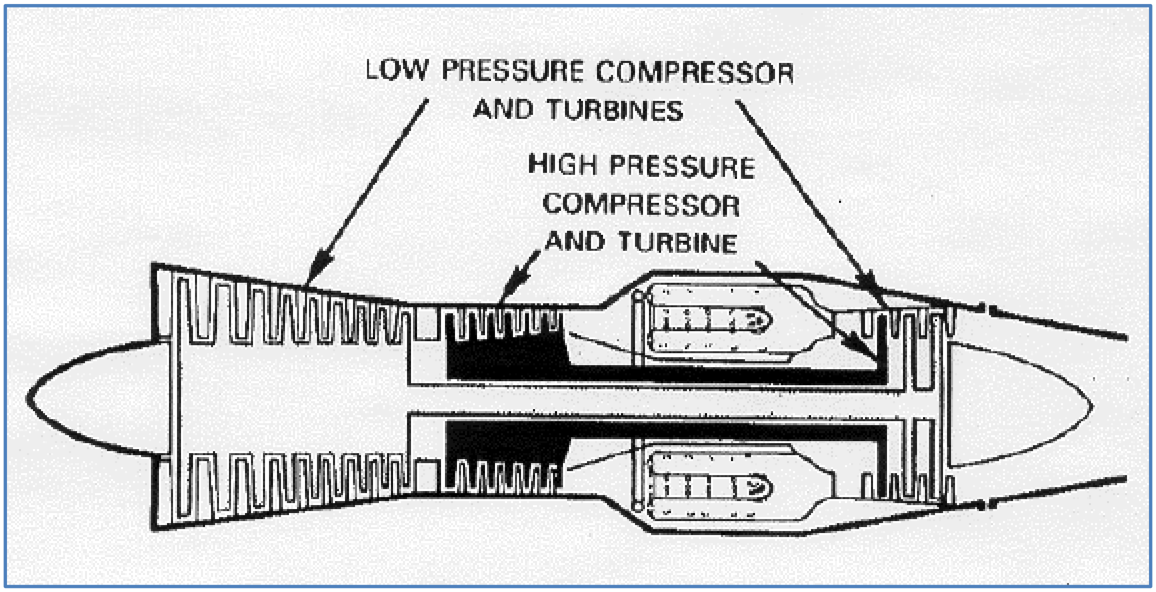


Figure 1.5 ,Twin Spool Turbo Jet

(Chiras, 2002)

1.2 Research Problem Statement

The control of turbo-jet engines is the general problem that will be investigated in this research. A twin-spool turbo-jet engine model presented by Mueller (1967) shall provide the theme problem. It represents a multivariable system with two inputs and two outputs. The fuel flow rate and the nozzle area are the manipulated input variables, while high and low pressure spool speeds, are the outputs variables.

The pilot controls the aircraft's jet engines, especially during the critical flight modes such as at takeoff and landing. The reliability and the quick response of the jet engine's control system are required; for the safe landing of airplanes.

The multivariable system presents new restrictions on the classical control problem; such as the output interaction. If any input is altered; this would affect both outputs. Any attempt to design a control system should reduce the output coupling which lowers the stability margins.

Random disturbances on the control system of turbo-jet engine are to be expected due to air turbulence and different flight modes during operation. This would affect the performance of the aircraft and the control system should be designed to limit the effect of these disturbances.

Aircraft are portable vehicles with limited power sources arising from the jet engine's fuel supply. The engines do not provide the thrust power only; but also the aircraft with electricity supply that is required to power many systems such as air conditioning, lighting, navigation and even the engine's control system itself. A lot of research and developments have been achieved minimizing the fuel consumption of jet engines from a mechanical point of view. However, few have dealt with the problem of the energy consumption for gas turbine control system minimizing wear, maintenance, noise and heat whilst reducing the deterioration of the controller system components.

1.3 Research Aims and Objectives

This research work is motivated by the desire to take a full advantage of recent advances in multivariable controller design emphasizing the minimization of the controller effort required. This was first presented by Whalley and Ebrahimi (2004). Their methodology is applied here to the design of a Least Effort Controller for the twin-spool turbo-jet engine model presented by Mueller (1967).The proposed controller would attempt to solve the classical control problems such as closed loop stability and to improve the transient and steady state responses. Also output interactions must be restricted to a maximum of 10 percent and the disturbance rejection properties of the system should be enhanced.

Another important purpose of this research is to evaluate the new methodology among other well-known multivariable control techniques. A comprehensive comparison study is provided with two methodologies from the British and from the American schools, named Inverse Nyquist Array method and Optimal Control method, respectively. The case study would be in respect of Mueller's twin spool turbo-jet engine, and the proposed controller will be compared with previous designs presented by McMorran (1971).

Comparisons shall be studied in detail concerning special areas of interest as clarified in the research problem statement. These include general control performance, disturbance recovery and dissipated control energy. In addition, highlights will be stated of the difficulties of each approach and the resulting controller complexity level, for implementation purposes.

1.3 Research Dissertation Organization

Chapter two in general is a literature review of previous work in the field of modeling and multivariable control of gas turbines. It will be shown that there are two theoretical schools; the British and American, which dealt with the problem of multivariable regulation.

Chapter three concludes the control methodologies that will be applied to Mueller's model. The mathematical derivation and analysis would be reviewed in view of the recent technique known as Least Effort control in accordance to Whalley and Ebrahimi (2004). Moreover, an outline of the Inverse Nyquist Array method and optimal control is presented including the derivation of formulas for comparison purposes. This chapter provides theoretical basis for the next chapter.

Chapter four deals with the implementation of control methodologies, first separately and then by the discussion of the simulation results comparing them collectively. A detailed comparison study is presented concerning the difficulties of each approach, then, the control objectives and finally the energy dissipated by each controller is presented.

Chapter five concludes the research and discusses the advantages of the new presented control methodology and recommendations for future work and development.

Chapter II

Literature Review

2.1 Introduction

A gas turbine was first designed in the 1930s and tested for operational use in 1941. The earliest control system was a manual hydraulic lever system to manage the fuel flow rate, which was proportional to the demanded engine power. This simple hydro-mechanical system regulates the required amount of fuel to generate the desired power (or speed). This is based upon the pilot's demand via a throttle valve, which is controlled by a hydraulic lever enabling the required amount of fuel to be delivered to the engine's combustion chamber. As shown in figure 2.1, a process diagram explaining the functional logic of fuel control system is presented (Jaw and Garg, 2005).

The valve is usually called an actuator. Its opening position changes according to the fuel input command. On the other hand, the output is the speed of the turbine shaft, as a direct indicator of the power or thrust required.

Consequently, gas turbines were linked with control theory developments. Engineers took advantage of control theory and applied different methodologies to improve the performance of the engine. Especially, in case of the multivariable systems as in the twin and triple spool jet engines, where more outputs are available with more restrictions and coupling control problems.

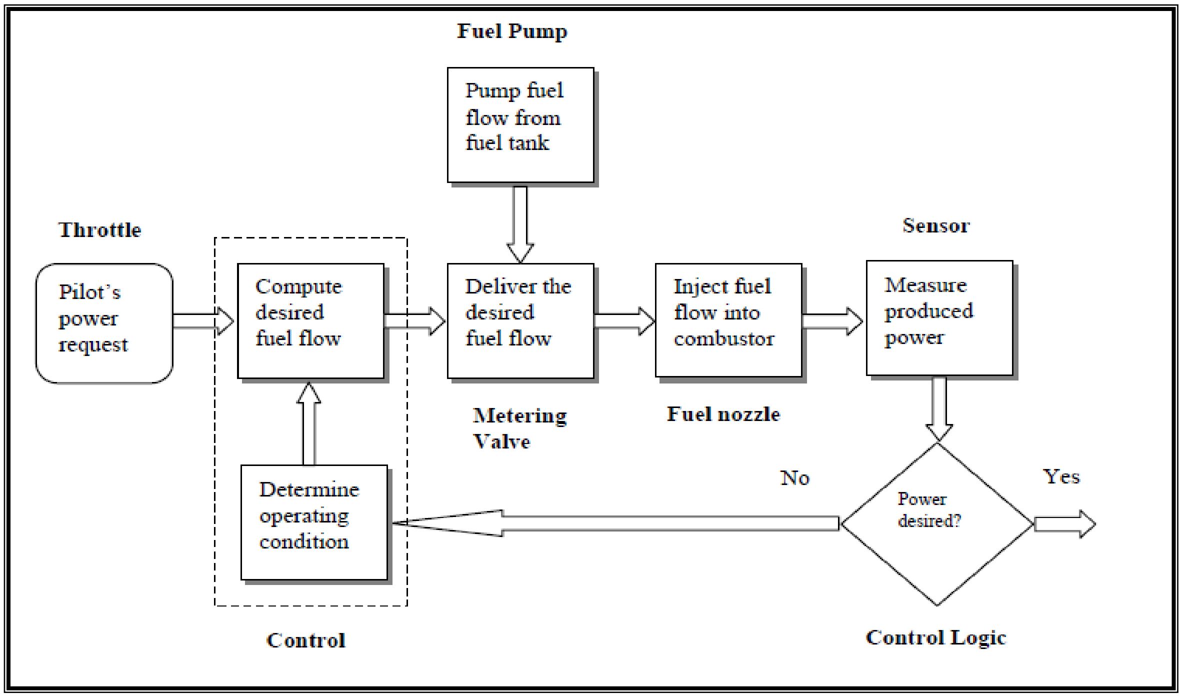


Figure 2.1 , Functional Process Diagram of a Simple Engine Control System . (Jaw &Garg, 2005)

In order to review gas turbine, multivariable regulation issues arising in the past 50 years, a review of the basic classical control concepts such as feedback, stability and the closed loop response will be given. Then the problem will be extended to the principle, two multivariable control theoretical approaches undertaken.

In the British school, frequency domain techniques were exploited whereas the American school developed time domain synthesis techniques.

Previous work on gas turbine multivariable control will be reviewed in accordance with each theoretical school, and comparisons are presented.

2.2 Control Fundamentals Review

It is often said that "necessity is the mother of invention", and control theory was developed to bridge the gap between engineering innovations and practical needs. Before World War II, most of the engineers and scientists dealt with the control problem during the design of their inventions. However, there was no basic theory to follow and everyone tackled problems from varying perspectives. Trial and error techniques were used and the approaches were entirely application-oriented. However, they succeeded in identifying the input-output relationships, understanding the closed loop response by using negative feedback to track the error between the output and input, and even discovering graphical ways to investigate the stability of systems. All these advances were accelerated by wartime pressures to develop and improve gun fire control systems. This formed the basis of classical control theory.

Hence classical control theory was born during World War II. It grew and developed in an environment saturated with long industrial experience and strong academic progress. These fundamentals could be summarized in the two theories of feedback and stability, which lean heavily on differential equations and Laplace transformations. In this section, reviews of historical developments of these theories are presented.

2.2.1 Feedback Control Theory

The feedback concept was introduced by ancient civilizations. Many examples could be found in old innovations such as water clock, mechanical clock, water or wind mills,… etc. The invention of steam engines resulted in all the innovations arising from the industrial revolution.

This introduced prime movers and new power generation systems, and gave the birth to classical mechanical engineering theory.

In 1788, James Watt invented a feedback mechanism in order to regulate steam engine speed. It consists of mechanical governor to measure output speed from the engine that was connected to a throttle valve to control the flow of steam into the engine (Bennett, 1993). This is clearly shown in figure 2.2.

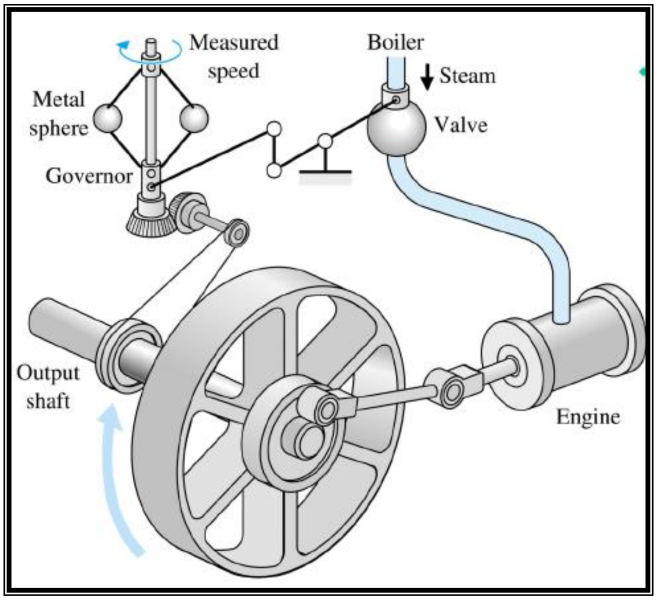


Figure 2.2,Watt's Mechanical Governor for Speed Regulation of Steam Engine.
(Dorf and Bishop, 2008)

Mayr (1970) discussed criteria to determine whether a certain engineering system is employing a feedback mechanism or not. His description of feedback included concept, purpose and structure. The purpose is to carry out commands based upon reference inputs; and the controlled system maintains the controlled variable, which is the output in response to reference commands, in spite of external disturbances. To do so, the system would include a sensor and a comparator, at least one of which can be distinguished as a physically separate element to read the output and compare it with the reference input. This structure presented closed loop control with negative feedback (Mayr, 1970).

In the example of Watt's steam engine, the input to the engine is steam flow, where the output is the shaft speed; the mechanical governor represents the sensor, the throttle valve and comparator.

By the end of 1920s, it was becoming clear to many engineers that improvements to control systems could be achieved if the measurement and control function were separated by a unit that could both amplify and modify the signal produced by the measuring device (Bennett, 1993).

This unit was defined by Harold Hazen from MIT in United States, who was the first to write a scientific paper about the theory of servo-mechanisms in 1934. He began to describe the differences between open-loop control and closed loop control in the amount of energy required to drive the controller mechanism. So the need of servo-mechanism is mainly in closed-loop systems, where the output is measured by instruments that usually fail in delivering the power required to the control mechanism. So according to Hazen: “some intermediate device for amplifying the power of the measuring instrument, while preserving its indications, was essential. When the output element of such a device is so actuated as to make the difference between the output and input indications tend to zero the device is called a follow up mechanism or servo-mechanism" (Hazen , 1934).

Servomechanisms, which use mainly feedback amplifier theory, progressed and developed especially during World War II. A group of engineers, scientists and mathematicians contributed in researches dealing with the improvement of position control involved in the military applications.

Another important field of feedback theory is the controller itself, or what is become known as the control law. After closing the loop, and feeding back its output signal through a feedback amplifier and finally comparing it with a reference input; a control action should be employed to achieve the desired transient dynamics and steady state response (final output value).

With the extensive industrial experience, it was found that proportional, integral and derivative actions known as (PID) could be an effective controller. Each action by itself could deal with a specific control problem. Proportional actions result in speeding up the response of systems, which simply means amplifying the input signal by a determined gain element. While integral action was used to enhance precision of outputs signal by eliminating steady state errors; which is the difference between the desired and actual final output values. Finally, the derivative action is effective for tracking systems where anticipating and predicting of the output value is required.

The first mathematical foundation of PID controller was discovered after deep observation of a helmsman steering a ship by Nicolas Minorsky in 1922 as stated "An Efficient helmsman keeps the ship accurately on her course by exerting a properly timed meeting and easing action on the rudder". Minorsky linearized the motion of the ship and showed that control action can be the summation of error, integral of error and derivative of error (Bennett, 1984).

Process industries also played an important role in developing PID control mechanisms. Industrial researches by the instrumentation companies like Leeds and Northrup, Taylor and Foxboro in the USA, resulted in introducing a wide range of PID control systems based upon either electrical or mechanical mechanisms. For example in 1939, Taylor Instrument Company presented the " Fulscope" pneumatic controller. At the same year, Foxboro introduced its PID version known as the "Stabiliog" pneumatic controller (Bennet, 1993).

Practical questions arose after the introduction and experience with PID controllers, such as "How can the appropriate controller adjustment be quickly determined for any control application?" and "How can a setting of a controller be determined before it is installed in an existing application?". These questions were answered by J.G. Zeigler and N.B. Nichols from Taylor instrument companies in 1942 in their paper titled "Optimum settings for automatic controllers ". They presented simple and successful procedures for PID tuning. Their procedures depend on finding the limiting gain for marginal stability and combining estimates of PID gains based on the limiting gain. According to the authors, these procedures were tested successfully on existing plant operations with many types of instruments from different manufacturers (Zeigler and Nichols, 1942).

Ziegler and Nichols procedure for PID tuning requires the investigation of stability, specifically the critical gain that makes the system marginally stable. A brief explanation and historical review of stability theory follows.

2.2.2 Stability Theory

A stable system is defined as a dynamic system with a bounded response to a bounded input. Instability, on the other hand, is defined as unbounded response to a bounded input. This behavior is not only expected upon closing the loop of a system; it can be found also in many open loop systems (Dorf and Bishop, 1993).

First attempts to understand the situation of stability were done by mathematicians and scientists regarding some particular applications. For example, G.B. Airy, a British astronomer in 1840 developed a feedback system for pointing a telescope. His system was a speed controller used for turning the telescope automatically in order to compensate for the earth's rotation, affording the ability to study a specific star for long period of time. He discovered that by the design of the feedback control loop, massive oscillations were present in the system. He was the first to discuss the unstable behavior of closed-loop systems, and the first to implement differential equations in the analysis (Lewis, 1992).

Maxwell in 1868 analyzed the stability of Watt's steam engine governor. He investigated the mathematics governing the motion of the engine, and stated that it represents a uniform steady motion which could be introduced by linearized differential equations. Then by analyzing different types and arrangements of governors, he concluded that to maintain stability for the solution of differential equations, the real parts of the roots, for the characteristic equation, should be negative. However, he could not completely prove that for equations with higher degrees than the third, and he addressed the problem of how could the coefficients of the characteristic polynomial affect its roots and so stability (Maxwell, 1868).

In 1877, E. J. Routh solved Maxwell problem by a computational technique to determine directly from the coefficients of the characteristic equation polynomial, if the characteristic equation detecting roots are stable or not (Routh, 1877).

Unaware of Routh’s work, two decades later another mathematician named A. Hurwitz, confirmed Routh’s technique. Today it is known as Routh-Hurwitz stability criteria. It is considered as a necessary condition but not a sufficient condition for the stability of control systems (Clark, 1992).

This approach provided a basic understanding of stability problem, by linearizing differential equations that present the system transfer function, and investigating the roots of characteristic equation (poles of closed loop transfer function). However, “A severe drawback with the approach was that the analysis gave no guidelines for modifying an unstable system to make it stable" Nyquist (1932). When he (Nyquist) presented an entirely different approach that introduces the frequency domain analysis, and which could be used for design purposes and stabilizing unstable systems (Nyquist,1932). His theory had powerful effects on classical control, and was extended to the control of multivariable system problems.

The investigation of stability using Nyquist theory was entirely different in approach from Routh-Hurwitz criteria. It is based upon the frequency response of the system with its feedback controller (loop transfer function). This is accomplished by testing the system with sinusoidal (sine wave) input signals under steady state conditions. It is shown, in accordance with his criteria that the shape of the frequency response had a critical effect on tracing the causes of instability. Thus, he found that the stability problem could be avoided by analyzing the graph of frequency response characteristics, or specifically by the encirclements of that graph around that critical (-1,0) point of the complex plane. The first version of the theory was mathematically complicated, but it was shown later on that it could be easily obtained from the theorem of Caushy (Nyquist, 1932).

Despite the fact that the theory was concentrated on the negative feedback amplifier for telephone industry, it was shown later on that it could be generalized to any control system, even multivariable ones. Its powerful advantage lies in the fact of the analysis for the proposed system with feedback controller in frequency response, could be repeated by altering feedback controller till encirclements around the critical point is satisfied the stability condition. This introduced a new method for design of stable control systems (Nyquist , 1932).

Moreover, Hendrick Bode provided in 1940, a simpler presentation of Nyquist frequency response, effective in investigating gain and phase stability margins (Kline, 1993).

2.2.3 Classical Control Theory during 1940s – 1950s

In 1940, the National Defense Research Committee (NDRC) was established in the USA. The direct outcome of this organization had a strong impact, not only on control theory, but on the general technology as a whole. The atomic bomb, radars, land-based anti-aircraft guns and other machineries and technologies were invented. The organization consisted of leaders from various fields and backgrounds including industry, government and the universities (Mindell, 1995).

Harold Hazen, the servomechanism theorist was the head of division 7 in NDRC. This was the division that witnessed the birth of classical control theory. Its projects and researches included ground-based anti-aircraft fire control, airborne fire control systems, servomechanisms and data transmissions optical range finders and naval fire control with radar. Mindell (1995) concluded the team work of division 7 ,by saying :" The novel organizational conditions of the NDRC allowed them to see fire control, as a particular case of a general problem of control: a feedback problem, a stability problem, and a problem of representing the world in machines using electrical signals." (Mindell, 1995).

After the war, the environment was ready for more theoretical development. Scientists and engineers had studied more mathematics and science to understand the control problem as an abstract theory. New organizations specialized in control had been established. A common language of control had been standardized. Universities and academic institutions started to deal with control as a new branch of applied science and as an independent discipline.

As a direct result, in 1950, Walter Evans introduced a graphical approach to visualize the system properties with varying gain, called the root locus method. It is a powerful tool to observe the behavior of the roots of the characteristic equation representing an open loop system, once closing the loop in accordance to loop gain variation. He defined rules for constructing such a graph in the complex plane. It depends on the single-input and single-output representation of the system in the Laplace domain. By plotting values of all singularities (poles and zeroes) of (s) so that transfer function equals -1, for increasing loop gain from zero to infinity, it is found that the behavior the of the singularities could be represented by graphical techniques. This new perspective provided an effective way to visualize a system's behavior, and it had a great impact on designing control systems. Limits of stability were presented, dynamic behavior of non-minimum phase systems (systems with right hand plane zeroes) were easily understood and the transient response was finally analyzed (Evans, 1950).

By the end of the 1950s, a classical control theory was completed. Mathematical foundations were established. The abstract theory of single-input single-output systems was well defined. The experiences from the application of the theory were widely promoted. This is led to more investigations of system control problems.

One great issue is that of multivariable control. It introduced much complexity in mathematics and generated complicated responses. Interaction between outputs and instability behavior in some cases were typical issues. Thus, a new theory specialized in solving such problems was of great need to industry. Two main schools were introduced in the period 1960s to 1970s, the British school, which extended the classical control theory to include the multivariable case, and on the other land, the American school that introduced modern control theory bringing an entirely different approach to classical control theory. A Literature review of each school follows.

2.3. British School for Multivariable Control

Multivariable input-output systems introduced more restrictions and difficulties to the control problem. Macfarlane describes that by saying “The difficulties in such a multiple-loop feedback control system arise from interactive effects; the action of one feedback loop affects the action of the other. This interaction is normally such that the stability margins of the system are reduced, and it is essential to design multiple-loop systems in such a way as to avoid interactive effects, which are prejudicial to system stability. It is helpful to have some physical insight into the nature of these difficulties before embarking on any detailed analytical study” (Macfarlane, 1970).

Extensive work has been done to tackle the multivariable problem from a mathematical point of view. For classical control, a single-input single-output transfer function was represented by the Laplace transformation of differential equation, with single scalar input and output. Thus, for the multivariable case, it requires the involvement of matrix algebra. Inputs and outputs are represented by vectors, while transfer functions are represented by a matrix. This issue complicated the extension of classical theory to solve the multivariable case. Extensive scientific researches by British control theorists were presented in the 1960s and 1970s, resulting in provided new control theory for multivariable systems. The main contributions to this field were provided by Rosenbrock,H.H and Macfarlane,A.J . A review of their work follows.

2.3.1 Inverse Nyquist Array Method

Rosenbrock was interested in frequency domain techniques; this appears in his earlier papers in obtaining the transient response from frequency response and vice versa (Rosenbrock, 1958). This background let him introduce later on an extension theory for multivariable control in the frequency domain. However, substantial work dealing with involved mathematics of matrix algebra was essential. This appears in his papers for reduction of system matrices (Rosenbrock, 1967), transformations of linear constant system equations (Rosenbrock, 1967) and on linear system theory (Rosenbrock, 1967). In 1969, he presented the Inverse Nyquist Array method for designing multivariable control systems. His philosophy was to decouple the system or reduce coupling between outputs, and then a single-input single-output theory could be easily applied.

He suggested investigating the coupling and stability of a multivariable system; by set of Nyquist diagrams corresponding to the elements of the inverse open loop transfer function matrix. (Rosenbrock, 1969). The inverse Nyquist plane was investigated earlier in 1947 for single-input single-output and shown that stability could be investigated in a manner analogue to conventional Nyquist technique (Criss, 1947).

However, his method was applicable for systems which are completely decoupled (in terms of mathematics called diagonal transfer function matrix) or at least, with weak coupling (in terms of mathematics called diagonal dominant transfer function matrix). Investigation of stability and diagonal dominance graphically was clarified by imposing the Gershgorin circles theorem at all frequencies of interest (Gerschgorin,1931). It states that, diagonal dominance is achieved, when the union of Gershgorin circles where none of the diagonal elements of the inverse Nyquist array, enclose the origin of the complex plane. Meanwhile, the closed loop stability is achieved when the union of Gershgorin circles do not touch the negative real axis between the origin and the feedback gain negative values (Munro,1972).

This simple and direct methodology attracted many researchers to develop it and extend its theory. For example, it was first extended to include unstable open loop systems with dynamics in the feed path (McMorran,1970). Also the closed loop stability region was shown to be larger by using Ostrowski's theorem (Hawkins and McMorran, 1973) .Moreover, extension to include non-square multivariable systems (systems with non-equal number of inputs and outputs) was achieved later (Lourtie,1985).

The main problem that still consumes a great work from engineers to implement the Inverse Nyquist Array (INA) method is that of achieving diagonal dominance by a simple and a stable pre-compensator. Many suggestions had been introduced concerning this particular issue. Pseudo-diagonalisation technique was suggested by Hawkins (1972). Also Whalley suggested decoupling the system using spectral factorization technique, and then relaxing the decoupling by approximation of the compensators resulting in diagonal dominance (Whalley,1978). In 1981,Mees suggested an optimal constant diagonal scaling matrix, which would result in a system as near dominant as possible, at a particular frequency. Limebeer extended Mees results after two years, and showed its connection with his generalized diagonal dominance approach (Munro, 1985).

2.3.2 Characteristic Locus method:

Macfarlane, meanwhile, provided another frequency domain technique, called Characteristic Locus or CL method. In 1970, he suggested first a "commutative controller" claiming that "spectral analysis may be carried out to determine a set of system characteristic transfer functions." This spectral analysis depends on decomposition of transfer function matrix in Eigen structure frame. This special frame consists of Eigen vectors, or characteristic directions of the transfer function matrix, and the diagonal matrix containing Eigen values of transfer function matrix which are called Characteristic Values. Consequently, decoupling the system is achieved and single-input single-output theory could be implemented by selecting a compensator shares the same Eigen vector (direction) of the system. Thus, a compensator with this structure commutes in multiplication with transfer function matrix and this is why it is called Commutative Controller (Macfarlane, 1970). However, this new approach was criticized by Layton,J.M. in the same year, when he pointed out that mathematical difficulties will appear from computation of Eigen values like irrational polynomials, which expose the theory to be impractical (Layton,1970).

In 1971, Macfarlane presented the Characteristic Locus method and defended the impracticality of his approach by using the Commutative Controller. Characteristic Locus first referred to the Eigen values of the transfer function matrix which its computation would attract irrationality. He showed that “a detailed knowledge of the eigenvalues as functions of (s) is not required, but merely a computed graphical representation of them, thereby permitting their direct use for compensation purposes “(Belletrutti and Macfarlane, 1971). This graphical representation was clarified by choosing an Approximate Commutative Controller, which replaces the Eigen vectors matrices of transfer function matrix, by approximated real matrices that share the same direction at some frequency. A direct algorithm to accomplish this was proposed by Kouvaritakis in PhD thesis 1974. He called it the ALIGN algorithm, in order to align real constant matrices at some frequency to the same direction of original system’s Eigen vector matrices (Kouvaritakis, 1974).

A great deal of attention was given in implementing ALIGN algorithm, which led to more refinement and investigation by Macfarlane and Kouvaritakis in 1977, followed by

Edmunds and Kouvaritakis in 1979. Their work focused on refinement of the algorithm at all frequencies. However, it did not give better results than the approximation at fixed frequency. So a multi-frequency alignment approach was suggested, in order to approximate a compensator at some chosen frequency, then another approximate in some other frequency yielding at the end of an Approximated Commutative Compensator.

2.4 American School for Multivariable Control

In 1960s, a modern school of control was born in USA by the appearance of a new general mathematical expression of dynamic systems in the time domain, named the state space. This was followed by a development of algorithmic techniques for control systems synthesis including multivariable ones. The main contribution of this new approach was a direct result of Kalman,R.E. Thus, a review of his papers and basic theory description follows.

2.4.1 State Space Representation

In 1960, in the first International Federation of Automatic Control (IFAC) congress held in Moscow, Russia; R. E. Kalman presented his famous paper called:" On The General theory Of Control Systems "(Kalman, 1960). In this paper, he introduced an abstract theory of mathematical representation for dynamical systems. He defined the inputs as certain physical quantities manipulated to control. Outputs are measurable physical variables of the plant. While States are "the minimal record of the past history needed to predict future behavior. ". In other words, states are the time dependent variables. Mathematically any variable differentiated with respect to time is a state. Kalman re-represent the differential equation that describing the system dynamic behavior, relating inputs and outputs indirectly through states (Kalman,1960).

He presented the system as a set of first order differential equations, written in term of states. A general matrix form was provided to describe the system in two vector differential equations, first the state vector equation as a function of input vector, then the output vector equation as a function of the state vector and input vector. This is the general representation of a system known as the state-space model. The general matrix form represented variant time and multivariable systems easily, and this new perspective provided a powerful tool for the analysis of systems properties.

He showed that the unforced solution (with input equal zero) for state equation is called state transition matrix, which relates states and initial conditions.

2.4.2 Controllability and Observability

In his paper, Kalman defined two essential properties for any dynamic system, named Controllability and Observability. His new perspective using state space, relating inputs and states in the state equation, provided the concept of Controllability. Then relating separately between outputs and states (in strictly proper systems) in the output equation provided the concept of Observability. A state is said to be controllable if it is reachable by an input signal that can transfer it from its initial condition to any desired location in a finite time. When all states are controllable, then a system is completely controllable. Meanwhile, if an initial state could be found from the input signal and measures output in a finite time, it is said that this state is observable. When all initial states appear in each output, then the system is completely observable (Kalman, 1960).

In 1966, Chen,C.T. and Desoer,C.E. introduced simple conditions to check

Controllability and Observability of systems. They provided special matrices that by checking its rank; complete Controllability and Observability could be determined (Chen and Dosoer , 1966).

2.4.3 State Feedback and Observers

By the appearance of the state space representation, control laws were suggested using the state feedback instead of output feedback, in classical control theory. Since the fact that outputs are related to states, consequently, controlling states directly would result in controlling outputs indirectly. However, practical problems appeared with this approach, especially when not all states are measurable. Thus, observers were suggested to estimate the states that are not measurable. The main contributions to this field were achieved by D.G.Luenberger.

In 1964, Luenberger dealt with the problem of reconstruction of state estimates from the available output information. He provided procedures to estimate the entire state vector of a system. This is known as a full order observer. He showed that the observer itself represents a subsystem that its dynamics should be faster than the original system. Moreover, he provided a low order observer to estimate only the non-measurable states using output feedback alone (Luenberger, 1964).

2.4.4 Optimal Control

Optimization was well known to early mathematicians and scientists. It is an extension from calculus of variations, which deals with functions of more than one variable. Its concept mainly deals with minimizing or maximizing a special mathematical expression called performance index under some mathematical constraint. Weiner (1949) and Hall (1943) proposed the first implementation of optimization techniques in feedback control. They suggested designing a feedback, which minimizes a performance index of the integral of a squared tracking error, but no clear algorithm was suggested to achieve this (Bryson, 1996).

In 1960, Kalman R.E. developed the idea and exploited the performance index with state feedback to optimize a regulator for a control system. His paper was the first to introduce the matrix Riccati equation as an algorithm for computing the state feedback gain matrix. Although, the Riccati equation had emerged earlier in the calculus of variations, its use in control was new. He showed that by solving backward Riccati equation to steady state, an optimal state feedback gain matrix was obtained. In addition, he clarified that checking Controllability and Observability is essential for regulation problems, as these were used to obtain the asymptotic stability properties of the Riccati equation (Kalman, 1960). His new theory of control was known later on as Linear Quadratic Regulator (LQR), was applied to linear time-invariant and multivariable systems.

Although Kalman investigated stability of Riccati equation by Lypunov theory for non-linear systems, Macfarlane showed that optimal controlled systems are always stable by Nyquist criteria; and proved that the frequency response for systems with an optimal controller; would not penetrate the circle of unity, which centre is (0,0) in the complex plane (Macfarlane, 1970).

One more great practical achievement of Kalman was the presentation of his famous filter. This is was a great solution for the problem of random noise signals disturbing control systems. He extended Wiener’s filter from a state point of view (Kalman, 1960). Moreover, In 1961 Kalman and Bucy showed that by solving forward Riccati equation, the optimal filter gains could be determined. This was known as Linear Quadratic Estimator (LQE) (Kalman and Bucy , 1961).

LQR and Kalman filter theory both formed the basis for new control techniques in presence of noise and disturbances. A Linear Quadratic Gaussian (LQG) compensator was suggested by feeding back estimated states from the filter with the optimal gains of the regulator and Gaussian inputs (characterized by rational spectra) on disturbances (Bryson, 1996).

In the 1980s, new methods and theories like $\mathcal{H}_2$ and $\mathcal{H}_\infty$ were developed to overcome the shortcomings and poor robustness of LQG control (Maciejowski, 1989).

2.5 Gas Turbine Modelling and Control Review

Gas turbine industry grew in line with control theory development. In its early years, engineers developed simple hydro-mechanical speed feedback systems; to control the flow rate of the fuel into the engines. When classical control theory emerged, modeling and control techniques were applied. However, modification of the gas turbine structure in order to increase its mechanical efficiency resulted in the twin spool jet engines; opening the door for the application of multivariable control theory. A great number of papers and researches have dedicated themselves to gas turbine multivariable control. Many regulators were suggested and tested to satisfy the engine operation conditions. A review first for modelling gas turbines is given herein. Then a review of the control attempts and achievements according to each multivariable school follows.

2.5.1 Gas Turbine Modelling Review

In order to practice control on any system, a linear mathematical model should be developed to establish input-output relations. This model must predict the system behavior following any input change. Such techniques usually involve identification experiments to linearize the system around certain operation condition.

Although the control of gas turbines in the early years during the 1940s-1950s was very simple and not based upon a mathematical model, it defined important characteristics of an engine's dynamic behavior basing a foundation for future modelling. To understand the gas turbine as a system, inputs and outputs shall be defined. Fuel was considered as the source of power and the feeding element, which could be manipulated independently, so the flow rate of fuel into combustion chamber, in jet engine, was considered as an input variable. The jet engine objective is to produce power, which can be converted into thrust. A direct measurement of thrust is impractical; alternatively, speed of the output spool was suggested because it analogous to thrust, and is easy to measure accurately by rotational speed sensors.

In 1948, at National Advisory Committee for Aeronautics (NACA) at Lewis Laboratory USA, this renamed later the National Aeronautics and Space Administration (NASA), conducted tests and experiments on typical turbo engines. Otto and Taylor reported the first gas turbine model. They determined the dynamic characteristics of a single spool turbojet engine, by developing a general equation for the transient behavior. Their approach was based on experimental data by verification of calculations based on special engine charts at specific equilibrium operating points. A linear differential equation for the engine acceleration as a function of fuel flow rate and speed when operating in steady state conditions was obtained. Derivation of transfer function relating speed and fuel flow was achieved by considering the thermodynamic and flow processes to be quasi-static. It was noticed that speed responses slower than reciprocating engines for step changes in the fuel flow rate were obtained. Thus, the turbojet engine transfer function was approximated to be a first order linear lag (Otto and Taylor, 1950).

For twin spool jet engine, one of the earliest multivariable models was presented by G. S.

Mueller in 1967. He linearized the equations relating the fuel flow rate and the jet nozzle area, as inputs, and the low and high spools speeds, as the outputs (Mueller, 1971) .It represents a linear multivariable system. His model had been controlled by several techniques, and they will be discussed in detail in this research.

The assumption of linearity is the simplest representation for a jet engine modelling .The dynamics are described by mathematical expressions involving time constants and partial derivatives, the values of which can be determined, based upon the system analysis or engine tests. However, the assumption of a linear systems limits the validity of the simulation model to small perturbations about a single operating point and hence the scope of the control strategy. Any alteration to the engine means new values must be determined for the time constants and partial derivatives, which necessitates further analysis or tests (Sanghi, et al., 2000).

This problem of validity for a limited operating range was overcome by thermodynamic modelling, at the cost of significantly increased complexity. Fawke and Saravanamuttoo (1970) were first to examine the derivation of thermodynamic jet engine models based on the engine physics. They validated their thermodynamic models using slowly ramped acceleration and deceleration input signals.

Another paper by same authors (1973) dealt with implementing digital computers in the simulation of the dynamic response of twin spool models, for experimental investigations (Fawke and Saravanamuttoo, 1973).

Thompson (1974) noted that engine models did not include thermal effects are tended to under-predict engine response times by as much as 20-30%.

More work on identification, estimation, simulation and model reduction had been undertaken in the field of gas turbine modeling.

In 1982 , the staff of Lewis Research Centre in NASA J. Zeller , B. Lehtinen and W. Merril prepared a paper for the 20th Aerospace Science Conference , held in the USA regarding "The Role of Modern Control Theory in the Design of Controls for Aircraft Turbine Engines " , this paper covered the 1970s activities and researches. In their review in the area of model identification work, three papers were highlighted. (Michael and Farrar, 1977) about "Large-Signal Estimation for Stochastic Nonlinear Multivariable Dynamic Systems ", where an F100 turbofan computer model was discussed and an algorithm to identify its parameters was given. The 2nd paper was by R.L.DeHoff (1977; 1979), where a single-input engine model was given, based on closed-loop flight data. Last highlighted paper was by W. Merrill (1981), that identified a multivariable model for F100 turbo fan based on actual closed-loop engine altitude test data (Merril, et al., 1982).

Moreover, approaches for reducing complexity of models were studied , for example in work conducted for Rolls-Royce , Jackson (1988) showed that "the higher order nonlinear thermodynamic models could be reduced to linear models with the same order as the number of engine shafts for a given stationary operating point "(Chiras,2002).

Recent developments in computers and advances in system identification techniques, both led to more systematic approaches for fitting models to engine test data. Evans (1998) used frequency-domain techniques. Meanwhile, Hill (1998) used time-domain techniques to estimate linear models. Comparisons with linearized thermodynamic models have been studied at different operating points by Evans et al. (1999, 2000a) (Chiras, 2002).

Nonlinear gas turbine engines dynamics arise from the variations about the operating point and due to thermal effects. General nonlinear models were suggested to act globally. A recent research by Chiras,N. (2002),estimated nonlinear models where the dynamics vary with operating points, was presented based on engine data which is not influenced by thermal effects .

The research undertaken herein requires linear models of gas turbine to apply linear control theory. A review of linear identification techniques and research was already given. Thus, nonlinear modeling is omitted owing to applicability. A review of Gas turbine control developments follows.

2.5.2 Gas Turbine Control Review

The jet engine is designed to operate across a wide envelope. Conditions of flight altitude can vary from sea level to 50,000 ft. Air speeds may go beyond Mach 3. Meanwhile, temperature fluctuations according to weather and altitudes occur. All these are challenges facing the control system designer. In figure 2.3, the scope of the control in accordance with the operating and safety requirements is presented. It shows the fuel flow rate as an input, versus engine shaft speed as an output. The operating envelope is the shaded area bounded by the maximum and the minimum fuel flow limit, idle and at maximum power. The maximum fuel flow limit is for the safety of the engine. Over limit cause surge and overheat condition. Consequently, minimum fuel flow must be such as to prevent engine flameout. Idle and maximum power requirements according to the flight mode, cruising, takeoff and landing are also important (Jaw and Garg, 2005).

This operating envelope was interpreted for the classical gas turbine control system limits. Fuel metering valves were designed to operate between flow limits (Jaw and Garg, 2005).

In 1948, the first control law for the J47 jet engine model was designed by frequency domain techniques. In the growth phase between the 1950s-1960s, control designs were dominated by frequency response techniques for single-input single-output systems. Gain and phase stability margins were investigated, and step response methods for testing the entire operating envelope were used. It was not until the 1970s, that multivariable design techniques were used to control jet engines (Jaw and Garg, 2005).

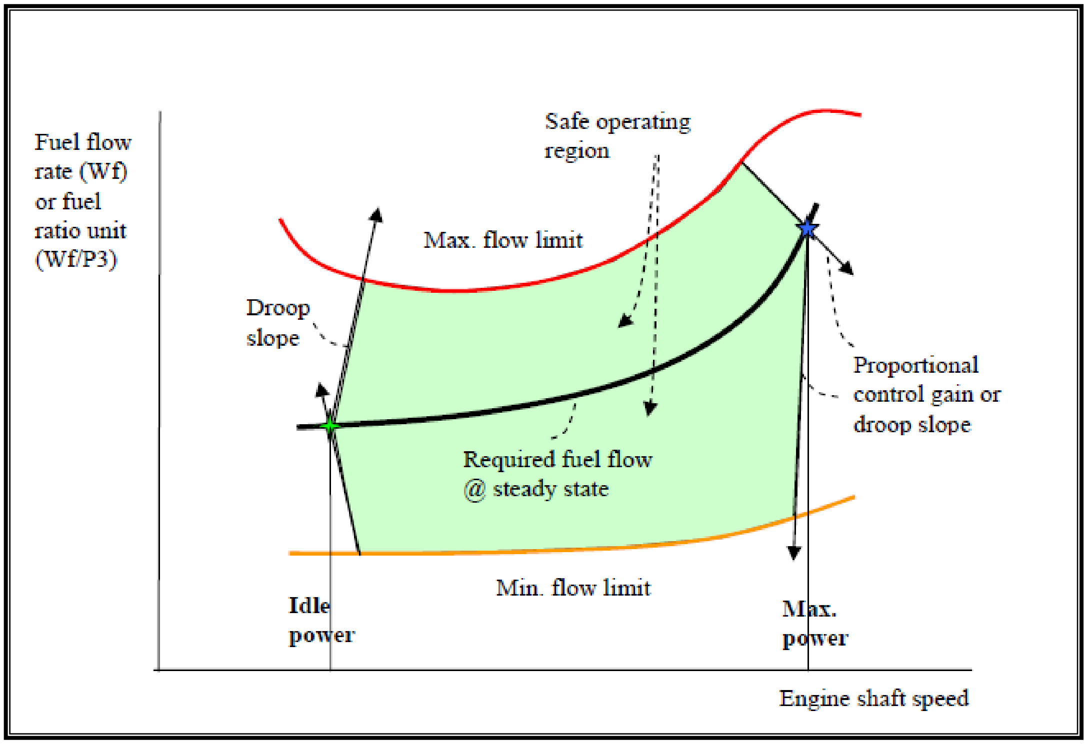


Figure 2.3, Gas Turbine Engine Fuel Control System Operating Envelope. (Jaw and Garg, 2005)

A review will be followed for the gas turbine multivariable control according to; first the British school that used frequency domain techniques, and then the American school using time domain synthesis methods.

In the period between 1970 and 1982, more than twenty papers and researches dealt with multivariable gas turbine control using the British school methods. Some of them are highlighted here (Merrill et al., 1982).

The earliest paper that applied the Inverse Nyquist Array (INA) method was by

McMorran (1970). He used Mueller's twin spool jet engine model (1967) and designed a simple controller that maintained stability and reduced interaction between output speeds. He showed that INA method is superior compared to the optimal controller designed by Mueller (McMorran, 1970). His work will be reviewed here in detail, and criticized in comparisons to the control technique proposal in this research.

In 1971, in a conference held in Manchester University on multivariable control system design and applications, a paper was presented applying Inverse Nyquist Array (INA) and Characteristic Locus (CL) methods on gas turbine control problems (Macfarlane et al., 1971).

It was not until 1976, that the Americans began to take advantage of British school techniques. NASA supported a great number of researches and scientific papers regarding alternatives for gas turbine multivariable control. Inverse Nyquist Array and Characteristic Locus methods were applied on a F100 turbofan engine model with two inputs, two outputs and five states by Sain et al. (1976) . Leininger reported to NASA in 1977, an application of multivariable direct and inverse Nyquist array methods for the F100 gas turbine model, with three inputs and three outputs. Diagonal dominance was achieved by a minimization algorithm. (Leininger , 1977). In 1985, in a final technical report for NASA submitted by Michael K.Sain , a series of researches presented alternatives for jet engine control covering ten years of work from 1975-1985. Great attention was given to the problem of achieving diagonal dominance, in order to apply Nyquist array methods. Algorithms and graphical techniques were suggested. It was reported that Column dominance was achieved for four-input four-output model within 30 minutes, using the graphical Complex Acceptability Region for Diagonal Dominance (CARDIAD) methodology (Schafer, 1980).

In 1981, Brown of General Electric Company designed a multivariable regulator, for a complex engine for Vertical and/or Short Take Off and Landing (V/STOL) aircraft, consists of twelve inputs. Approximate diagonalisation was achieved using so called K/Q algorithm (Brown, 1981).

In 1990, Munro applied the INA method for an automotive gas turbine. The model presented a twin spool engine with two inputs of fuel and nozzle actuator area, while outputs were the generator speed and the turbine inlet temperature. The engine also includes a heat exchanger fitted at the exhaust outlet to increase the thermal efficiency of the system, and a set of variable geometry nozzles fitted upstream of the power turbine to assure a high inlet temperature value to improve fuel consumption. Inverse Nyquist Array methods were applied and a proportional action pre-compensator and a proportional plus integral action controller was determined (Munro, 1990).

In 1991, a decoupling compensators and a series of SISO controllers were proposed to control gas turbine engines over an entire operating envelope by (Perez and Nwokah , 1991).

In a recent paper by (Nobakhti et al., 2003), an evolutionary design methodology to achieve diagonal dominance and synthesize a pre compensator, for a Rolls Royce Spey jet engine, was presented. The model was a three-input three-output system.

The most recent paper by Chugtai and Munro (2004), provided a diagonal dominance algorithm using Linear Matrix Inequalities (LMI). A systematic approach was proposed, and applied on the same three-input three-output model of a Rolls Royce Spey jet engine; showing that it is comparable with the results obtained by Nobakhti et al. (2003) , and more achievable in a systematic way.

On the other hand, American school synthesis techniques were implemented more frequently on jet engines than British methods.

More than forty papers dealing with Linear Quadratic Regulation LQR techniques were reported between 1973 – 1982 (Merrill et al., 1982). The earliest paper which will be highlighted here is by Michael and Farrar (1973). Under the sponsorship of the Office of Naval Research in the USA, Michael and Farrar applied optimal control method on linear models, among several operating points of F100 jet engine, at sea level static conditions. A nonlinear feedback control was constructed from a package of linear optimal controls (Michael and Farrar, 1973).

Another early work was reported in the same year. A Master's thesis by Browles (1973) studied a GE J-85 engine and obtained a linearized model in state space form. Linear Quadratic Regulation was implemented, to control mainly the fuel flow rate with limited control of the exhaust nozzle area (Bowles,1973).

In 1975, many papers were reported such as (Merrill , 1975) and (Elliot and Seitz, 1975). The work of Weinberg (1975) is highlighted, where a development of a widely used procedure was provided for presenting linear models in state space form, from a nonlinear F100 engine simulation using perturbation techniques. A multivariable controller was suggested using LQR method.

More control inputs than the previous papers were regulated using optimal control, which was presented by Beattie and Spock (1976). The use of integral action in LQR control for a variable cycle engine was suggested by Slater (1976) .

In 1977, a technical report for NASA by Szucet al. dealt with F100 engine multivariable control synthesis program. A complete package of control was provided to operate over the entire flight envelope of the engine. 30 Linear models for the complete operating envelope of the F100 were controlled by a package of 6 LQR designs. Integral action was combined with each design. A schedule for LQR and Integral gains in order to meet the operational conditions was proposed. Logic control was digitally implemented for engine protection and to define the maximum and minimum limits of the inputs to the control system. Testing by simulation of the system across the operating envelope was done and a satisfactory performance was reported (Szuc et al. 1977).

This great work by NASA known as Multivariable Control Synthesis (MVCS) program encouraged many engineers and control designers to initiate a forum in 1977 regarding alternatives for jet engine multivariable controller design. Fourteen papers were presented in this forum dealing with same model, as a theme problem. These papers were presented in a published book by Sain et al. (1978).The theme model was in respect of a Pratt and Whitney F100 after burning, turbo fan, twin spool engine with a low by-bass ratio and axial gas flow. Different types of multivariable controllers were employed. Seven papers used the British school methods, the rest were designed by the American techniques. Highlights of some papers, from each school follow.

From the British school, Kouvaritakis and Edmonds (1978) described designs using multivariable Root Locus and Characteristic Locus techniques. Rosenbrock and Munro (1977) suggested Inverse Nyquist Array methods to solve the control problem, meanwhile from the American school, Merrill (1978) proposed an output feedback regulator procedure to design a multivariable controller and compared it with LQR designs from the MVCS program (Merrill et al.,1982).

Optimal control theory expansions and extensions to LQG and $\mathcal{H}_\infty$ were used for the design of multivariable controllers for jet engines. In 1989, Dellinger and Alouani proposed designs of multivariable control system to GE-21 engine. Their design utilized Linear Quadratic Regulator (LQR) theory followed by Loop Transfer Recovery (LTR) to generate a robust Linear

Quadratic Gaussian (LQG) controller. Results showed that the response was quicker and better, than that proposed by Kapasouris (1984) for the same model (Dellinger and Alouani, 1989).

In addition, a two degree of freedom $\mathcal{H}_\infty$ control design, for Rolls Royce twin spool reheated turbo fan engine, was suggested by Samar and Postlethwaite (1994). In a recent paper by Hardt et al. described a numerical approach to $\mathcal{H}_2$ and $\mathcal{H}_\infty$ optimal nonlinear control applied on jet engine compressor (Hardt et al.2000).

Many other papers that dealt with multivariable control of gas turbine were presented using different techniques other than British and American ones. A recently developed technique, proposed for this dissertation, was first presented by Prof. R. Whalley in 2004.

A linear multivariable automotive gas turbine model was investigated by so-called Least Effort Control technique presented by Whalley and Ebrahimi (2004). The model presented a two-input two-output system. Fuel flow and nozzle area were inputs, while turbine gas inlet temperature and shaft speed were the outputs. The new factor employed in this technique is the minimum control effort strategy. The procedure first required reducing the complexity of the model in the frequency domain. Remote singularities were omitted to simplify the proposed model. An inner loop designed by Root Locus, in a way that the transfer function matrix is equivalent to a single transfer function. Optimization methods were employed to minimize a performance index comprising the control energy dissipation and absolute minimum gains are used to design the outer loop (Whalley and Ebrahimi , 2004).

2.5.5 Gas Turbine Multivariable Control Comparisons:

Comparisons between British and American schools methodologies are dependent on the application and the control system specifications. Each school and each technique has its own philosophy and approach in dealing with control systems with their advantages, disadvantages and tradeoffs. A review for some of control theorist's opinions and comparisons studied follows.

The philosophy of the British school is seen as an extension of classical control theory. The entire approach is dependent on frequency domain techniques. Direct input-output relations are built using the Laplace transformations of the linear differential equation, known as the transfer functions. Consequently, it is only applicable for time-invariant linear systems (Lewis,1992). The basic concept of Root Locus and Nyquist is to determine the closed loop performance in terms of open loop information. The extension of this concept to multivariable systems requires graphical investigations, by closing one loop at a time. This difficulty was overcome by two main techniques which are the Inverse Nyquist Array and Characteristic Locus methods. These graphical methods are presented as "true design methods, in which the engineer tries a controller, assesses the results, improves the design and tries again until satisfied. " (Dutton et al.,1997). Lewis described this design process as an engineering art. It proved that it can solve problems of interaction and disturbance suppression (Lewis,1992).

On the other hand, the American modern school's philosophy was entirely different. It was built on indirect mathematical relation between inputs and outputs through states in real time, as known as state space representations. This perspective resulted in a powerful tool of analysis when the essential properties of the system were investigated and its controllability and observability was determined. Also time variant systems could be represented easily. Noise and disturbances could be included into the control problem. The optimal control theory, based on state feedback provided a powerful tool for the synthesize of a stable controller. However, Rosenbrock and McMorran criticized the optimal control by describing it as "such control is usually not feasible, and if feasible it can have serious defects " (Resonbrock and McMorran , 1971). These defects were also argued by Dutton , Thompson and Barraclough who pointed out that state space methods use internal models of the system , and synthesize controllers using an algorithm based on these models. So modeling errors could result in poor control (Dutton et al., 1997). This poor robustness resulted in more researches developing $\mathcal{H}_\infty$ from LQG control. However, in aerospace applications, such as turbo jet engines, highly accurate models after extensive researches and tests over the entire operating envelope are achieved. Optimal control theory was said to be implemented by the aerospace industry where state access was often available.

In the field of gas turbine multivariable control, comparisons between the two schools were reported by NASA in 1977. The F100 engine's model presented a theme problem, and a final conclusion was widely understood. British school methods succeeded in developing control systems which were much simpler than those by American methods. However, this conjecture was not substantiated since all proposed designs were good only for one operating point, and not the entire envelope (Merrill et al., 1982).

The new proposed technique, based on Least Effort Control was compared to the classical British techniques such as Characteristic Locus and Inverse Nyquist Array methods in a paper presented by Whalley and Ebrahimi (2006). The application was for a hydraulic system with two inputs and two outputs. They showed that simpler controller could be determined and its closed loop response was comparable to INA and CL results. Moreover, a disturbance rejection analysis was provided to improve the system recovery characteristics. It was also shown that the procedure dissipated the least energy in achieving closed loop control, in comparison to all the designs considered (Whalley and Ebarahimi, 2006).

Another comparison between Least Effort Control and $\mathcal{H}_\infty$ was proposed by Whalley et al. (2010). The case study presented a deep mine shaft winder. They showed that Least Effort control resulted in a simple controller compared to the sixth order compensator arising from $\mathcal{H}_\infty$ control method. The control energy was minimized and good closed loop responses and disturbance recovery rates were achieved.

In this research, the Least Effort Control will be applied on a multivariable turbo jet engine model, introduced by Mueller (1967). Comparisons with Inverse Nyquist Array and Optimal Control presented by McMorran (1970) for the same model will be critically studied.

Mueller's model was used as a case study in many papers. McMorran (1970) applied INA and Optimal Control. Al-Assadi and Marouf (1988) proposed a computation of optimal controller parameters for the model. Tsay (2009) applied a sequential design method also on the model. Moreover, an algorithm for a PID controller, for the same model, was suggested recently by Suma and Appukuttan (2010).

2.6 Summary

The problem of jet turbine engine, multivariable control was reviewed in view of recent control theory developments. Basic concepts of feedback control and internal structure of such a system was defined by way of early steam engines. It was shown that jet turbine and early feedback system analogues could be formed. Servomechanism theory was introduced to integrate the understanding of feedback systems. Descriptions of open loop and closed loop responses were given and PID control law explanations were provided. Stability theory on the other hand was reviewed from a mathematical point of view. Routh-Hurwitz Criteria and Nyquist theory were presented. Root locus techniques were also explained. It was shown that this theory was sufficient to develop classical control methods. The historical view of early applications in World War II was described.

This presentation of single-input single-output (SISO) theory, which formed a basis for introducing the multivariable system problems, was considered. It was shown that multivariable control had two main philosophies. These were known as the British and American School approaches. A review for each one was given and a description of the main methodologies will be presented. The British School followed an extension of classical theory and provided the Inverse Nyquist Array and Characteristic Locus methods, based on frequency domain analysis. On the other hand, the American school presented the techniques of optimal control theory, based on time domain and state space analysis.

The link between gas turbine and control theory was achieved by reviewing the researches and work dealing with gas turbine control problem. This was sectioned into two connecting parts. First the modeling, then the control was identified. Linearization methods and system identification techniques were discussed. The complete scope of control then was clarified by defining the operating envelope. The application of control theory was reviewed according to a time line. From the early feedback system to classical control theory, then the two multivariable control schools followed by the latest methodology of Least Effort Control. Comparisons between different methodologies including difficulties, challenges, advantages and disadvantages were highlighted.

The problem in this research will be to present a multivariable control system design for a jet engine. The new proposed methodology of Least Effort Control will be employed and comparison with previously applications by one method from each school will be studied.

Chapter III

Research Methodology

3.1 Gas turbine Mathematical Model

In 1967, G. S. Mueller presented a linear model of a twin spool turbojet. A brief outline of his work will be derived here. The modeling according to Mueller was based on the theory of one-dimensional flow of gas; considering the thermodynamic processes such as compression, combustion and expansion, and the fluid dynamics such as the mixing of gas streams, area change, drag and friction and heat transfer. He summarized it by saying “The method is general, in that any gas turbine can be modeled providing its gas flow processes can be defined. “ (Mueller , 1971) .

Mueller assumed that the unbalanced torque on each compressor-turbine assembly; to be a function of low and high pressure spool speeds, the engine fuel flow rate, and the nozzle area. Then Newton's second law for rotating inertias could be applied resulting in :

$$\dot{N}_H = \frac{60 g_c}{2\pi I_H} T_H\left(N_H, N_L, A_J, Q_f\right) \qquad 3.1$$

$$\dot{N}_L = \frac{60 g_c}{2\pi I_L} T_L\left(N_H, N_L, A_J, Q_f, P_1\right) \qquad 3.2$$

In order to linearize these equations, expanding of right-hand side as a Taylor series, omitting second or higher degree terms and introducing perturbation variables yields:

$$\Delta\dot{N}_H = \frac{60g_c}{2\pi I_H}\left(\frac{\partial T_H}{\partial N_H}\Delta N_H + \frac{\partial T_H}{\partial N_L}\Delta N_L + \frac{\partial T_H}{\partial A_J}\Delta A_J + \frac{\partial T_H}{\partial Q_f}\Delta Q_f\right) \qquad 3.3$$

$$\Delta\dot{N}_L = \frac{60g_c}{2\pi I_L}\left(\frac{\partial T_L}{\partial N_H}\Delta N_H + \frac{\partial T_L}{\partial N_L}\Delta N_L + \frac{\partial T_L}{\partial A_J}\Delta A_J + \frac{\partial T_L}{\partial Q_f}\Delta Q_f + \frac{\partial T_L}{\partial P_1}\Delta P_1\right) \qquad 3.4$$

ΔA_J and ΔQ_f are actual inputs to the system ; however these quantities could not be changed instantaneously , Mueller suggested that demands for these variables would be more reasonable . He assumed that dynamics relating these to the demand values are the 1st order lags:

$$\Delta\dot{A}_J = -\frac{1}{\tau_A}\Delta A_J + \frac{1}{\tau_A}\Delta A_{J,d} \qquad 3.5$$

$$\Delta\dot{Q}_f = -\frac{1}{\tau_Q}\Delta Q_f + \frac{1}{\tau_Q}\Delta Q_{f,d} \qquad 3.6$$

Considering $\Delta A_{J,d}$ and $\Delta Q_{f,d}$ as the system inputs and ΔN_H and ΔN_L as system outputs, from equation 3.3, 3.4, 3.5 and 3.6, a state space representation of the system could be developed easily as:

$$\begin{bmatrix} \dot{N}_H \\ \dot{N}_L \\ \Delta\dot{A}_J \\ \Delta\dot{Q}_f \end{bmatrix} = \begin{bmatrix} k_1\frac{\partial T_H}{\partial N_H} & k_1\frac{\partial T_H}{\partial N_L} & k_1\frac{\partial T_H}{\partial A_J} & k_1\frac{\partial T_H}{\partial Q_f} \\ k_2\frac{\partial T_L}{\partial N_H} & k_2\frac{\partial T_L}{\partial N_L} & k_2\frac{\partial T_L}{\partial A_J} & k_2\frac{\partial T_L}{\partial Q_f} \\ 0 & 0 & -\frac{1}{\tau_A} & 0 \\ 0 & 0 & 0 & -\frac{1}{\tau_Q} \end{bmatrix} \begin{bmatrix} \Delta N_H \\ \Delta N_L \\ \Delta A_J \\ \Delta Q_f \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \frac{1}{\tau_A} & 0 \\ 0 & \frac{1}{\tau_Q} \end{bmatrix} \begin{bmatrix} \Delta A_{J,d} \\ \Delta Q_{f,d} \end{bmatrix} + \begin{bmatrix} 0 \\ d \\ 0 \\ 0 \end{bmatrix} \qquad 3.7$$

$$\begin{bmatrix} \Delta N_H \\ \Delta N_L \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta N_H \\ \Delta N_L \\ \Delta A_J \\ \Delta Q_f \end{bmatrix} \qquad 3.8$$

where $k_1 = \frac{60g_c}{2\pi I_H}$, $k_2 = \frac{60g_c}{2\pi I_L}$ and d may considered as a disturbance input to the system.

From experimental and theoretical results, the transfer function matrix in the Laplace domain, between high/ low pressure spool speed vector and nozzle area / fuel flow rate vector is :

$$\begin{bmatrix} \Delta N_H(s) \\ \Delta N_L(s) \end{bmatrix} = \begin{bmatrix} \frac{1.496}{s+2} & \frac{951.5(s+1.898)}{s^2+3.225s+2.525} \\ \frac{8.52}{s+2} & \frac{1240(s+2.037)}{s^2+3.225s+2.525} \end{bmatrix} \begin{bmatrix} \Delta A_J \\ \Delta Q_f \end{bmatrix} \qquad 3.9$$

Mueller clarified the numerical-model parameters derivation, first by experimental results at particular running condition by the identification of g_{11} and g_{21} by testing the nozzle area input, and second from the theoretically analysis with the same conditions to identify g_{12} and g_{22}.

Meanwhile, although the dynamics of the inputs which are the nozzle actuators and fuel pump should be at least of second order for acceptability, Mueller assumed time constants of 0.1 and 0.01 to represent adequately the relationship between demanded and actual variables as following:

$$\begin{bmatrix} \Delta A_J \\ \Delta Q_f \end{bmatrix} = \begin{bmatrix} \frac{10}{s+10} & 0 \\ 0 & \frac{100}{s+100} \end{bmatrix} \begin{bmatrix} \Delta A_{J,d} \\ \Delta Q_{f,d} \end{bmatrix} \qquad 3.10$$

Combining both equation 3.9 and equation 3.10 yields a direct input output relationship

$$\begin{bmatrix} \Delta N_H(s) \\ \Delta N_L(s) \end{bmatrix} = \begin{bmatrix} \frac{1.496}{s+2} & \frac{951.5(s+1.898)}{s^2+3.225s+2.525} \\ \frac{8.52}{s+2} & \frac{1240(s+2.037)}{s^2+3.225s+2.525} \end{bmatrix} \begin{bmatrix} \frac{10}{s+10} & 0 \\ 0 & \frac{100}{s+100} \end{bmatrix} \begin{bmatrix} \Delta A_{J,d} \\ \Delta Q_{f,d} \end{bmatrix} \qquad 3.11$$

And a block diagram of equation 3.11 is shown in figure 3.1.

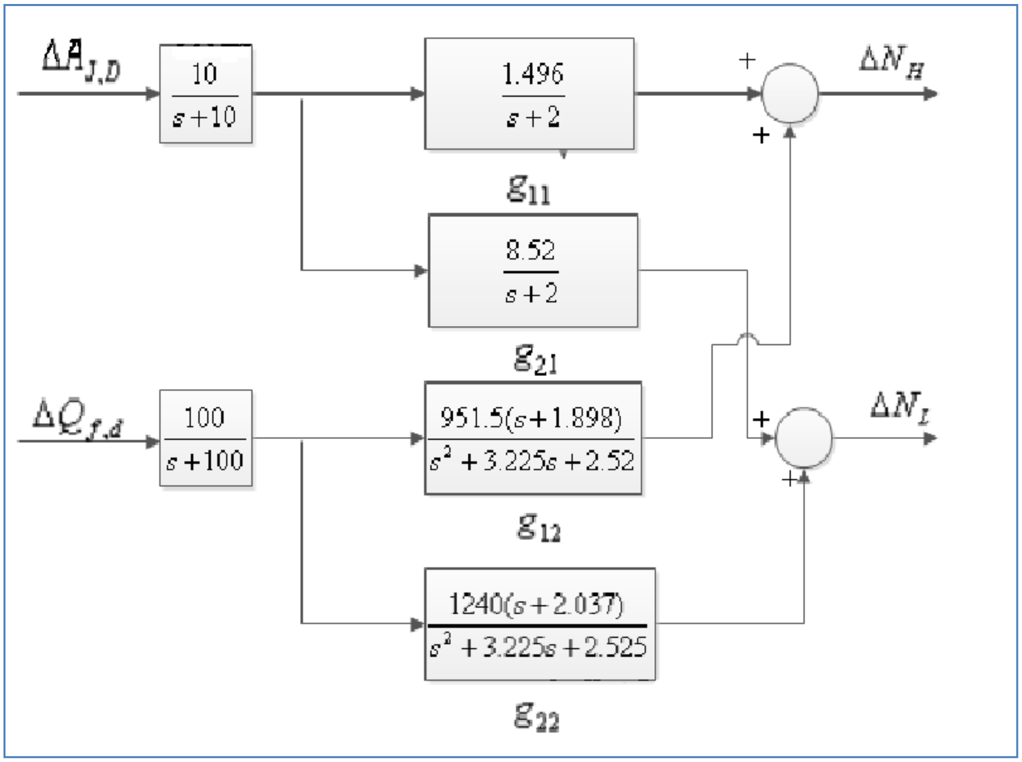


Figure 3.1,Block diagram for open loop transfer function matrix

State space representation of system is also obtained and numerical parameters for equation 3.7 are defined as following:

$$\begin{bmatrix} \dot{N}_H \\ \dot{N}_L \\ \Delta\dot{A}_J \\ \Delta\dot{Q}_f \end{bmatrix} = \begin{bmatrix} -1.268 & -0.04528 & 1.498 & 951.5 \\ 1.002 & -1.957 & 8.52 & 1240 \\ 0 & 0 & -10 & 0 \\ 0 & 0 & 0 & -100 \end{bmatrix} \begin{bmatrix} \Delta N_H \\ \Delta N_L \\ \Delta A_J \\ \Delta Q_f \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 10 & 0 \\ 0 & 100 \end{bmatrix} \begin{bmatrix} \Delta A_{J,d} \\ \Delta Q_{f,d} \end{bmatrix} + \begin{bmatrix} 0 \\ d \\ 0 \\ 0 \end{bmatrix} \qquad 3.12$$

Mueller recommended his model by saying: "This model is useful as a bench mark for multivariable regulator-design methods, since it is of low enough order to allow easy computation, and yet is sufficiently representative of the real plant." (Mueller,1971).

3.2 Open Loop Response Analysis and Control Objectives

From previous section, an open loop transfer function matrix is introduced in equation 3.11, and could be implemented directly in the simulation model, as shown in the Appendix, based upon the block diagram of figure 3.1. Then simulation following a unit step change on first input $\Delta A_{J,d}$, an open loop response of the system for the low and high pressure spool speeds is given in figure 1.6. Following a unit step change on second input$\Delta Q_{f,d}$, the simulation open loop response of the system is shown also in figure 3.3.

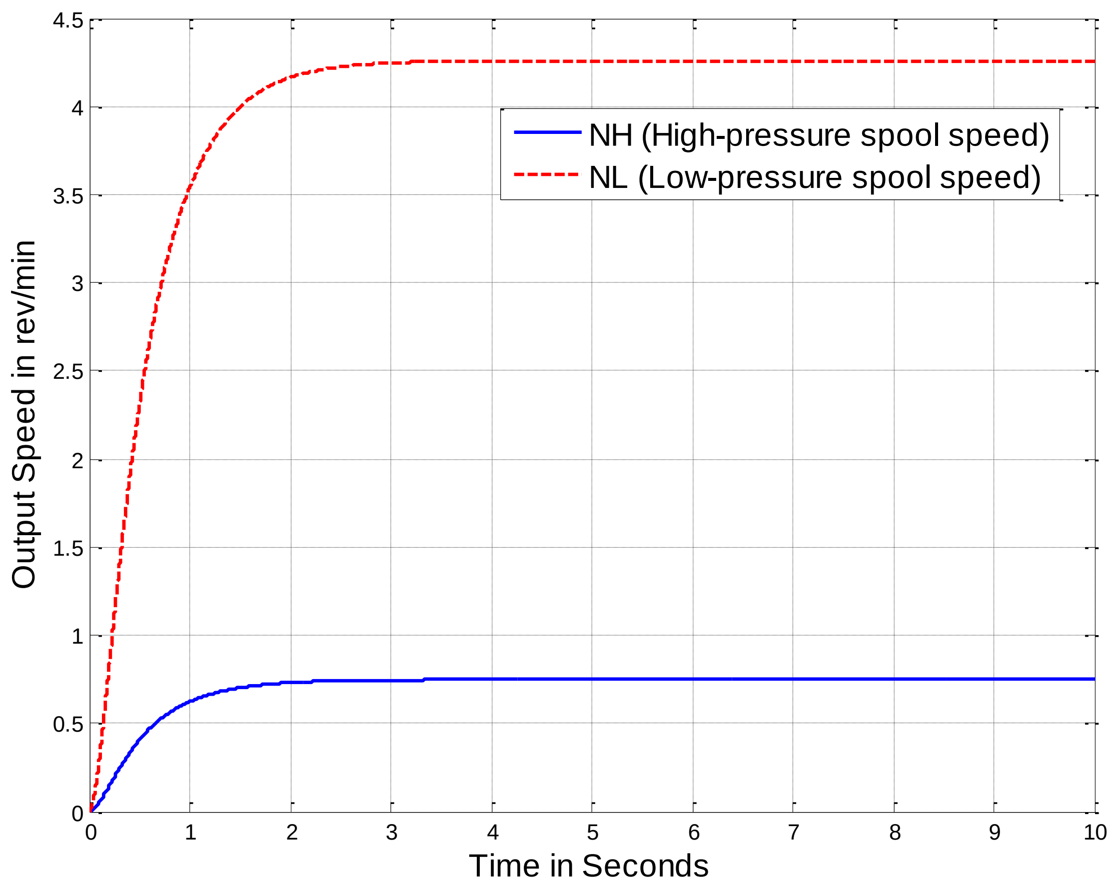


Figure 3.2,Open Loop Responses Following a Unit Step Change on $\Delta A_{J,d}$

It can be seen from figure 3.2 that this results in over damped behavior for both outputs, and a settling time of almost 3 seconds with large interaction of almost 570%, for the second output which is the low-pressure spool speed $\Delta N_L(s)$.

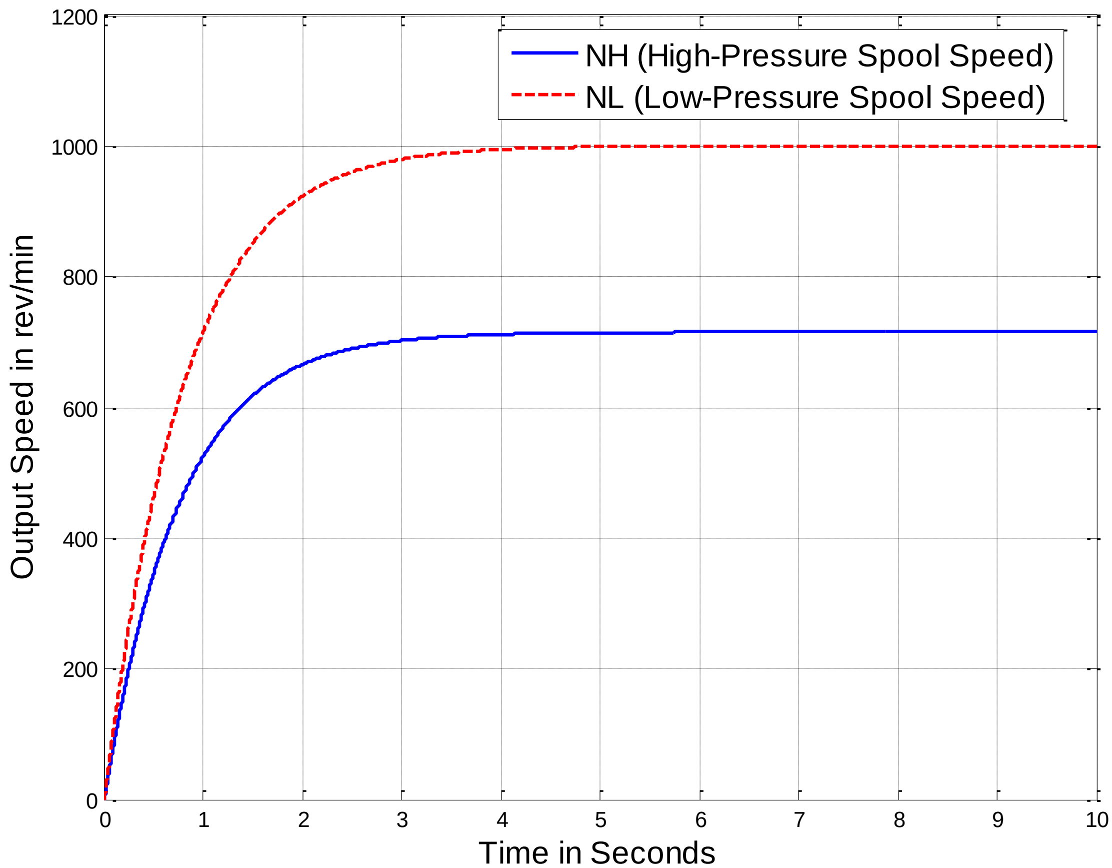


Figure 3.3, Open Loop Response Following a Unit Step Change on $\Delta Q_{f,d}$

Meanwhile it can be concluded from figure 3.3 that the results are also over damped for both outputs, a settling time about 4 seconds and about 75% interaction for the first output $\Delta N_H(s)$ is also acknowledged.

Disregarding the strong coupling in the system model, it is stable and well behaved. Therefore, the control strategy should focus on reducing coupling in the system as much as possible. In other words, altering any input of the system will not cause a significant change in the remaining outputs. A ten percent limit on the interaction between outputs, for any input changes, will be acceptable. Meanwhile, the closed loop response should be stable, and well behaved. Overshoots of less than twenty percent will be permissible. Speeding up system response is required, in other words settling time must be minimized. Since the system model represents a jet engine, random disturbances are expected. Air turbulence and changes in the aircraft flight mode (e.g. takeoff , cruise) are natural disturbances , not to mention the robustness of the system and any change in model parameters which could affect the response. Consequently, an investigation of disturbance recovery response is required. One more important issue to be investigated in this study is the energy consumed by the control action. Since the system model represents an aircraft, a limited source of power is available (e.g. Fuel). So that any approach to minimize fuel consumption should be encouraged in the design of the control system.

3.3 Least Effort Control Method

In the design procedure outlined here, a dual loop approach is adopted, where the inner loop will be utilized to secure acceptable stable dynamics, the outer loop and the pre-compensator will primarily be employed to satisfy disturbance-output conditions. The derivation of formula herein is in accordance with Whalley and Ebrahimi (2006).

If the open loop output equation in the Laplace Domain is

$$y(s) = G(s)u(s) + \delta(s) \quad 3.13$$

and the control law for the proposed feedback is

$$u(s) = k(s)[\bar{r}(s) - h(s)y(s)] + P(r(s) - Fy(s)) \quad 3.14$$

where in equations 3.13 and 3.14 there are m independent inputs, disturbances, and outputs and $F = Diag(f_j, f_j, \dots . f_j)$, $0 < f_j < 1$, $1 \leq j \leq m$

and the inner loop controller is

$$k(s)[\bar{r}(s) - h(s)y(s)]$$

will be used to satisfy the specified dynamic behavior of the closed loop system. While the outer loop controller is given by

$$P(r(s) - Fy(s))$$

will be utilized to secure the required steady state interaction and disturbance recovery.

with $\overline{r}(s) = 0$, the closed-loop equation becomes

$$y(s) = (I_m + G(s)(k(s) >< h(s) + PF))^{-1} \times (G(s)\,\mathrm{Pr}(s) + \delta(s)) \quad 3.15$$

where in equation 3.15, $\|G(s)(k(s) >< h(s) + PF)\|_\infty$ is finite for all s on the D contour.

If a steady-state matrix S_s is now selected such that

$$y(0) = S_s r(0)$$

Then from equation 3.15 for $s = 0$:

$$P = (G(0)^{-1} + k(0) >< h(0))S_s(I - FS_s)^{-1} \qquad 3.16$$

For steady state, decoupling the S_s matrix should be equal to the identity matrix ($S_s = I_m$). Usually, to achieve a low interaction, S_s would have diagonal elements of unity and off diagonal elements that are less than unity ;$|s_{i,j}| \ll 1, \quad 1 \leq m, \quad i \neq j$.

Consequently, substituting $S_s = I_m$ for P from equation 3.16, results in equation 3.15 to become:

$$y(s) = \{I_m + G(s)[k(s) >< h(s) + (G(0)^{-1} + k(s) >< h(s))(I_m - F)^{-1}F]\}^{-1} \times$$
$$\{G(s)\,\mathrm{Pr}(s) + \delta(s)\} \qquad 3.17$$

At low frequencies, as shown in the Appendix

$$G(s)P \cong \frac{1}{1-f}(I_m + G(s)k(0) >< h(0))$$

Consequently, equation 3.17, on approaching steady-state conditions becomes

$$y(s) = I_m r(s) + S(s)\delta(s) \qquad 3.18$$

where the low frequency sensitivity matrix

$$S(s) = (1 - f)(I_m + G(s)k(s) >< h(s))^{-1}, 0 < f < 1$$

Evidently, from equation 3.18, the steady-state non-interaction will be achieved. Moreover, as f is increased, $f < 1$, there will be increasing steady-state disturbance rejection , with imposition of the stability constraint.

For implementation purposes, a conventional multivariable regulator structure comprising a forwards path $K(s)$ and feedback path compensator $H(s)$ can easily be computed from equation 3.15.The input-output relation would be:

$$y(s) = (I_m + G(s)K(s)H(s))^{-1}[GK(s)r(s) + \delta(s)] \quad 3.19$$

On comparing equations 3.15 and 3.19, evidently

$$K(s) = P \quad 3.20$$

and

$$K(s)H(s) = k(s) >< h(s) + PF$$

hence

$$H(s) = P^{-1}k(s) >< h(s) + F \quad 3.21$$

enabling the employment of established feedback structures.

From equations 3.20 and 3.21, it is clear that the forward path compensator $K(s)$ and the feedback compensator $H(s)$ are constant, full rank $m \times m$ matrices, respectively. Moreover, the feedback matrix $H(s)$ is a stable, proper $m \times m$ minimum phase realization which could be easily constructed from passive elements.

“In view of the previous theory, the design strategy to be adopted here will be to adjust the inner-loop $k(s)$ and $h(s)$ vectors to provide ‘well behaved’ dynamic conditions. Thereafter, with the pre-compensator, configured to produce acceptable steady-state output coupling , the outer-loop feedback gain f becomes the ultimate design parameter enabling final dynamic and disturbance suppression characteristics to be achieved“ (Whalley and Ebrahimi ,2006).

- Inner Loop Analysis

The open loop system $G(s)$ is assumed to be an $m \times m$ linear, regular, proper, or strictly proper realization and admits a general factorization of:

$$G(s) = L(s)\frac{A(s)}{d(s)}R(s)\Gamma(s) \qquad 3.22$$

where $L(s), A(s), R(s), \Gamma(s)$, and the elements of $\frac{A(s)}{d(s)} \in H_{\infty},\ s \in \mathbb{C}$

In equation 3.22, $L(s)$ contains the left (row) factors of $G(s)$, while $R(s)$ contains the right (column) factors, and $\Gamma(s)$ contains the transformed finite time delays, such that the $m \times m$ matrices comprising equation 3.22 are

$$L(s) = Diag(\lambda_j(s)/p_j(s))$$

$$R(s) = Diag(\rho_j(s)/q_j(s))$$

$$\Gamma(s) = Diag(e^{-sT_j}), \qquad 1 \le i, j \le m$$

Noting that $A(s)$ is a non-singular matrix of rational functions, such that $\det A(s) \ne 0$, with elements

$$a_{ij}(s) = a_{ij}s^{m-1} + b_{ij}s^{m-2} + \cdots + \gamma_{ij} \qquad 1 \le i, j \le m \qquad 3.23$$

As the input-output-disturbance relationship is

$$y(s) = G(s)u(s) + \delta(s) \qquad 3.24$$

and if the inner-loop controller is

$$u(s) = k(s)[\bar{r}(s) - h(s)y(s)] \qquad 3.25$$

Then, combining equations (3.24) and (3.25) yields

$$y(s) = (I_m + G(s)k(s) >< h(s))^{-1}(G(s)k(s)\bar{r}(s) + \delta(s)) \qquad 3.26$$

The finite time delays in $\Gamma(s)$ could be ordered with $T_i \geq T_j$, $1 \leq j \leq m$, $i \neq j$, so that the forward path gain vector can be arranged as

$$k(s) = [k_1(s)e^{-s(T_i - T_j)}, k_2(s)e^{-s(T_i - T_j)}, \ldots.., k_1(s), \ldots\ldots, k_m(s)e^{-s(T_i - T_j)}]^T \qquad 3.27$$

Because

$$h(s) = (h_1(s), h_2(s), \ldots.., h_m(s)) \qquad 3.28$$

and if

$k_j(s) = k_j\emptyset_j(s)$ and $h_j(s) = h_j x_j(s)$ $\quad 1 \leq j \leq m$ where $\emptyset_j(s)$ and $x_j(s)$ are proper or strictly proper , stable , realizable , minimum phase functions , then they could be selected such that equation 3.26 becomes

$$y(s) = \left(I_m + e^{-sT_i}n(s)L(s)\frac{A(s)}{d(s)}k(s) >< h(s)\right)^{-1} \times \left(n(s)L(s)\frac{A(s)}{d(s)}ke^{-sT_i}r(s) + \delta(s)\right) \qquad 3.29$$

where

$$k = (k_1, k_2, \ldots.., k_m)^T \qquad 3.30$$

and

$$h = (h_1, h_2, \dots, h_m) \tag{3.31}$$

$$d(s) = s^k + a_1 s^{k-1} + \cdots + a_0$$

and

$$\deg\Big(n(s)a_{i,j}(s)\Big) < k \quad 1 \le i, j \le m$$

The determinant required in equation 3.29 is

$$\det\left[I_m + e^{-sT_i} n(s)L(s)\frac{A(s)}{d(s)}k(s) >< h(s)\right] = 1 + e^{-sT_i} n(s) < h\frac{A(s)}{d(s)}k > \tag{3.32}$$

The inner product in equation 3.32, may be expressed as

$$< hA(s)k >= [1, s, \dots, s^{m-1}] \begin{bmatrix} \gamma_{11} & \gamma_{12} & \cdots & \gamma_{mm} \\ \vdots & \vdots & \ddots & \vdots \\ b_{11} & b_{12} & \cdots & b_{mm} \\ a_{11} & a_{12} & \cdots & a_{mm} \end{bmatrix} \begin{bmatrix} k_1 h_1 \\ k_2 h_1 \\ \vdots \\ k_m h_m \end{bmatrix} \tag{3.33}$$

If in equation 3.33, the gain ratios are

$$k_2 = n_1 k_1, k_3 = n_2 k_1, \dots . k_m = n_{m-1} k_1 \tag{3.34}$$

and

$$< hA(s)k >= b(s) \tag{3.35}$$

Then equation 3.35 implies that

$$k_1[Q]h = (b_{m-1}, b_{m-2}, \dots, b_0)^T \tag{3.36}$$

where

$$Q = \begin{bmatrix} \gamma_{11} + \gamma_{12} n_1 + \gamma_{1m} n_{m-1} & \vdots & \gamma_{21} + \gamma_{22} n_1 + \gamma_{2m} n_{m-1} & \vdots & \cdots & \gamma_{m1} + \gamma_{m2} n_1 + \gamma_{mm} n_{m-1} \\ \vdots & & \vdots & & & \vdots \\ b_{11} + b_{12} n_1 + b_{1m} n_{m-1} & \vdots & b_{21} + b_{22} n_1 + b_{2m} n_{m-1} & \vdots & \cdots & b_{m1} + b_{m2} n_1 + b_{mm} n_{m-1} \\ a_{11} + a_{12} n_1 + a_{1m} n_{m-1} & \vdots & a_{21} + a_{22} n_1 + a_{2m} n_{m-1} & \vdots & \cdots & a_{m1} + a_{m2} n_1 + a_{mm} n_{m-1} \end{bmatrix}$$

and b_j, $0 \leq j \leq m-1$, are the coefficients of $b(s)$, given in equation 3.35 Providing the weak constraint , that $n_1, n_2, \ldots, n_{m-1}$ can be selected in equation 3.36 so that the matrix is invertible , then a unique solution for $(h_1, h_2, \ldots, h_m)k_1$ exists.

Selecting a suitable $b(s)$ function and gain ratios $n_1, n_2, \ldots, n_{m-1}$,an improvement in the closed-loop dynamics arising from equation 3.28 could be achieved. If equation 3.36 is solved, then the vector h can be computed once an arbitrary value for k_1 has been decided. Moreover, the inner-loop design procedure is equally valid for non-square system models.

- Optimization

An approach for designing closed-loop systems, using the transfer function matrix and output measurements alone has been introduced, now the optimization of this process could be considered. An indication for that the freedom to do this exist arises from the arbitrary choice of $n_1, n_2, \ldots, etc$ for the gain ratios.

To detect the absolute minimum control effort required, for disturbance suppression, under closed-loop conditions, a performance index representing the energy consumed should be defined. This should exploit the arbitrary choice, gain selection in designing the closed loop system. Then an optimization routine could be determined to find the minimum of that performance index against the gain ratios.

The control effort, at time t is proportional to

$$(|k_1h_1| + |k_2h_1| + \cdots |k_mh_1|)|y_1(t)| + (|k_1h_2| + |k_2h_2| + \cdots |k_mh_2|)|y_2(t)| + \cdots$$
$$+ (|k_1h_m| + |k_2h_m| + \cdots |k_mh_m|)|y_m(t)|$$

Hence, the control energy costs, under these conditions, are proportional to

$$E(t) = \int_{t=0}^{t=T_f} \left(\sum_{i=1}^{m} k_1^2 \sum_{j=1}^{m} h_j^2 y_j^2(t)\right) dt \qquad 3.37$$

Then for arbitrary changes in the transformed output vector $y(t)$, following arbitrary disturbance changes

$$J = \sum_{i=1}^{m} k_1^2 \sum_{j=1}^{m} h_j^2 \qquad 3.38$$

would minimize the required control energy, given by equation 3.37 . If the relationships

$$k_2 = n_1k_1 , k_3 = n_2k_1 \ldots\ldots k_m = n_{m-1}k_1$$

are adopted , then equation 3.38 can be written as

$$J = (k_1)^2(1 + n_1^2 + n_2^2 + \cdots + n_{m-1}^2) \times (h_1^2 + h_2^2 + \cdots + h_m^2) \qquad 3.39$$

and $h_1^2 + h_2^2 + \cdots + h_m^2 < h, h >$. The closed-loop determinant is given by equation 3.32 with the inner product equated to $b(s)$, as in equation 3.35 then from equation 3.36

$$h = k_1Q^{-1}b \qquad 3.40$$

Upon substituting for h from equation (3.40), equation (3.39) becomes

$$J = (1 + n_1^2 + n_2^2 + \cdots + n_{m-1}^2)b^T(Q^{-1})^TQ^{-1}b \qquad 3.41$$

To find the minimum value for J, assuming , for example , that $m = 3$

$$J = (1 + n_1^2 + n_2^2) b^T (Q^{-1})^T Q^{-1} b$$

where J is minimized when

$$\frac{\partial J}{\partial n_1} = 0\,, \frac{\partial J}{\partial n_2} = 0\,, and \frac{\partial^2 J}{\partial n_1^2} \frac{\partial^2 J}{\partial n_2^2} - \left(\frac{\partial^2 J}{\partial n_1 \partial n_2} \right)^2 > 0\,, if \frac{\partial^2 J}{\partial n_1^2} > 0$$

For $m > 3$, a numerical optimization routine could be employed to establish the values of $n_1, n_2 \dots, n_{m-1}$ which minimize J. This completes the inner-loop analysis.

- Disturbance Rejection Analysis

The employment of the minimum control effort would not, in general, achieves disturbance recovery conditions. To achieve that, the outer-loop feedback gain f could be manipulated between $0 < f < 1.0$, as indicated by equation 3.18. For further analysis see Whalley and Ebrahimi (2006).

❖ Stability Of Combined System

The stability of the combined system includes inner and outer loop should be investigated. Inner loop is designed by root locus method, giving a great chance to stabilize systems in closed loop. If the inner loop is stable, the outer loop will be also stable but with conditions. The stability condition is dependent on the denominator of the input-output relationship for the complete, closed-loop system which is given by equation 3.19. Consequently, the outer –loop feedback gain matrix F is given by

$$F = Diag(f_j, f_j, \dots . f_j)\,, \quad 0 < f_j < 1 \quad, \quad 1 \le j \le m$$

Then the denominator of equation 3.19 may be computed from

$$det\left\{I_m + G(s)\left[\frac{k(s) >< h(s)}{(1-f)} + \frac{G(0)^{-1}f}{(1-f)}\right]\right\}$$

From this expression, it is clear that the elements of the feedback-compensator matrix $\left[\frac{k(s)><h(s)}{(1-f)} + \frac{G(0)^{-1}f}{(1-f)}\right]$ become infinite as $f \to 1$. In other words, this would always result in closed-loop system instability.

Moreover, confirmation of the relative stability for the closed-loop system can be easily computed using the characteristic locus method, presented by Macfarlane. This is evident from the establishment of the pre- and feedback compensator in equation 3.20 and 3.21 and hence, the conventional multivariable feedback structure, indicated by equation 3.19 (Whalley and Ebrahimi , 2006).

3.4 Inverse Nyquist Array Method

The British well known multivariable systems controller design methodology known as the Inverse Nyquist Array (INA) method was developed by Rosenbrock (1969). An outline of its mathematical foundations and design procedure will be highlighted.

The multivariable system is represented by $m \times m$ transfer function matrix $G(s)$, which is assumed to be controlled by a $m \times m$ transfer function matrix $K(s)$. Closing m feedback loops is resulting in the block diagram in figure 3.4.

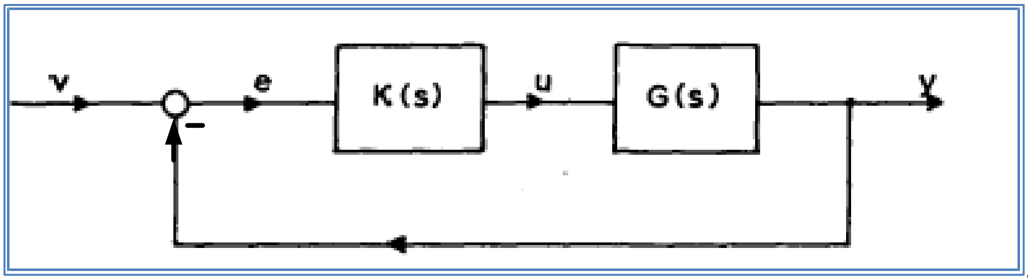


Figure 3.4, General form of multivariable control system

(Rosenbrock, 1969)

Many assumptions are necessary before applying INA method. First, all elements of $K(s)$ and $G(s)$are rational polynomial functions in the transform s and both matrices are invertible. Second, the system is open loop stable. Third, the controller matrix $K(s)$ elements are expected to be stable compensators (all its zeroes and poles are in the open left half (s)-plane), which induce diagonal dominance and reduce the system output coupling. SISO controllers could be then employed for each loop independently, to give the required closed loop dynamics.

The open loop transfer function matrix is

$$Q(s) = G(s).K(s) \quad 3.42$$

For notation, inverted matrixes will be denoted by

$$G^{-1} = \hat{G}\,, K^{-1} = \hat{K}\ ,\ Q^{-1} = \hat{Q} \quad 3.43$$

and the elements of the inverses could be written $\hat{q}_{i,j}$…. etc

From figure 3.4, it can be seen that

$y = GKe = Q(v - y)$ 3.44

So that the closed loop transfer function matrix is

$H(s) = \{I_m + Q(s)\}^{-1}Q(s)$ 3.45

and

$\widehat{H}(s) = I_m + \widehat{Q}(s)$ 3.46

For design purposes, a more general expression for equation 3.46 could be defined. Let F be a matrix when all of its elements are zeroes except for unity on the principal diagonal, for each closed loop function. Then from figure 3.43 :

$y = GKe = Q(v - Fy)$ 3.47

So that the transfer function matrix relating output to input is

$$R = (I_m + QF)^{-1}Q$$

and so

$R^{-1} = \widehat{R} = F + \widehat{Q}$ 3.48

when $= I_m$, $\widehat{R} = \widehat{H}$ while if $F = 0, \widehat{R} = \widehat{Q}$

Rosenbrock states that "The Inverse Nyquist Array (INA) is the set of m^2diagrams representing the elements $\hat{q}_{ij}(j\omega) of\ \widehat{Q}_{ij}(j\omega)$. " (Rosenbrock, 1969).

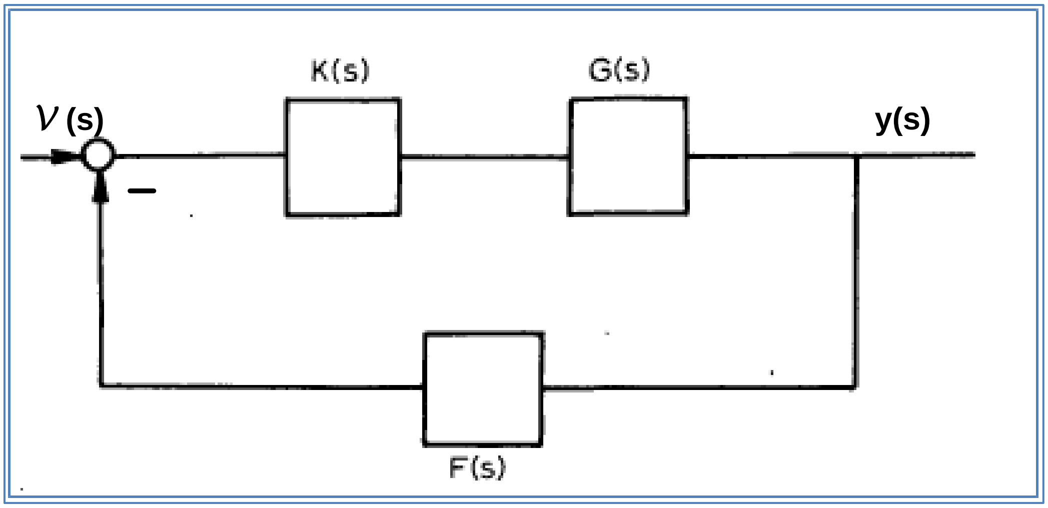


Figure 3.5, Closed Loop Transfer Function Matrix with Feedback

(Hawkins and McMorran , 1973)

These sets of diagrams are used for the investigation of the so called diagonal dominance of open loop transfer function matrix Q. An $m \times m$ rational matrix $\hat{Q}(s)$ is row diagonal dominant on the Nyquist D-contour if

$$|\hat{q}_{ii}(s)| > \sum_{j=1\,,j\neq i}^{m} |\hat{q}_{ij}(s)| \qquad 3.49$$

Column dominance is investigated in a way similar to equation 3.49, For $i = 1, \dots., m$ for all s on D-contour, the diagonal dominance of $\hat{Q}(s)$ is investigated graphically by superimposing a set of Gershgorin circles (1932) ,with center on the diagonal elements at radius

$$d_i(s) = \sum_{j=1\,,j\neq i}^{m} |\hat{q}_{ij}(s)|$$

If each of the bands generated by theses circles excludes the origin at any frequency; then $\hat{Q}(s)$ is row (or column) dominant (Munro, 1972). As shown in figure 3.44 , the Inverse Nyquist

Diagram of $\hat{q}_{ii}(s)$ and all union of bands generated by Gershgorin circles exclude the origin at all frequencies , resulting in row (or column) dominance.

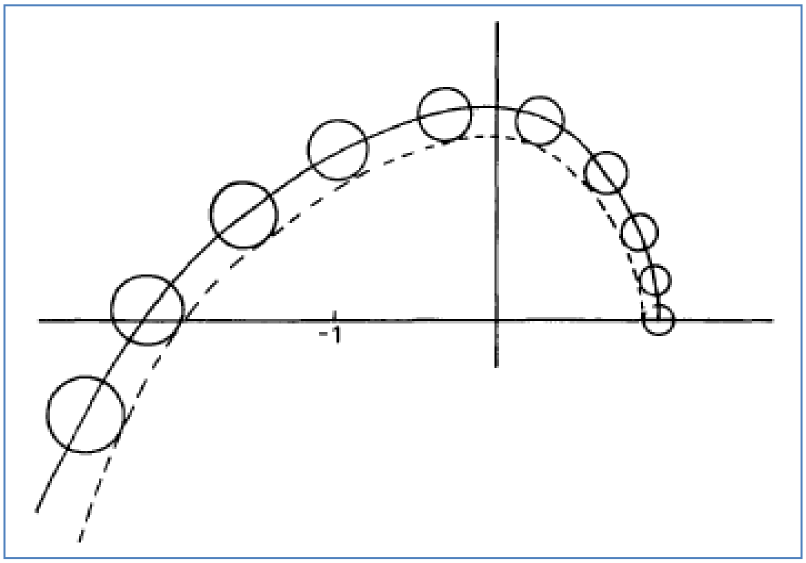


Figure 3.6, Inverse Nyquist Diagram of $\hat{q}_{ii}(s)$

(Hawkins and McMorran , 1973)

❖ Stability

Rosenbrock had shown that stability of the overall system could be given by the generalized Nyquist Theorem.

$$\sum_{i=1}^{m} \hat{N}_{q_i} - \sum_{i=1}^{m} \hat{N}_{h_i} = z_o - z_c \qquad 3.50$$

where $\hat{N}_{q_i}$ and $\hat{N}_{h_i}$ are the number of origin encirclements; by the mapping of the D-contour for $\hat{q}_{ii}$and $\hat{h}_{ii}$, respectively, while z_o and z_c are the number of right-half s-plane zeroes of the open and closed loop system , respectively.

Once diagonal dominance is achieved for the open loop system by the pre-compensator $K(s)$, each loop could be controlled independently from the other. The diagonal element in controller $k_d(s)$ could be designed to control an ith loop using classical single-loop theory such as Nyquist, Root Locus, PID … etc.

$$h_i^{-1}(s) = h_{ii}^{-1}(s) - f_i \qquad 3.51$$

where $h_i^{-1}(s)$ is the inverse transfer function which is seen in the iith loop open, and all other loops are closed. While $h_{ii}^{-1}(s)$ is the inverse transfer function, which is seen between input and output i, with all loops closed. It can be shown that equation 3.51 is contained within the band swept out by the Gershgorin circles, and this is true for all feedback gain values f_i in each o loop i between zero and the design value (Munro, 1972).

Once feedback gains f_i have been selected, so that closed loop stability is achieved in terms of the larger Gershgorin bands; the region of uncertainty could be reduced by drawing Ostrowski bands, which are defined by a set of circles centered on $\hat{q}_{ii}(s)$ with radii

$$r_i(s) = \emptyset_i(s) d_i(s)$$

where

$$\emptyset_i(s) = max \frac{d_j(s)}{|\hat{q}_{ii}(s)| + f_j} \qquad i = 1,2, \dots . m \text{ , } j = 1,2, \dots m \text{ , } i \neq j \qquad 3.52$$

This could be used to expand stability region and select higher feedback gains f_i to improve closed loop response.

3.5 Optimal Control Method

Kalman presented a new mathematical expression for the system dynamics known as the state space representation (Kalman, 1960).

For linear, continuous and time invariant system

$$dx/_{dt} = \dot{x}(t) = Ax(t) + Bu(t) \quad 3.53$$

$$y(t) = Cx(t) + Du(t) \quad 3.54$$

where x is a $n \times 1$ state vector ,$u(t)$ is a $m \times 1$ input vector and $y(t)$ is a $r \times 1$ output vector, where A is $n \times n$ matrix , B is $n \times m$ matrix , C is a $r \times n$ matrix and D is $r \times m$ matrix.

For transforming a system from state space to Laplace domain equation 3.53 becomes

$$\mathcal{L}\{\dot{x}(t)\} = sx(s) + sx(0) = Ax(s) + Bu(s) \quad 3.55$$

Let initial conditions $x(0) = 0$,

Then $x(s) = (sI - A)^{-1}Bu(s)$ 3.56

$$\mathcal{L}\{y(t)\} = y(s) = Cx(s) + Du(s) \quad 3.57$$

and for a strictly proper system ,$D = 0$, then substituting 3.56 in 3.57 yields

$$y(s) = Cx(s) = C(sI - A)^{-1}Bu(s)$$

Then a direct input-output relationship which is known as the transfer function matrix $G(s)$ is given by

$$G(s) = \frac{y(s)}{u(s)} = C(sI - A)^{-1}B \quad 3.58$$

This new perspective of the systems, led to view the system characteristic behavior in more general form. Eigen values of matrix A which is computed from roots of $\det(sI - A)$ presents a general form of system poles in Laplace domain. Cancellation of some modes (poles and zeroes) that may occur when transforming from state space to Laplace domain led to investigation of new properties of the system known as Controllability and Observability.

$\Phi(s) = (sI - A)^{-1}$is the Laplace transform of $\Phi(t) = \exp(At)$ which is called the state transition matrix .

Kalman defined Controllability by saying " A state x of a plant is said to be controllable if there exists a control signal $u_1(t)$, defined over a finite interval $0 \ll t \ll t_1$ such that $\Phi(t_1; x, 0) = 0$, in general , the time t_1, will depend on x . If every state is controllable, then the plant (system) is said to completely controllable " (Kalman, 1960). In other words, if there an unconstrained control signal that could transfer any initial state to any desired location in a finite time, if the state is controllable.

Controllability could be investigated by checking the rank of a special matrix $\mathbb{C}$:

$$\mathbb{C} = [B\ AB\ A^2B \dots .\ A^{n-1}B]$$

If $\mathbb{C}$ is full rank (n) , then the system is completely controllable.

On the other hand, a system is said to be completely observable if only and only if there exists a finite time t_2 , such that an initial state $x(0)$ could be determined from the measurement of the output signal $y(t)$given the control $u(t)$in a time interval $0 \ll t \ll t_2$ (Dorf and Bishop , 1993)

Observability could be investigated by checking the rank of a special matrix Θ :

$\Theta = [C^T C^T A^T \dots . C^{T^2} A^{T^{n-1}}]$

If Θ is full rank (n) , then the system is completely observable .

Then a full state feedback design is proposed for closed-loop system control. The first assumption is that all states are controllable, observable and measurable. That means that the full state vector $x(t)$ is well defined. Then a control law depending on state feedback, given by

$$u(t) = -Kx(t) \qquad 3.59$$

could be defined for closed loop control, substituting equation 3.59 in equation 3.53 yields ,

$$\dot{x}(t) = Ax(t) - BKx(t) = (A - BK)x(t) \qquad 3.60$$

Then the system behavior depends on the determinant of the characteristic equation:

$$\det(\lambda I - (A - BK)) = 0 \qquad 3.61$$

If the roots of the equation 3.61 are in the left hand complex plane, then the system is stable in closed loop. So given the matrices A and B , the design problem is to find the matrix K that will place the poles of the closed loop system in a suitable (s)-plane location. As shown in block diagram in figure 3.7.

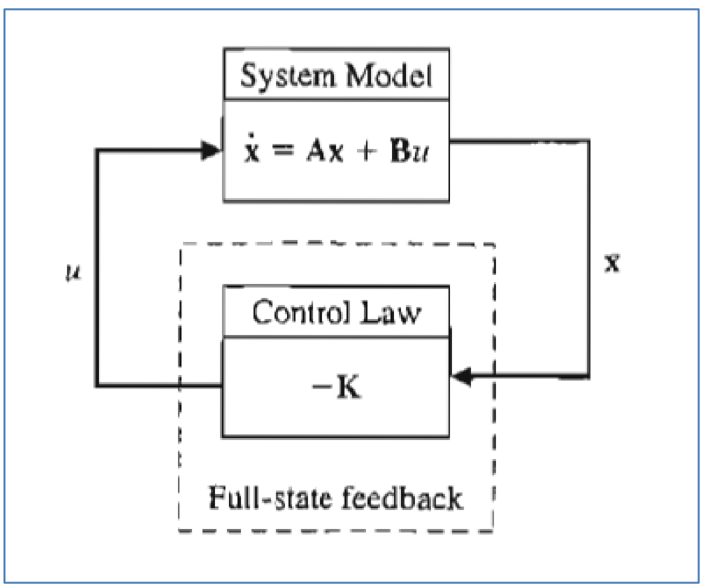


Figure 3.7, Full State Feedback Control

(Dorf and Bishop , 1993)

In case that not all of the states are measurable, which is common in many systems such as gas turbine engines; an observer could be designed to estimate the state variables which are not measurable.

According to Luenberger (1964), the full order observer estimating the whole state vector as shown in figure 3.54, is given by

$$\dot{\bar{x}}(t) = A\bar{x}(t) + Bu(t) + L(y(t) - C\bar{x}(t)) \quad 3.62$$

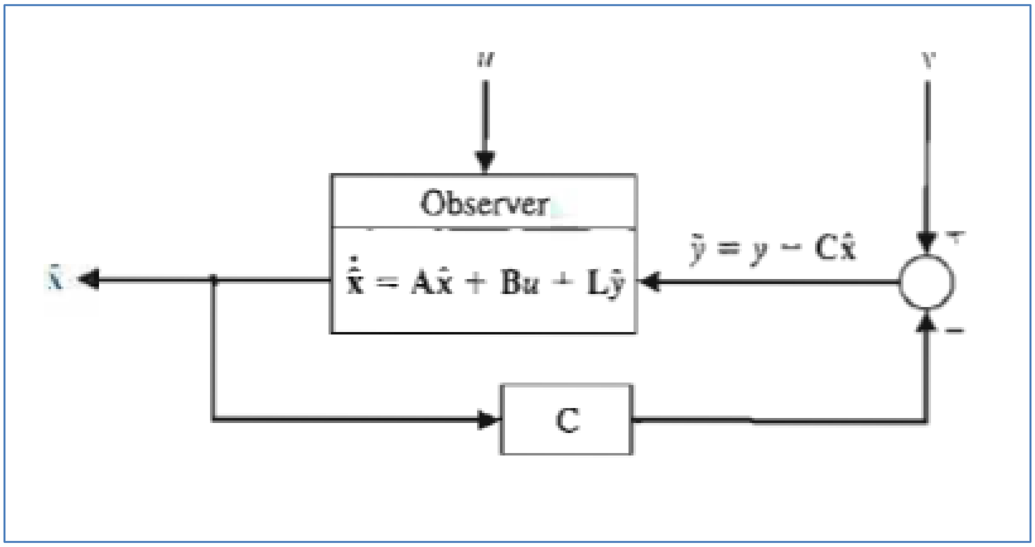


Figure 3.8, Full State Feedback Observer Block Diagram

(Dorf and Bishop , 1993)

Where $\bar{x}(t)$denotes the estimated state vector, and L the observer gain matrix that will be determined in the design procedure. In order to do so, some analysis is required. The estimation error could be given by

$$e(t) = x(t) - \bar{x}(t) \quad 3.63$$

The observer design requires that the error is eliminated when $t \to \infty$, $(t) \to 0$. Now if the system is completely observable, an observer gain matrix L could be found so that the tracking error is asymptotically stable. Derivation with respect to time of equation 3.63 yields

$$\dot{e}(t) = \dot{x}(t) - \dot{\bar{x}}(t) \quad 3.64$$

Then substituting equation 3.5.1 and equation 3.5.10 in equation 3.5.12 results in

$$\dot{e}(t) = Ax(t) + Bu(t) - A\bar{x}(t) - Bu(t) - L(y(t) - C\bar{x}(t))$$

$$\dot{e}(t) = (A - LC)e(t) \quad 3.65$$

To guarantee that $e(t) \to 0$ when $t \to \infty$, the roots of the characteristic equation of 3.65, given by $\det\big(\lambda I - (A - LC)\big) = 0$ 3.66

should be located in the left hand, complex plane . This is could be achieved by a proper design of the observer gain matrix.

The observer and the control law could be arranged in many ways to control a linear system by state feedback. First, the control law will be

$$u(t) = -K\bar{x}(t) \quad 3.67$$

then substituting this law in equation 3.62 yields ,

$$\dot{\bar{x}}(t) = A\bar{x}(t) - BK\bar{x}(t) + L\big(y(t) - C\bar{x}(t)\big)$$

$$\dot{\bar{x}}(t) = (A - BK - LC)\bar{x}(t) + Ly(t) \quad 3.68$$

The equation 3.68 is representing the compensator, which consists of the full order observer and the controller, as shown in the block diagram of figure 3.9.

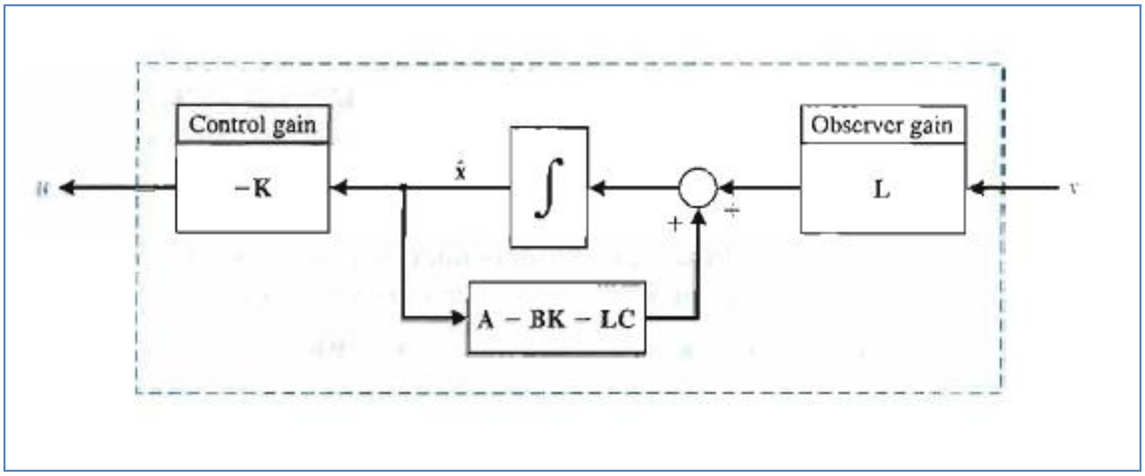


Figure 3.9, Full Order Observer and Controller Block Diagram

(Dorf and Bishop , 1993)

Recall equation 3.65

$$\dot{e}(t) = (A - LC)e(t)$$

and substituting the control law equation 3.67 in the system model equation 3.53 yields

$$\dot{x}(t) = Ax(t) - BK\bar{x}(t) \qquad 3.69$$

Following a substitution of $\bar{x}(t) = x(t) - e(t)$ in equation 3.69 results in

$$\dot{x}(t) = Ax(t) - BKx(t) + BKe(t) = (A - BK)x(t) + BKe(t) \qquad 3.70$$

Writing equations 3.70 and 3.65 in matrix form yields

$$\begin{bmatrix} \dot{x}(t) \\ \dot{e}(t) \end{bmatrix} = \begin{bmatrix} A - BK & BK \\ 0 & A - LC \end{bmatrix} \begin{bmatrix} x(t) \\ e(t) \end{bmatrix} \qquad 3.71$$

The characteristic equation of 3.71 is

$$det\big(\lambda I-(A-BK)\big).\det\big(\lambda I-(A-LC)\big)=0 \qquad 3.72$$

So the stability of the overall closed system depends on the poles of the observer and the system structure itself. This is achievable by the separation principle of designing the gain matrices K and L independently. By long experience, it is recommended to place the poles of the observer ten times faster than the system poles, in order to that, the observer dynamics are quicker than the system dynamics, giving a sufficient time to eliminate the estimation error $e(t)$.

The compensator in equation 3.68, which is shown in figure 3.9, could be realized as a transfer function by Laplace transformation.

$$\mathcal{L}\{\dot{\bar{x}}(t)=(A-BK-LC)\bar{x}(t)+Ly(t)\}=s\bar{x}(s)-\bar{x}(0)=(A-BK-LC)\bar{x}(s)+Ly(s)$$

Putting initial conditions $\bar{x}(0)=0$ and rearranging results in

$$\bar{x}(s)=(sI-\{A-BK-LC\})^{-1}Ly(s) \qquad 3.73$$

Also Laplace transformation applied on the control law yields

$$\mathcal{L}\{u(t)=-K\bar{x}(t)\}=u(s)=-K\bar{x}(s) \qquad 3.74$$

Then substituting equation 3.73 into equation 3.74 yields,

$$u(s)=-K(sI-\{A-BK-LC\})^{-1}Ly(s)$$

The compensator transfer function would be

$$C(s) = \frac{y(s)}{u(s)} = \frac{1}{-K(sI - \{A - BK - LC\})^{-1} L} \qquad 3.75$$

Many arrangements for the compensator could be implemented to control the system; the one proposed for this research, is shown in figure 3.10, where the observer is driven by the tracking error between the reference input and the output.

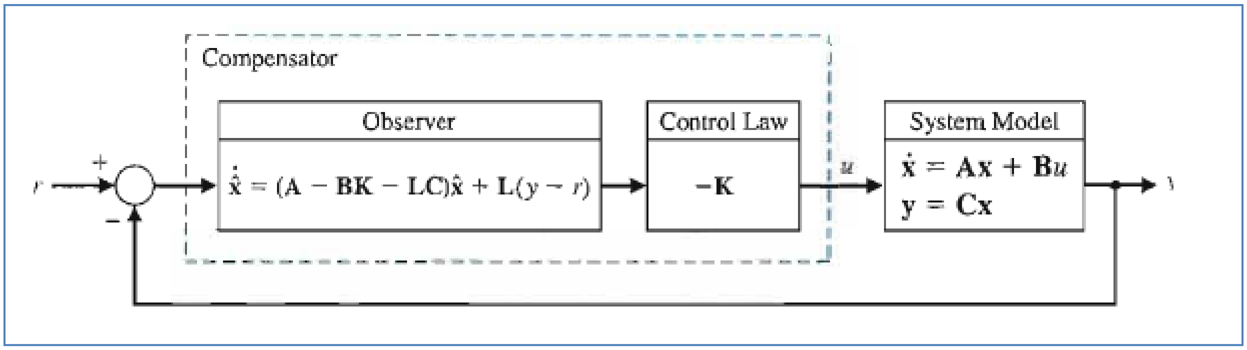


Figure 3.10,Compensator Arrangement Block Diagram

(Dorf and Bishop, 1993)

The observer gain matrix L, could be designed by poles assignment technique. An algorithm to accomplish this is programmed in Matlab Software. Meanwhile, to find a control law for the estimated state feedback; an optimal control is proposed to synthesize a gain matrix K. The performance index is given by

$$J = \int_0^\infty (x^T Q x + u^T R u) dt \qquad 3.76$$

where Q $(n \times n)$ and $R(m \times m)$ are the weighting matrices of the state vector and the inputs vector, respectively. These should be positive definite or semi definite symmetric matrices, selected arbitrarily by the designer. Only a systematic trial and error approach could be employed to synthesize a controller, minimizing this performance index thereby.

An alternative index which is adopted in this research, where the state vector is replaced by output vector is given by

$$J = \frac{1}{2}\int_0^\infty (y^T Q y + u^T R u)dt \qquad 3.77$$

and the gain matrix K is determined by

$$K = R^{-1} B^T P \qquad 3.78$$

where P is the solution of the Riccati Matrix Equation given by

$$A^T P + PA - PBR^{-1}B^T P + Q = 0 \qquad 3.79$$

This is called a Linear Quadratic Regulator (L.Q.R.) controller. Numerical Algorithms were developed and adopted in Matlab software for solving the equation 3.79.

Macfarlane (1970) proved that optimal control is always stable. He investigated the stability of the optimal system in the frequency domain, and showed that the Nyquist frequency response never penetrates the unity disc, as shown in figure 3.11.

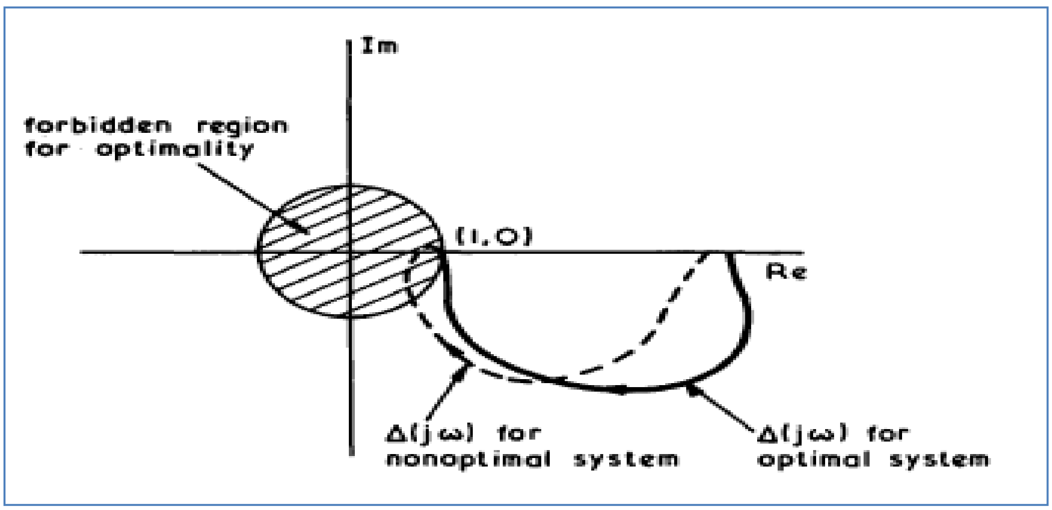


Figure 3.11, Frequency Response of Optimal Control System
(Macfarlane, 1970)

Chapter IV

Simulation Results and Discussion

4.1 Least Effort Controller

According to the methodology presented in the last chapter, a direct implementation of will be presented here.

The Transfer Function matrix representing the gas turbine model, according to equation 3.11 is

$$G(s) = \begin{bmatrix} \dfrac{1.496}{s+2} & \dfrac{951.5(s+1.898)}{s^2+3.225s+2.525} \\ \dfrac{8.52}{s+2} & \dfrac{1240(s+2.037)}{s^2+3.225s+2.525} \end{bmatrix} \begin{bmatrix} \dfrac{10}{s+10} & 0 \\ 0 & \dfrac{100}{s+100} \end{bmatrix}$$

To put it in the form of equation 3.10 which is $G(s) = L(s)\frac{A(s)}{d(s)}R(s)\Gamma(s)$, the transfer function matrix could be multiplied by $\begin{bmatrix} \frac{s+10}{s+100} & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} \frac{s+100}{s+10} & 0 \\ 0 & 1 \end{bmatrix} = 1$

$$G(s) = \begin{bmatrix} \dfrac{1.496}{s+2} & \dfrac{951.5(s+1.898)}{s^2+3.225s+2.525} \\ \dfrac{8.52}{s+2} & \dfrac{1240(s+2.037)}{s^2+3.225s+2.525} \end{bmatrix} \begin{bmatrix} \dfrac{10}{s+10} & 0 \\ 0 & \dfrac{100}{s+100} \end{bmatrix} \begin{bmatrix} \dfrac{s+10}{s+100} & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \dfrac{s+100}{s+10} & 0 \\ 0 & 1 \end{bmatrix}$$

Multiplying the first three matrices and rearranging in the form $G(s) = A(s)/d(s)$ according to equation 3.11 yields

$$G(s) = \frac{\begin{bmatrix} 14.96(s^2+3.225s+2.525) & 95150(s+1.898)(s+2) \\ 85.2(s^2+3.225s+2.525) & 124000(s+2.037)(s+2) \end{bmatrix}}{(s^2+3.225s+2.525)(s+2)(s+100)} \begin{bmatrix} \dfrac{s+100}{s+10} & 0 \\ 0 & 1 \end{bmatrix}$$

where $L(s) = I$, $\Gamma(s) = I, R(s) = \begin{bmatrix} \frac{s+100}{s+10} & 0 \\ 0 & 1 \end{bmatrix}$,

$$A(s) = \begin{bmatrix} 14.96(s^2 + 3.225s + 2.525) & 95150(s + 1.898)(s + 2) \\ 85.2(s^2 + 3.225s + 2.525) & 124000(s + 2.037)(s + 2) \end{bmatrix}$$

and $$d(s) = (s^2 + 3.225s + 2.525)(s + 2)(s + 100)$$

The rational matrix $A(s)$ is not as required in equation 3.24. So an approximation is employed, in order to simplify the transfer function matrix by an approximate cancellation for zeroes and poles, keeping at the same time, as much as possible, same transient and steady state values for the original transfer functions.

$$g_{11} = \frac{14.96(s^2 + 3.225s + 2.525)}{(s^2 + 3.225s + 2.525)(s + 2)(s + 100)} \approx \frac{15.418s + 18.887}{(s + 100)(s^2 + 3.225s + 2.525)}$$

$$g_{21} = \frac{85.2(s^2 + 3.225s + 2.525)}{(s^2 + 3.225s + 2.525)(s + 2)(s + 100)} \approx \frac{87.8082s + 107.565}{(s + 100)(s^2 + 3.225s + 2.525)}$$

$$g_{12} = \frac{95150(s + 1.898)(s + 2)}{(s^2 + 3.225s + 2.525)(s + 2)(s + 100)} = \frac{95150(s + 1.898)}{(s + 100)(s^2 + 3.225s + 2.525)}$$

$$g_{22} = \frac{124000(s + 2.037)(s + 2)}{(s^2 + 3.225s + 2.525)(s + 2)(s + 100)} = \frac{124000(s + 2.037)}{(s^2 + 3.225s + 2.525)(s + 100)}$$

Then the approximated transfer function matrix is

$$g(s) = \frac{A(s)}{d(s)} \approx \frac{\begin{bmatrix} 15.418s + 18.887 & 95150s + 180594.7 \\ 87.8082s + 107.565 & 124000s + 252588 \end{bmatrix}}{(s^2 + 3.225s + 2.525)(s + 100)}$$

According to equation 3.33 :

$$< hA(s)k > = [h_1 h_2] \begin{bmatrix} 15.418s + 18.887 & 95150s + 180594.7 \\ 87.8082s + 107.565 & 124000s + 252588 \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \end{bmatrix}$$

$$= (15.418\, s + 18.887)k_1 h_1 + (95150s + 180594.7)k_2 h_1$$

$$+ (87.8082s + 107.565)k_1 h_2 + (124000s + 252588)k_2 h_2$$

$$= [1\ s] \begin{bmatrix} 18.887 & 180594.7 & 107.565 & 252588 \\ 15.418 & 95150 & 87.8082 & 124000 \end{bmatrix} \begin{bmatrix} k_1 h_1 \\ k_2 h_1 \\ k_1 h_2 \\ k_2 h_2 \end{bmatrix}$$

According to equation 3.34 , the gain ratio n is substituted to formulate the matrix Q:

$k_2 = nk_1$, then substituting for k_2 yields

$$Q = \begin{bmatrix} 18.887 + 180594.7n & 107.565 + 252588n \\ 15.418 + 95150n & 87.8082 + 124000n \end{bmatrix}$$

Now to design the inner loop, root locus methods will be employed according to equation 3.35, to design a single transfer function, which is equivalent to the closed loop multivariable system.

$$< h\frac{A(s)}{d(s)}k >= \frac{b(s)}{d(s)}$$

where $d(s) = (s^2 + 3.225s + 2.525)(s + 100) = (s + 1.34)(s + 1.8866)(s + 100)$

this equation gives the system poles. Let $b(s) = s + 1.5$, so that this stable zero at ($s = -1.5$) will attract the fastest pole of the system which is ($s = -1.34$) away from the imaginary axis; resulting in reducing the settling time , which assuring closed loop stability. The Root Locus plot of $\frac{b(s)}{d(s)}$ is shown in figure 4.1.

Now, substituting Q , $b = \begin{bmatrix} 1 \\ 1.5 \end{bmatrix}$ into equation 3.36, assuming that $k_1 = 1$ yields the inner loop feedback gain values which are

$$h = \frac{Q^{-1}b}{k_1} = [-0.8854e - 7\ 0.5035e - 7] \qquad 4.1$$

To find k_2,in order to complete the inner loop design , an optimization routine to find the absolute minimum control effort, at a specific gain ratio n will be employed . According to equation 3.41, the performance index is given by

$$J = (1 + n^2)b^T(Q^{-1})^T Q^{-1}b$$

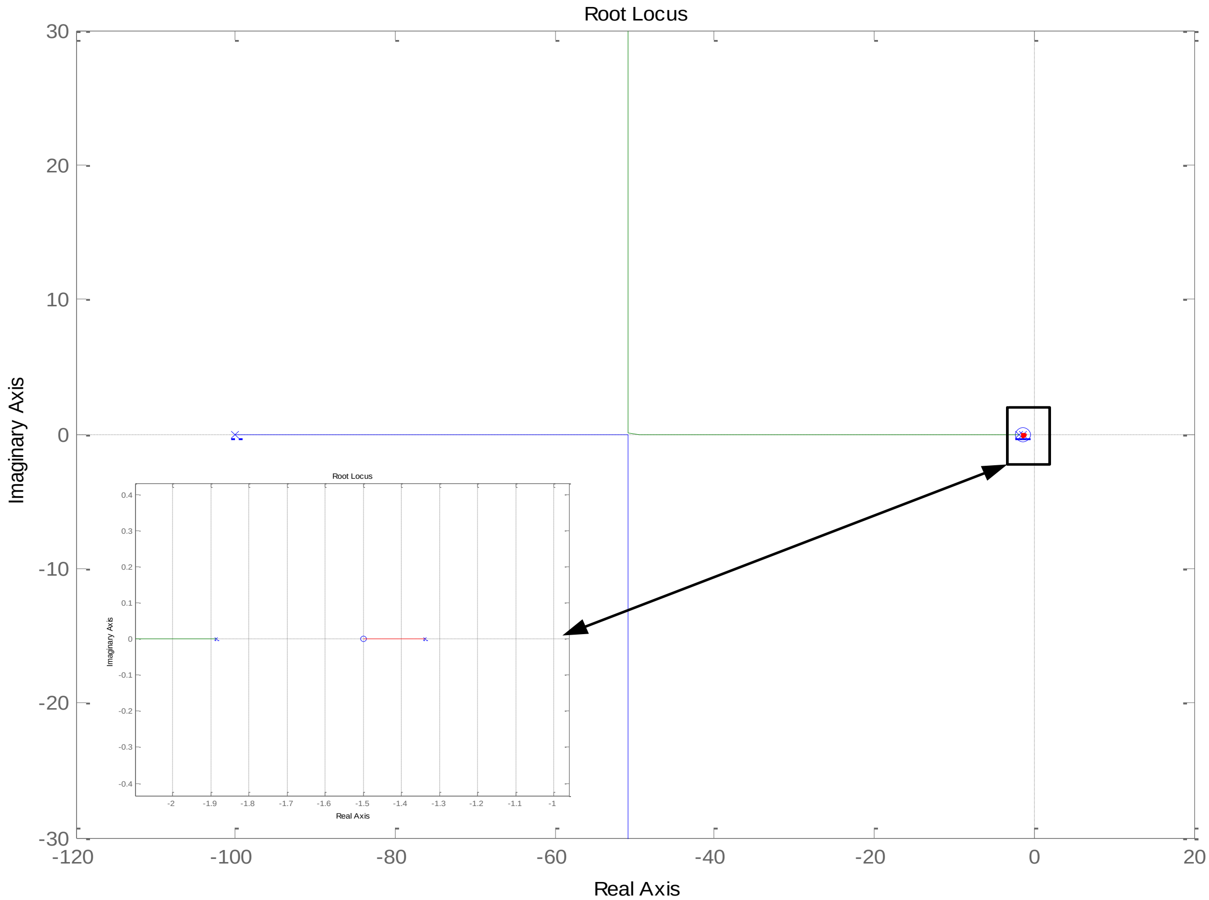


Figure 4.1,Root Locus Plot for $b(s)/d(s)$

Substituting b and Q yields the performance index to be:

$$J = \frac{10000(1+n^2)(0.6010696973e11-0.3536975881e15n+0.5868075922e18n^2)}{(3697-0.4070472010e13n+0.1640005400e16n^2)^2} \quad 4.2$$

As shown in figure 4.2 , the performance index J against gain ratio n according to equation 4.2 .

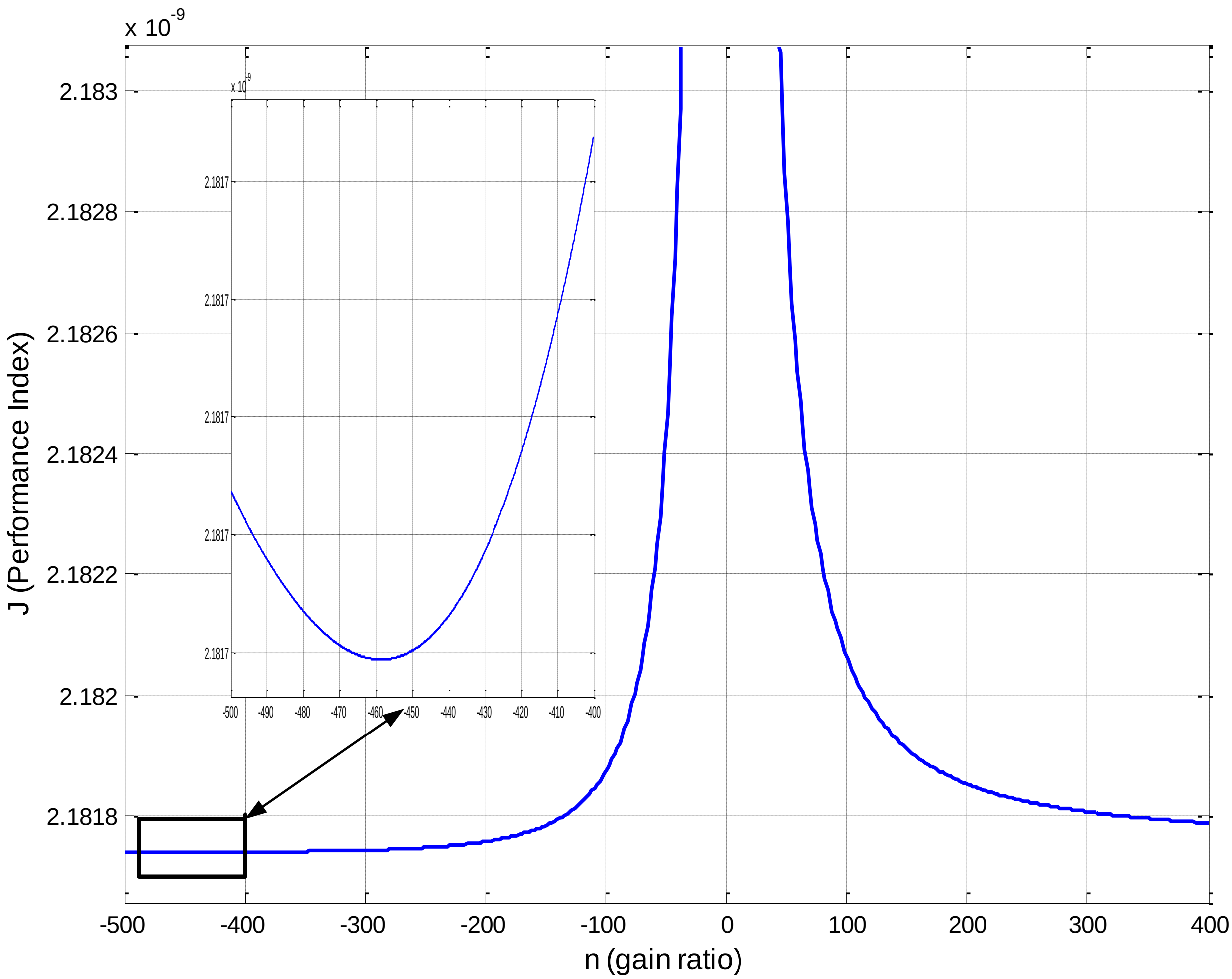


Figure 4.2,The Performance Index J versus Gain Ratio n

Now, to find the absolute minimum value of the performance index, differentiating the function with respect to gain ratio and equating to zero is employed:

$$\frac{\partial J}{\partial n} = 0$$

Reveals that there are four supremum values for J at:

$n = -458.58735\ where\ J = 0.218e - 18$ (minimum)

$n = 0.0002853738 \pm 0.000825i$ (not real number)

$n = 0.0003333764503\ where\ J = 0.0000537$ (Maximum)

Selecting $\underline{n = -458.58735}$ to be the gain ratio proposed for the inner-loop, results in:

$k_2 = nk_1 = -458.58735$ completing the design.

So that, the inner loop forward gain values are

$$k = \begin{bmatrix} 1 \\ -458.58735 \end{bmatrix} \qquad 4.3$$

Now to design the outer loop, all matrices involved in computing the feed forward and feedback gains are calculated:

since

$$G(s) = < h\frac{A(s)}{d(s)}k > R(s)$$

Then

$$k(s) = k.R(s) = \begin{bmatrix} 1 \\ -458.58735 \end{bmatrix} \begin{bmatrix} \frac{s+100}{s+10} & 0 \\ 0 & 1 \end{bmatrix}$$

$$k(s) = \begin{bmatrix} \frac{s+100}{s+10} \\ -458.5874 \end{bmatrix} \qquad 4.4$$

$$k(0) = \begin{bmatrix} 10 \\ -458.5874 \end{bmatrix} \qquad 4.5$$

$$h(s) = h(0) = h = [-0.8854e-7 \;\; 0.5035e-7] \qquad 4.6$$

The steady state values for the open loop transfer function matrix is given by

$$G(0) = \begin{bmatrix} 0.748 & 715.2265 \\ 4.26 & 1000.3 \end{bmatrix} \qquad 4.7$$

Meanwhile, the steady state interaction due to output coupling could be minimized in closed loop to be maximum 10 percent by assuming that

$$S_s = \begin{bmatrix} 1 & 0.1 \\ 0.1 & 1 \end{bmatrix} \qquad 4.8$$

Now, the outer loop feedback gains should be selected with $0 < f < 1$ to complete the loop design calculations. Investigation of the effect of increasing feedback gains from 0.1 to 0.9 on closed loop response and disturbance suppression will be studied in order to select the best control system strategy.

First, assume that

$$f_1 = 0.1 \text{ , then } F = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix} \qquad 4.9$$

The feed forward gain values for the outer loop could be calculated by substituting equations 4.5 to 4.9 into equation 3.16 which is

$P = (G(0)^{-1} + k(0) >< h(0))S_s(I - FS_s)^{-1}$ yielding:

$$P = \begin{bmatrix} -0.4457 & 0.2924 \\ 0.0021 & -0.0002 \end{bmatrix} \qquad 4.10$$

The feedback gain matrix H could be computed by substituting equations 4.4,4.6 ,4.9 and 4.10 into equation 3.21 which is :

$H = P^{-1}k(s) >< h(s) + F$ yielding:

$$H = \begin{bmatrix} \frac{0.12349(s+10)}{s+10} & \frac{-0.013361(s+10)}{s+10} \\ \frac{0.03581(s+10)}{s+10} & \frac{0.0796(s+10)}{s+10} \end{bmatrix} = \begin{bmatrix} 0.12349 & -0.013361 \\ 0.03581 & 0.0796 \end{bmatrix} \qquad 4.11$$

This is the simplest form of a feedback gain matrix.

Substituting the gain values of equations 4.10 and 4.11 in the simulation model by Matlab Simulink software, as shown in Appendix , and simulating the system response results in the closed loop responses, following a unit step change on the reference inputs of jet nozzle cross area $r_1(s)$, first , then on the fuel flow rate $r_2(s)$,are as shown in figures 4.3 and 4.4, respectively.

It can be shown from figure 4.3 , that the output closed loop responses with low outer-loop feedback gains like $f = 0.1$; are quite good, but not adequate. Stability and well behaved outputs are achieved, however, the two outputs N_H and N_L are restricted by a slight steady state error. It is clear also that the first output $N_H$response is over damped with a long rising time (about 3 seconds). Meanwhile the second output N_L is restricted to less than 15 percent of the first output N_H with acceptable oscillations but long settling time also (about 3 seconds).

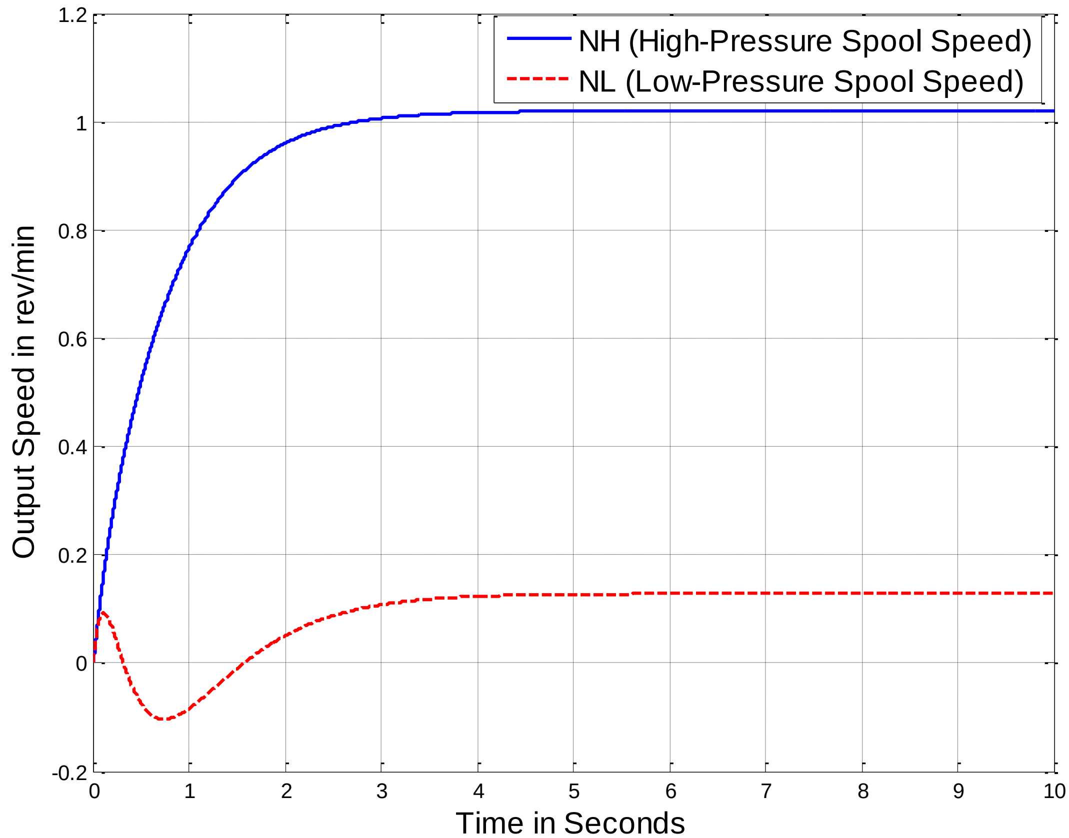


Figure 4.3, Closed Loop Response with $f_1 = 0.1$ Following a Unit Step Change on $r_1(s)$

This is still true for the closed loop response following a unit step change on the second reference input $r_2(s)$ as shown in figure 4.4, confirming that low outer loop feedback gain values would introduce errors in the outputs steady state values. Also the second output N_L response is over damped with a rising time of about 2 seconds, meanwhile, the first output N_H is restricted to less than 10 percent of the second output N_L with slight overshoot and relatively long settling time (about 2 seconds).

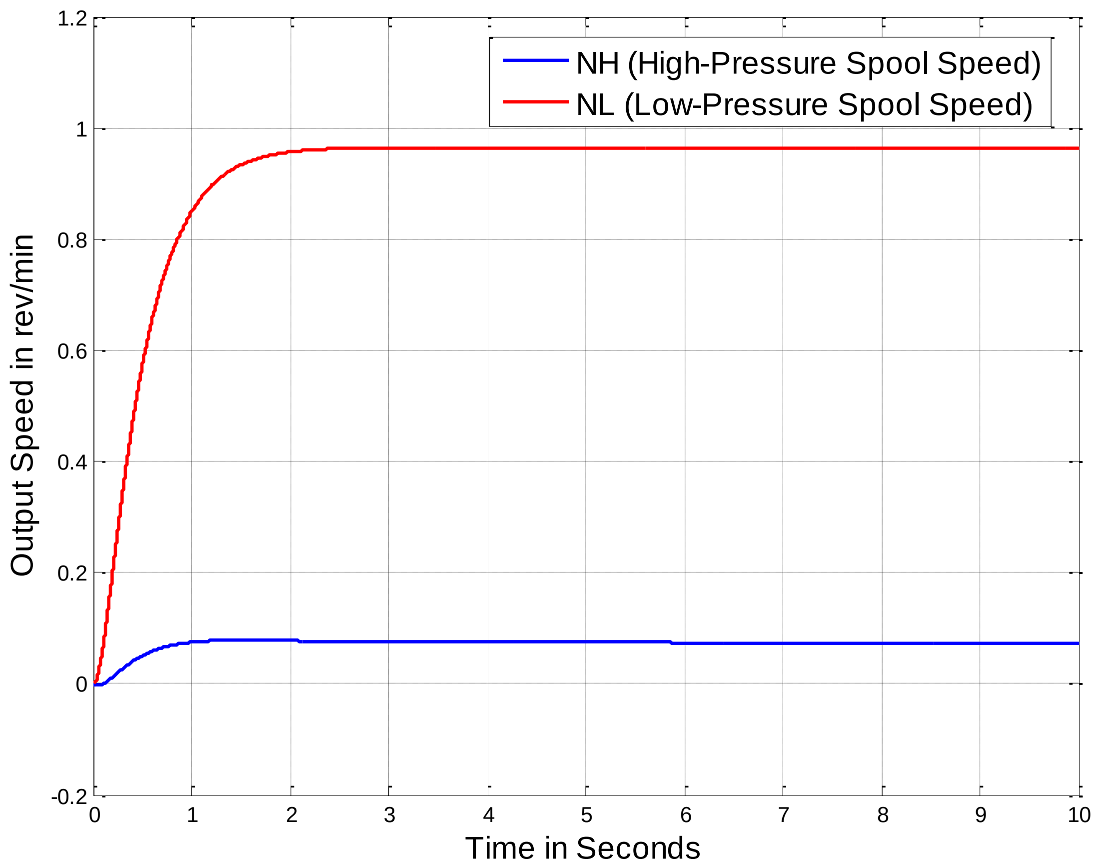


Figure 4.4, Closed Loop Response with $f_1 = 0.1$ Following a Unit Step Change on $r_2(s)$

Now if the outer loop feedback gain is increased to 0.5 , the feed forward and feedback gain matrices will be:

$$f_2 = 0.5 \text{ , then } F = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix} \quad 4.12$$

$$P = \begin{bmatrix} -0.76221 & 0.459 \\ 0.00373 & 0.0000523 \end{bmatrix} \quad 4.13$$

$$H = \begin{bmatrix} \frac{0.51127(s+10)}{s+10} & \frac{-0.00641(s+10)}{s+10} \\ \frac{0.0187(s+10)}{s+10} & \frac{0.489(s+10)}{s+10} \end{bmatrix} = \begin{bmatrix} 0.51127 & -0.00641 \\ 0.0187 & 0.489 \end{bmatrix} \quad 4.14$$

Substituting gain values of equations 4.13 and 4.14 into the simulation model, by Matlab Simulink software as shown in Appendix, the simulation of the system results in the closed loop responses, shown in figures 4.5 and 4.6, following a unit step change on $r_1(s)$ and $r_2(s)$ respectively.

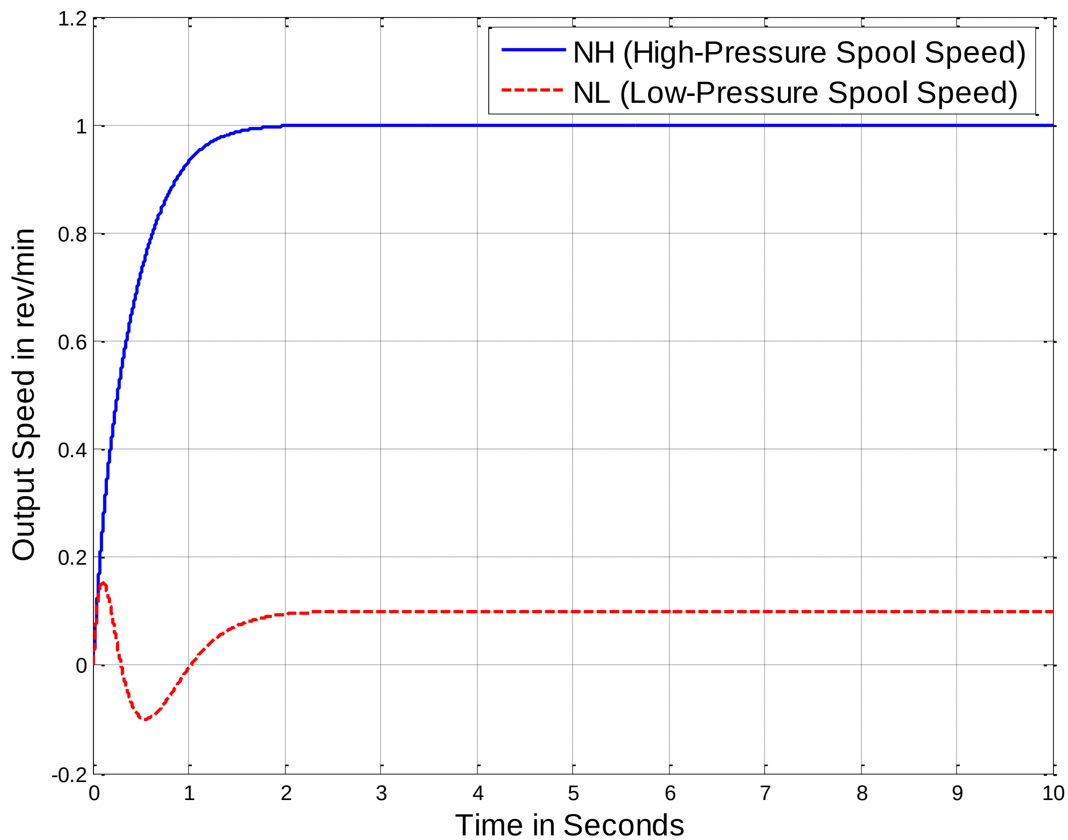


Figure 4.5, Closed Loop Response with $f_2 = 0.5$ Following a Unit Step Change on $r_1(s)$

It is clear from figure 4.5 that increasing the outer loop feedback gain to 0.5 results in the elimination of the steady state errors in both outputs. Also speeding up the system response is achieved. This is evident from comparing the rise time of the first output N_H which is about 2 seconds with the one of figure 4.3 which is 3 seconds. Moreover, the settling time of the second output is minimized to 2 seconds, compared with 3 seconds, using$f = 0.1$.

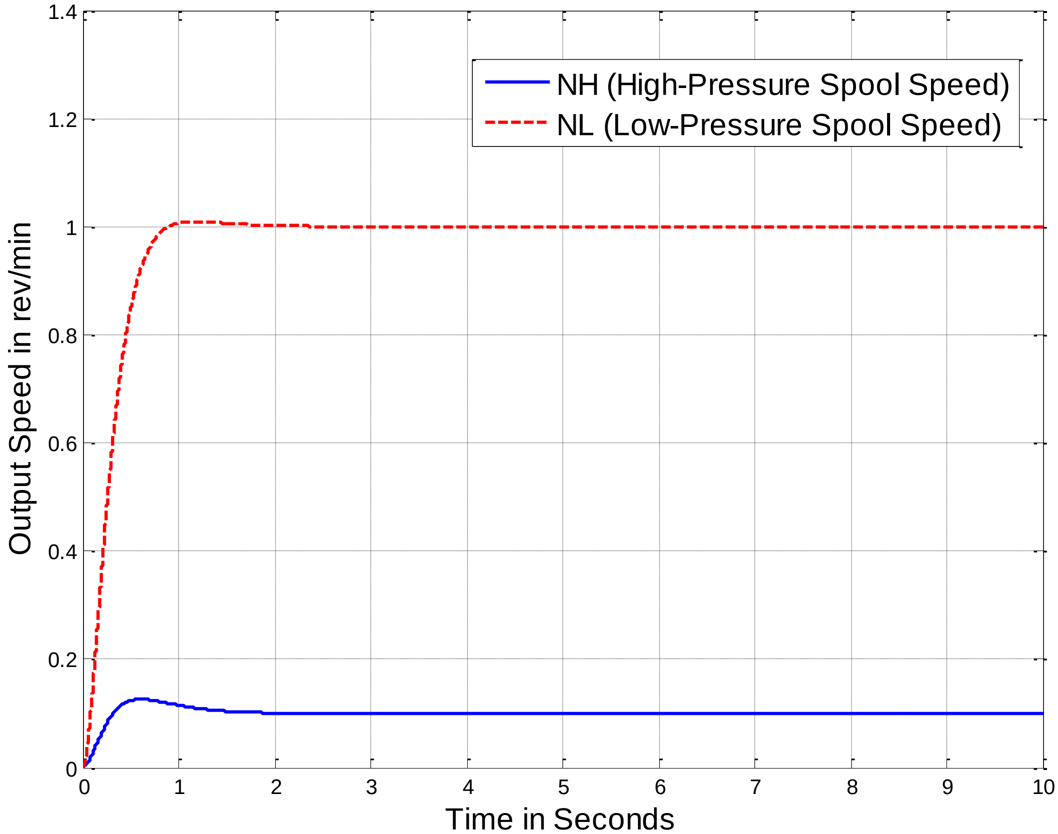


Figure 4.6, Closed Loop Response with $f_2 = 0.5$ Following a Unit Step Change On $r_2(s)$

This is still true for figure 4.6, where using $f = 0.5$ results also in speeding up the system response and eliminating the steady state errors. It is noticeable, that the second output response N_L, is so closed to critical damped behavior, with a very slight overshoot and short rise time (about one second only). Also the first output N_H response overshoots and settles down in two seconds, and is restricted by design to 10 percent interaction with second output N_L.

To see the maximum effects of increasing the outer loop feedback gain :

$$f_3 = 0.9, \text{ then } F = \begin{bmatrix} 0.9 & 0 \\ 0 & 0.9 \end{bmatrix} \quad 4.5$$

$$P = \begin{bmatrix} -8.588 & -5.053 \\ 0.0904 & 0.0797 \end{bmatrix} \quad 4.6$$

$$H = \begin{bmatrix} \frac{0.899(s+10)}{s+10} & \frac{0.000542(s+10)}{s+10} \\ \frac{0.00162(s+10)}{s+10} & \frac{0.8991(s+10)}{s+10} \end{bmatrix} = \begin{bmatrix} 0.899 & 0.000542 \\ 0.00162 & 0.8991 \end{bmatrix} \quad 4.7$$

Substituting the gain values of equations 4.6 and 4.7 into the simulation model of Matlab-Simulink software as shown in Appendix , and simulation the system results in closed loop responses in figures 4.7 and 4.8 following a unit step change on first and second reference inputs, respectively.

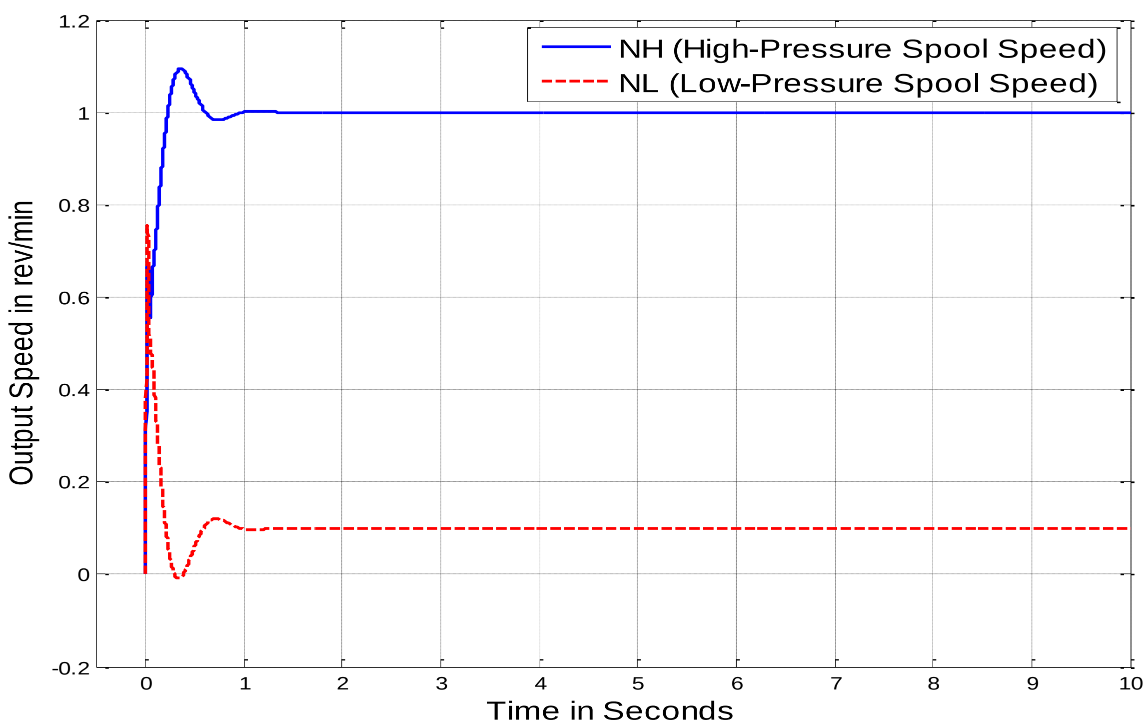


Figure 4.7, Closed Loop Response with $f_3 = 0.9$ Following a Unit Step Change on $r_1(s)$

From figure 4.7, it is clearly evident that increasing the outer loop feedback gain to $f = 0.9$, would result in speeding up the system, and attracting overshoot conditions. The first output N_H response is obviously under damped, with acceptable overshoot less than 20 percent, and very short rising time (about half a second). Meanwhile, the second output N_L overshoots but settles down very quickly (in less than a second) to the pre-designed value of 10 percent interaction.

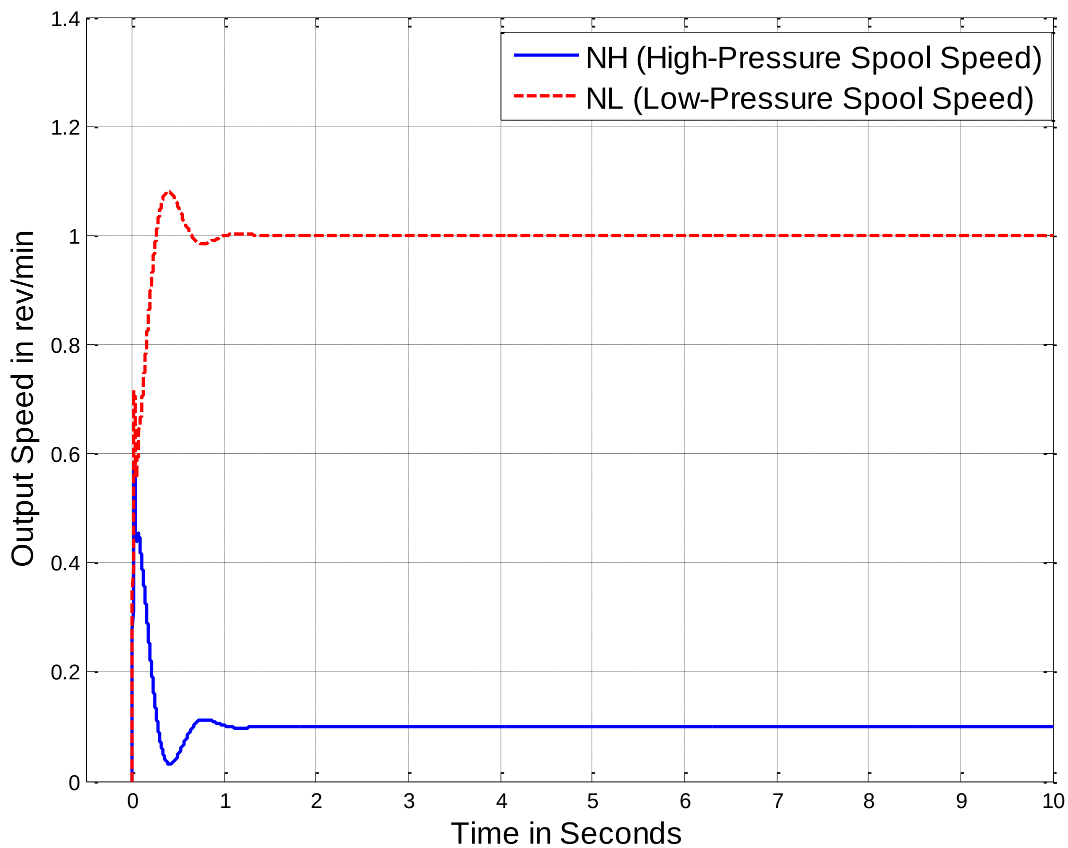


Figure 4.8, Closed Loop Response with $f_3 = 0.9$ Following a Unit Step Change on $r_2(s)$

On the other hand, following a unit step change on the second reference input $r_2(s)$ would result in a similar response. The second output N_H is clearly an under damped response with an acceptable overshoot of less than 10 percent and very short rising time of almost half a second. Also the first output N_L has quick dynamics in almost zero time that vanish quickly and settles down to its pre designed steady state interaction of 10 percent, in almost a second.

❖ Disturbance Rejection Analysis :

To investigate the effect of increasing outer loop feedback gain f from 0.1 to 0.9 on the system disturbance rejection, the closed loop response for the system following a unit step change on the first disturbance $\delta_1(t)$ and second disturbance $\delta_2(t)$, respectively will be compared between the three system designs using $f_1 = 0.1, f_2 = 0.5\ and\ f_3 = 0.9$, as shown in figures 4.9 and 4.10. The simulation models are shown in Appendix.

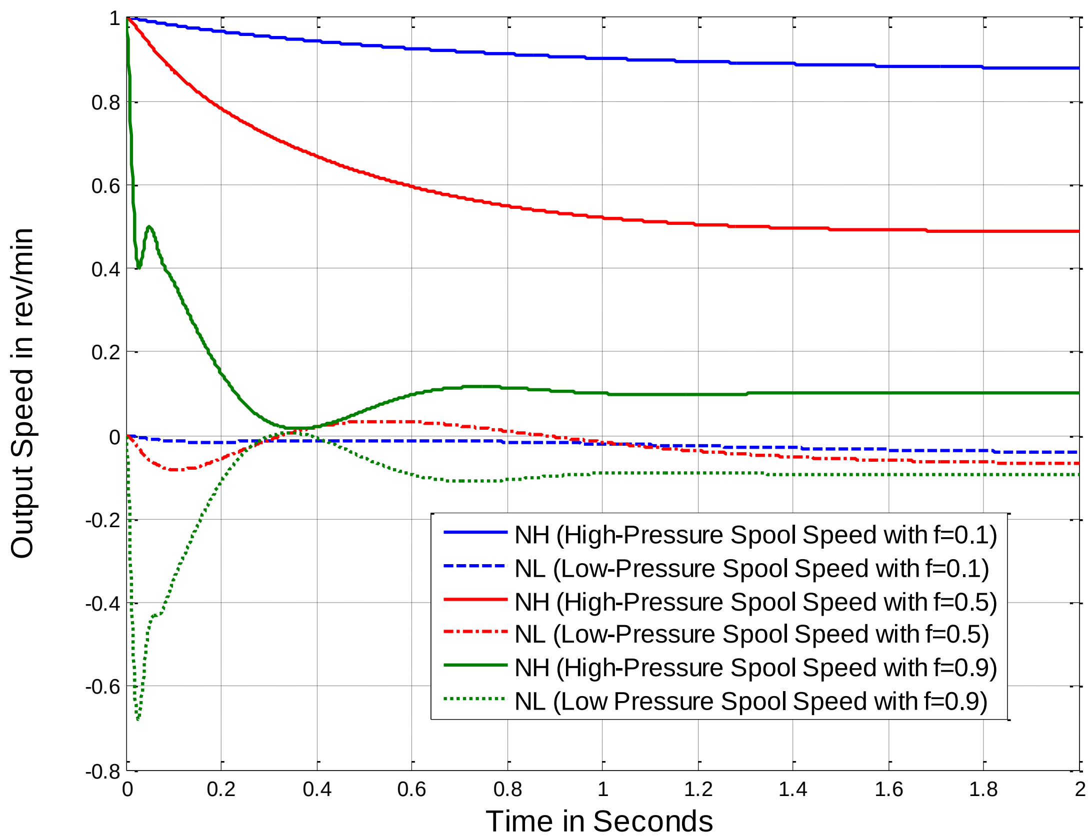


Figure 4.9, Closed Loop Response Following a Unit Step Change on $\delta_1(t)$

From figure 4.9, it is clearly shown that increasing the outer loop feedback gain f would increase the disturbance recovery rates. This is obviously seen in the first output N_H responses, where using $f = 0.1$ would result in poor disturbance recovery (about 10 percent only), while $f = 0.5$ would result in almost 50 percent recovery , both with settling time almost a second. On the other hand, using $f = 0.9$ would result in the best disturbance recovery response that is more than 90 percent is eliminated, in less than a second. The disturbance on the first output also affects the second output $N_L$responses; however these effects could be neglected since they are very small.

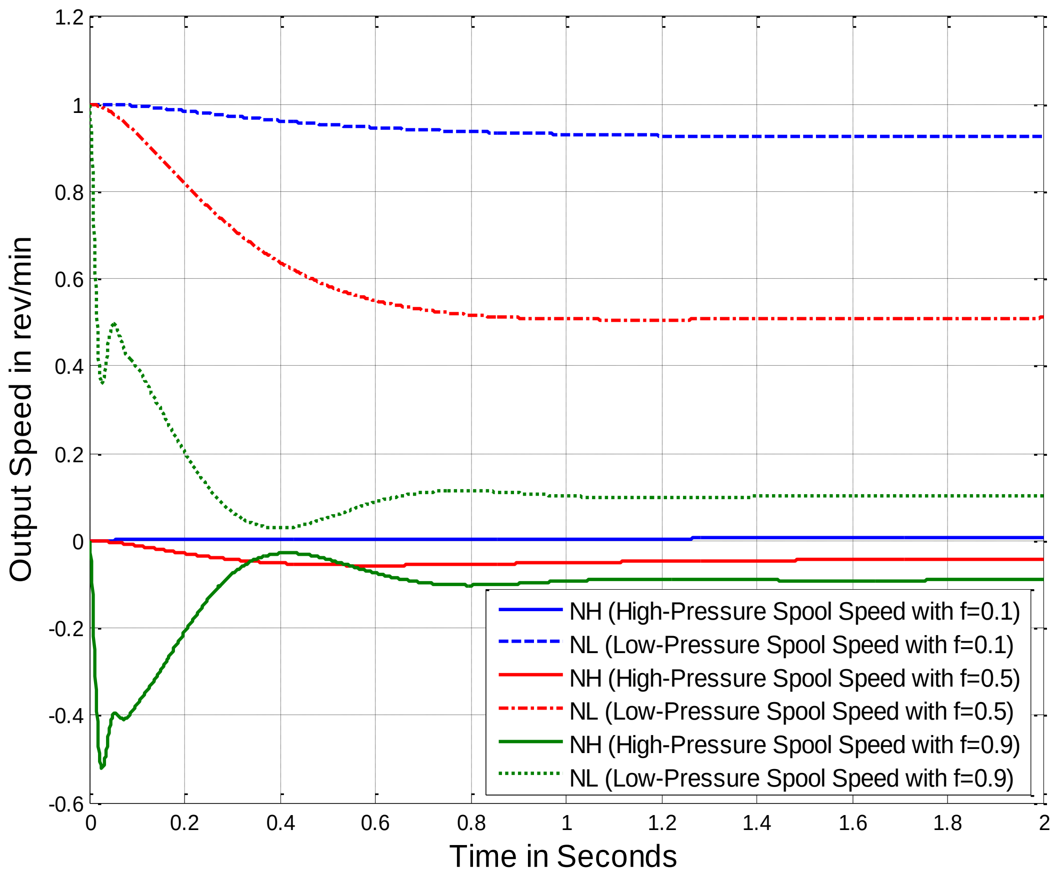


Figure 4.10, Closed Loop Response Following a Unit Step Change on $\delta_2(t)$

In a similar analysis, it can be shown also from figure 4.10 that increasing outer loop feedback gain, would result in better and quicker disturbance recovery responses.

It could be concluded that designing a Least Effort Control system requires selecting the outer loop feedback gains between $0.5 \leq f \leq 0.9$, in order to achieve the best closed loop response. This is to guarantee the minimum settling and rising time and also eliminating any steady state errors. Moreover, the system disturbance rejection ability increases, especially with high values such as $f = 0.9$.

On the other hand, disadvantages also could occur from increasing the outer loop feedback gain. This is evident from the truth that the system becomes unstable when $f = 1$. So that, selecting values close from $f = 1$ would result in introducing unacceptable dynamics in closed loop response. For comparison purposes, the design with $f = 0.9$ will be employed.

It should be noted that many other least effort designs could be considered with different $b(s)$ function and steady state S_s matrices.

4.2 Inverse Nyquist Array Controller

In 1970, P.D. McMorran designed a controller for the same gas turbine model, using the Inverse Nyquist Array (INA) method. His main work is reviewed here. McMorran suggested a pre compensator to achieve column diagonal dominance using inequalities and an analysis of the system using root locus technique. His pre compensator is given by :

$$K(s) = \frac{1}{s+158.5}\begin{bmatrix} 1000 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} -1.57(s+20.6) & 1.205(s+11.6) \\ 1.08(s+146.3) & -0.189(s+101.4) \end{bmatrix} \qquad 4.8$$

Since

$Q(s) = G(s)K(s)$ and its inverse :

$$\hat{Q}(s) = \frac{1}{6250(s+9.7)}\begin{bmatrix} 0.08s^3 + 8.88s^2 + 103.12s + 120 & -0.01s^2 - 1.91s \\ -5s^2 + 59.6s & 0.08s^3 + 13.82s^2 + 127s + 204.8 \end{bmatrix} \quad 4.9$$

Now, the Inverse Nyquist Array for equation 4.9 could be plotted by executing special m-file to create the Nyquist diagram, with Gershgorin circles superimposed. This m-file is given in the Appendix. In figure 4.8, the Nyquist diagram of the element $\hat{q}_{11}$ with imposed Gershgorin circles for the first column of $\hat{Q}(s)$, shows clearly that none of the circles enclose the origin , and so the system is first column diagonal dominant.

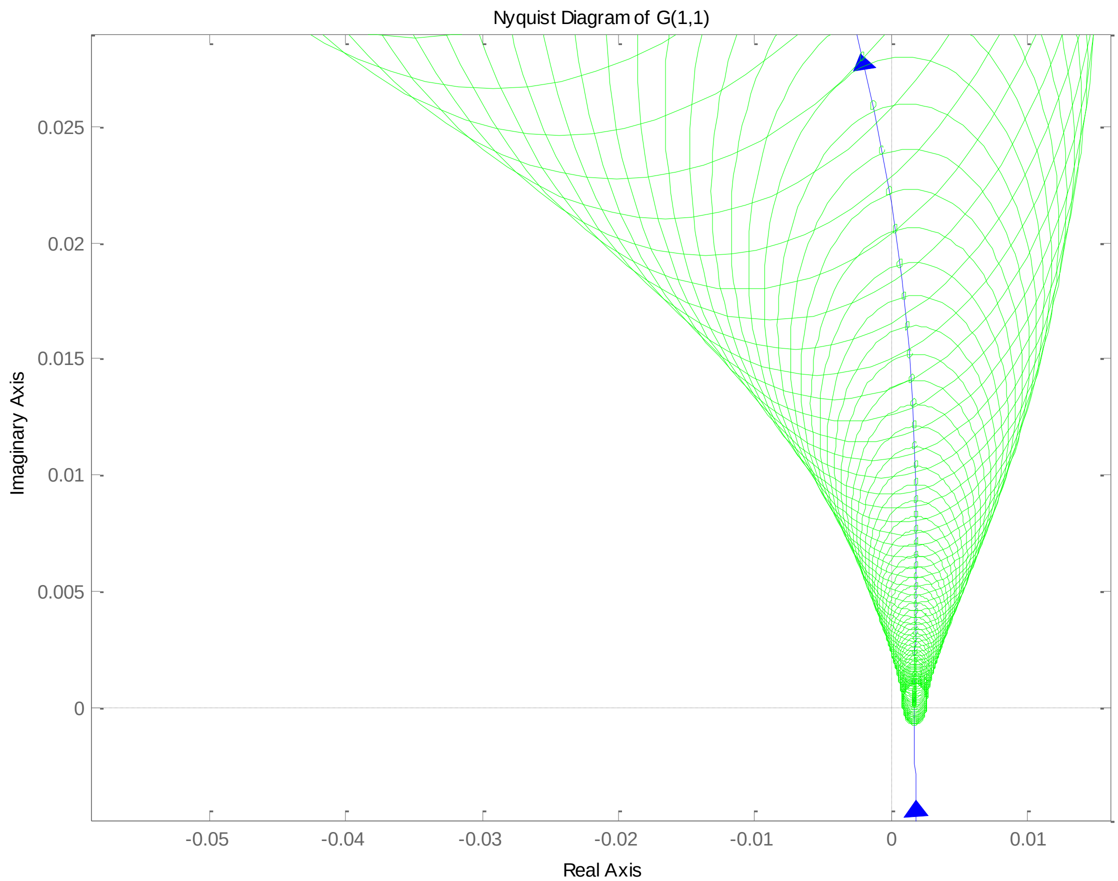


Figure 4.11, Nyquist Diagram of $\hat{q}_{11}$ with Gershgorin Circles

Also from figure 4.12 , the Nyquist diagram of the element $\hat{q}_{22}$ with imposed Gershgorin circles for the second column of $\hat{Q}(s)$, shows clearly that none of the circles enclose the origin , and so the system is completely column dominant.

Since diagonal dominance is achieved by this pre-compensator; a controller for each loop could be designed independently. It is noticed from figures 4.11 and 4.12 that the Nyquist diagrams never cross the negative real axis, yielding infinite stability margins, resulting in the freedom of designing the feedback loops, with infinite gains.

McMorran designed a proportional control for the first output, while a proportional and integral control for the second output. His controller is given by

$$k = \begin{bmatrix} k_1 & 0 \\ 0 & k_2(1+\frac{1}{Ts}) \end{bmatrix} \; where \; k_1 = 0.0647\,, k_2 = 0.1095\,, T = 0.178\; sec \qquad 4.10$$

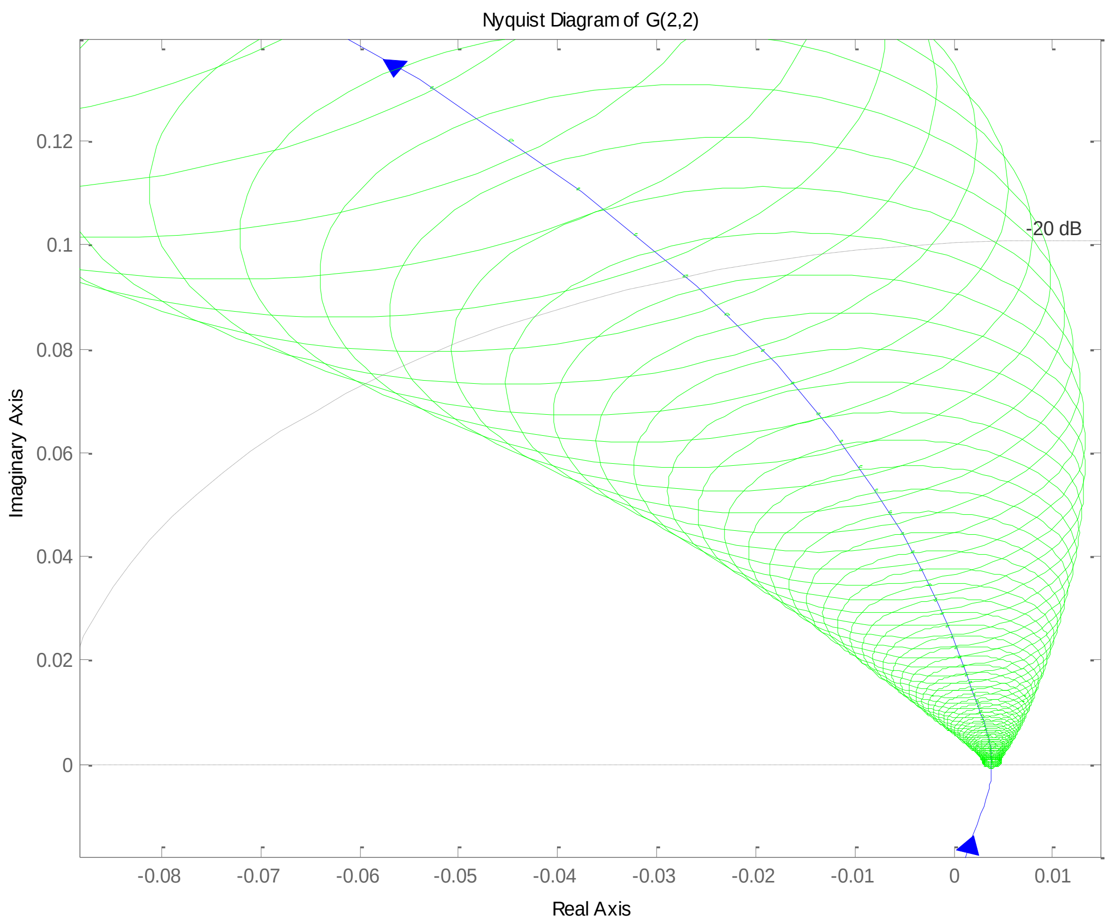


Figure 4.12, Nyquist Diagram of $\hat{q}_{22}$ with Gershgorin Circles

So the overall compensator and controller is given by

$$K(s) = \begin{bmatrix} \frac{-101.6s-2093}{s+158.5} & \frac{23.49s^2+404.4+1531}{s(0.178s+28.21)} \\ \frac{0.06988s+10.22}{s+158.5} & \frac{-0.003684s^2-0.3942s-2.099}{s(0.178s+28.21)} \end{bmatrix} \quad 4.11$$

Simulation model is shown in Appendix. Unity feedback is proposed with independent loops. The simulation of the system results in the closed loop response following a unit step change on $r_1(s)$ and $r_2(s)$, as shown in figures 4.14 and 4.15 , respectively.

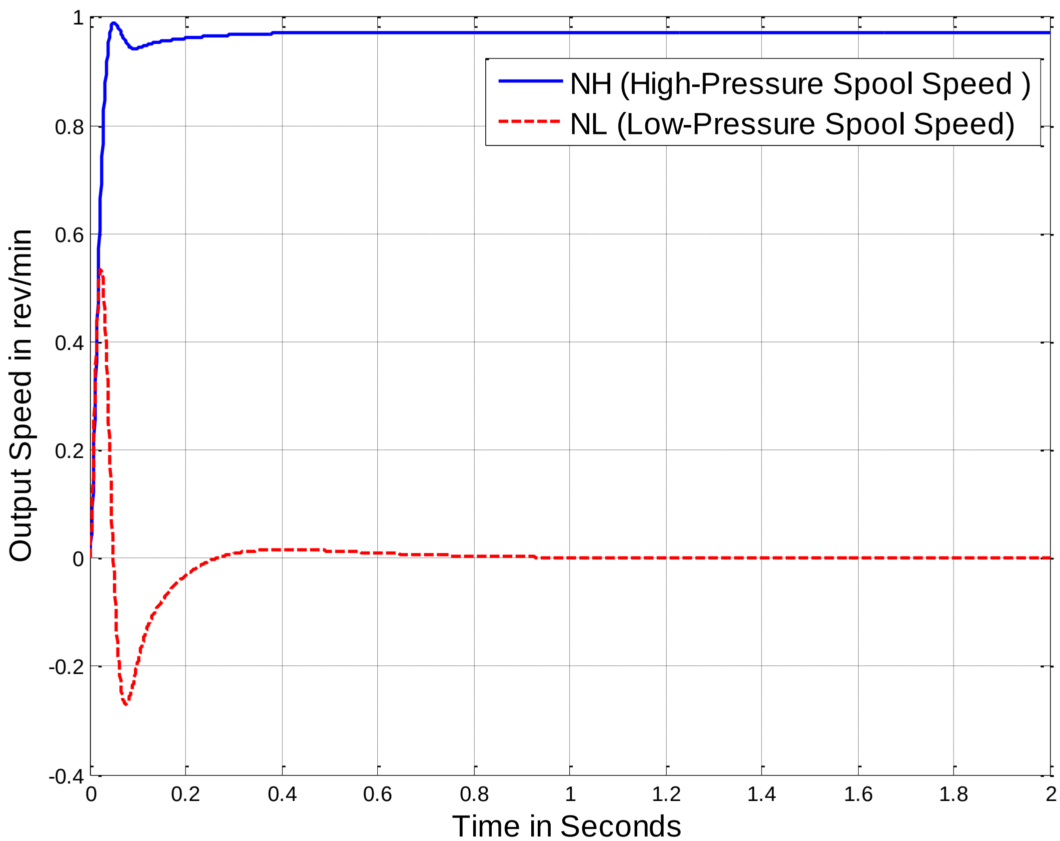


Figure 4.13, Closed Loop Response Following a Unit Step Change on $r_1(s)$

From figure 4.14, it can be seen that the system response is very quick (settling time about 0.25 seconds) and stable. However, there is a steady state error introduced in the first output response. Meanwhile, the second output response has acceptable dynamics that vanish quickly (about a second). It is noticeable, that the system is completely decoupled in steady state.

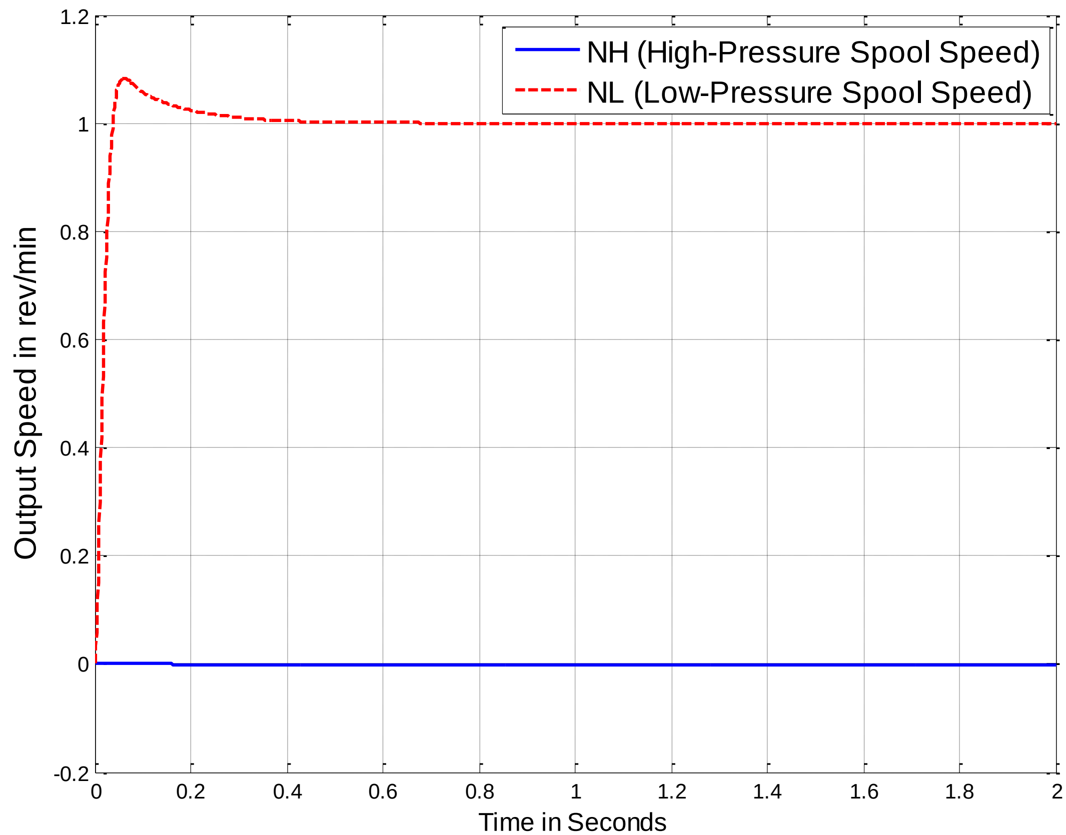


Figure 4.14, Closed Loop Response Following a Unit Step Change on $r_2(s)$

On the other hand, following a unit step change on second reference input $r_2(s)$; results in the response shown in figure 4.15. It can be clearly seen that both outputs are perfectly well behaved. The second output transient response is very attractive with a slight overshoot and short settling time (about half a second). Meanwhile, the first output is completely decoupled from the second loop which results in no response at all.

4.3 Optimal Controller

According to McMorran (1970) , Mueller designed an optimal control system for his gas turbine multivariable model . A review for his work follows.

The optimal control requires representing the system model in state space form , which is according to equation 3.12 :

$$\begin{bmatrix} \Delta\dot{N}_H \\ \Delta\dot{N}_L \\ \Delta\dot{A}_J \\ \Delta\dot{Q}_f \end{bmatrix} = \begin{bmatrix} -1.268 & -0.04528 & 1.498 & 951.5 \\ 1.002 & -1.957 & 8.52 & 1240 \\ 0 & 0 & -10 & 0 \\ 0 & 0 & 0 & -100 \end{bmatrix} \begin{bmatrix} \Delta N_H \\ \Delta N_L \\ \Delta A_J \\ \Delta Q_f \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 10 & 0 \\ 0 & 100 \end{bmatrix} \begin{bmatrix} \Delta A_{J,d} \\ \Delta Q_{f,d} \end{bmatrix}$$

where the states are the high and low pressure spool speeds $x_1 = \Delta N_H$ and $x_2 = \Delta N_L$, beside the actual jet nozzle area and fuel flow rate $x_3 = \Delta A_J$ and $x_4 = \Delta Q_f$. Inputs are the demanded value of jet nozzle area and fuel flow rate $u_1 = \Delta A_{J,d}$and $u_2 = \Delta Q_f$. The outputs will be the first two states only $y_1 = \Delta N_H$and $y_2 = \Delta N_L$, which means that the output equation is given by

$$\begin{bmatrix} \Delta N_H \\ \Delta N_L \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta N_H \\ \Delta N_L \\ \Delta A_J \\ \Delta Q_f \end{bmatrix}$$

From previous equations, it will be concluded that:

$$A = \begin{bmatrix} -1.268 & -0.04528 & 1.498 & 951.5 \\ 1.002 & -1.957 & 8.52 & 1240 \\ 0 & 0 & -10 & 0 \\ 0 & 0 & 0 & -100 \end{bmatrix} \quad 4.12$$

$$B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 10 & 0 \\ 0 & 100 \end{bmatrix} \quad 4.13$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \quad 4.14$$

$$D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \quad 4.15$$

To investigate the system Eigen values $\det(\lambda I - A)$yields that

$$\lambda_1 = -1.3417 \text{ , } \lambda_2 = -1.8833 \text{ , } \lambda_3 = -10 \text{ , } \lambda_4 = -100$$

which all are poles lie on the left hand complex plane, resulting in stability of the system in open loop response.

Testing Controllability first of the system, before trying to synthesize a controller is essential step. So substituting values of matrices A and B into Controllability Matrix $\mathbb{C}$ yields:

$$\mathbb{C} = [B \; AB] = \begin{bmatrix} 0 & 0 & 14.98 & 95150 \\ 0 & 0 & 85.2 & 124000 \\ 10 & 0 & -100 & 0 \\ 0 & 100 & 0 & -10000 \end{bmatrix} \quad 4.16$$

Since $\det(\mathbb{C}) \neq 0$, then the Controllability matrix is full rank and the system is completely controllable. In addition, testing Observability of the system, before trying to construct an observer for the states that are not measurable is an important issue. So substituting values of matrices A and C into Observability Matrix Θ, results in

$$\Theta = [C^T (CA)^T] = \begin{bmatrix} 1 & 0 & -1.268 & 1.002 \\ 0 & 1 & -0.04528 & -1.957 \\ 0 & 0 & 1.498 & 8.52 \\ 0 & 0 & 951.5 & 1240 \end{bmatrix} \quad 4.17$$

Since $\det(\Theta) \neq 0$, then the Observability matrix is full rank and the system is completely observable.

Now the system is ready for designing an observer and synthesizing a controller based on estimated state feedback.

According to McMorran (1970), Mueller designed a low order observer to estimate the two non-measurable states, which are $x_3 = \Delta A_J$ and $x_4 = \Delta Q_f$. He placed both poles of the observer at $s = -160$, in order to guarantee that the dynamics of the observer would vanish quickly before the control action is applied on the estimated state values.

Since there is no online copy for the original design by Mueller G.S, regarding the observer, and the only guidance for his approach is McMorran's review of his work that provided the arrangement of Mueller's Observer as shown in figure 4.15. A full order observer will be designed instead, by the same arrangement, and transformed to Laplace domain in order to meet the comparison study requirements in next section.

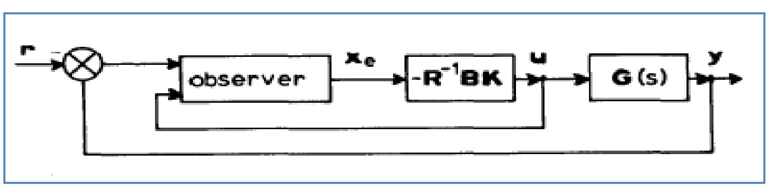


Figure 4.15, Observer Arrangement with Optimal Controller (McMorran,1970)

The observer and the controller could be designed separately. Mueller suggested an Optimal controller according to his own steady-state solution for Riccati Matrix Equation to minimize the performance index in equation 3.77, which is

$$J = \frac{1}{2}\int_0^\infty (y^T Q y + u^T R u)dt$$

With weighting matrices $Q = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$ and $R = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

and the state feedback gain matrix is

$$K = \begin{bmatrix} -1.869 & 1.437 & 0.6986 & 0.005312 \\ 1.908 & 2.512 & 0.05312 & 8.9800 \end{bmatrix} \qquad 4.18$$

The full order observer equation is assumed to be according to figure 3.10, which is similar to Mueller's arrangement, and given by

$$\dot{\bar{x}} = (A - BK - LC)\bar{x} + L(y - r) \qquad 4.19$$

The observer gain matrix L could be designed by poles shifting technique. The observer poles should be faster than the system poles by a considerable amount of time. It will be assumed that the observer poles would be shifted to $s =$ -160 , -180 , -200 , -220 . This guarantees the stability and the quick performance for the designed observer subsystem. An algorithm to compute the observer gain matrix is given by Matlab.

The observer gain matrix L that will move the Eigen values of the matrix $(A - LC)$

to -160 , -180 , -200 , -220 is found to be

$$L = \begin{bmatrix} 205.46 & 67.27 \\ -188.76 & 441.314 \\ -5740.25 & 4491.03 \\ 5.6 & 2.98 \end{bmatrix} \tag{4.20}$$

Now, substituting matrices $A, B, K, L\ and\ C$ into equation 4.19, yields the observer in state space form to be:

$$\begin{bmatrix} \dot{\bar{x}}_1 \\ \dot{\bar{x}}_2 \\ \dot{\bar{x}}_3 \\ \dot{\bar{x}}_4 \end{bmatrix} = \begin{bmatrix} -206.7 & -67.32 & 1.498 & 951.5 \\ 189.8 & -443.3 & 8.52 & 1240 \\ 5759 & -4505 & -16.99 & -0.05313 \\ -196.4 & -254.2 & -5.313 & -998 \end{bmatrix} \begin{bmatrix} \bar{x}_1 \\ \bar{x}_2 \\ \bar{x}_3 \\ \bar{x}_4 \end{bmatrix} + \begin{bmatrix} 205.46 & 67.27 \\ -188.76 & 441.314 \\ -5740.25 & 4491.03 \\ 5.6 & 2.98 \end{bmatrix} \begin{bmatrix} y_1 - r_1 \\ y_2 - r_2 \end{bmatrix} \tag{4.21}$$

$$\begin{bmatrix} \bar{x}_1 \\ \bar{x}_2 \\ \bar{x}_3 \\ \bar{x}_4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The observer is a sub system of two-inputs and four-outputs, could be transformed from state space form to Laplace domain, in a transfer function matrix form $O(s)$, which is given by :

$$O(s) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} . \left\{ sI - \begin{bmatrix} -206.7 & -67.32 & 1.498 & 951.5 \\ 189.8 & -443.3 & 8.52 & 1240 \\ 5759 & -4505 & -16.99 & -0.05313 \\ -196.4 & -254.2 & -5.313 & -998 \end{bmatrix} \right\}^{-1} . \begin{bmatrix} 205.46 & 67.27 \\ -188.76 & 441.314 \\ -5740.25 & 4491.03 \\ 5.6 & 2.98 \end{bmatrix}$$

$$O(s) = \frac{\begin{bmatrix} 205.5s^3+3.091e5s^2+2.503e8s+2.149e10 & 67.27s^3+7.796e4s^2-9.92e7s-8.492e9 \\ -188.8s^3-2.336e5s^2-9.699e7s-8.249e9 & 441.3s^3+5.939e5s^2+1.978e8s+1.706e10 \\ -5740s^3-7.426e6s^2-4.578e9s-2.813e8 & 4491s^3+5.8e6s^2+3.571e9s+7.768e9 \\ 5.6\,s^3+4.187e004s^2+3.711e006s-1.968e9 & 2.98s^3-1.473e5s^2-4.621e7s-2.632e9 \end{bmatrix}}{s^4+1665s^3+1.313e6s^2+3.408e8s+2.803e10} \tag{4.22}$$

Now, the overall compensator $C(s)$ including the observer and the Optimal controller could be computed by:

$$C(s) = K.O(s) = \frac{\begin{bmatrix} c_{n11} & c_{n12} \\ c_{n21} & c_{n22} \end{bmatrix}}{c_d} \tag{4.23}$$

where the common denominator is :

$$c_d = s^{16} + 6660s^{15} + 2.189e7s^{14} + 4.606e10s^{13} + 6.863e13s^{12} + 7.595e16s^{11} + 6.397e19s^{10} + 4.143e22s^9 + 2.066e25s^8 + 7.892e27s^7 + 2.285e30s^6 + 4.945e32s^5 + 7.837e34s^4 + 8.807e36s^3 + 6.635e38s^2 + 3.003e40s + 6.175e41$$

And the numerator elements are:

$$c_{n11} = -4665s^{15} - 2.94e7s^{14} - 9.146e10s^{13} - 1.813e14s^{12} - 2.527e17s^{11} - 2.587e20s^{10} - 1.986e23s^9 - 1.148e26s^8 - 4.959e28s^7 - 1.575e31s^6 - 3.595e33s^5 - 5.709e35s^4 - 6.017e37s^3 - 3.864e39s^2 - 1.258e41s - 1.15e42$$

$$c_{n21} = -336.9s^{15} - 1.699e6s^{14} - 4.184e9s^{13} - 6.391e12s^{12} - 6.588e15s^{11} - 4.684e18s^{10} - 2.25e21s^9 - 6.564e23s^8 - 6.222e25s^7 + 3.458e28s^6 + 1.769e31s^5 + 4.175e33s^4 + 5.942e35s^3 + 5.207e37s^2 + 2.6e39s + 5.684e40$$

$$c_{n12} = 3646s^{15} + 2.297e7s^{14} + 7.142e10s^{13} + 1.415e14s^{12} + 1.972e17s^{11} + 2.019e20s^{10} + 1.549e23s^9 + 8.957e25s^8 + 3.872e28s^7 + 1.232e31s^6 + 2.815e33s^5 + 4.483e35s^4 + 4.748e37s^3 + 3.076e39s^2 + 1.021e41s + 1.009e42$$

$$c_{n22} = 1502s^{15} + 8.131e6\, s^{14} + 2.162e10s^{13} + 3.627e13s^{12} + 4.219e16s^{11} + 3.55e19s^{10} + 2.203e22s^9 + 1.013e25s^8 + 3.456e27s^7 + 8.71e29s^6 + 1.609e32s^5 + 2.145e34s^4 + 2.004e36s^3 + 1.242e38s^2 + 4.574e39s + 7.566e40$$

The compensator, according to equation 4.23, shows clearly that it is a very complicated sub-system with two inputs and two outputs. The order of its denominator is 16, while at all numerators is 15th order. It is also noticeable, that the large coefficients weaken the practicality of using optimal control. Transforming any optimal controller from the time domain to frequency domain reveals unexpected complexity. Even if the observer was low order, it would not reduce the complexity of the overall compensator, to an acceptable manner.

The simulation model, according to this compensator (in Laplace domain) is shown in the Appendix. Unity decoupled feedback is proposed. Another simulation model, in the state space form, was built and tested also; to check the transformation and simulation results. The model in Laplace domain is adopted for comparison purposes.

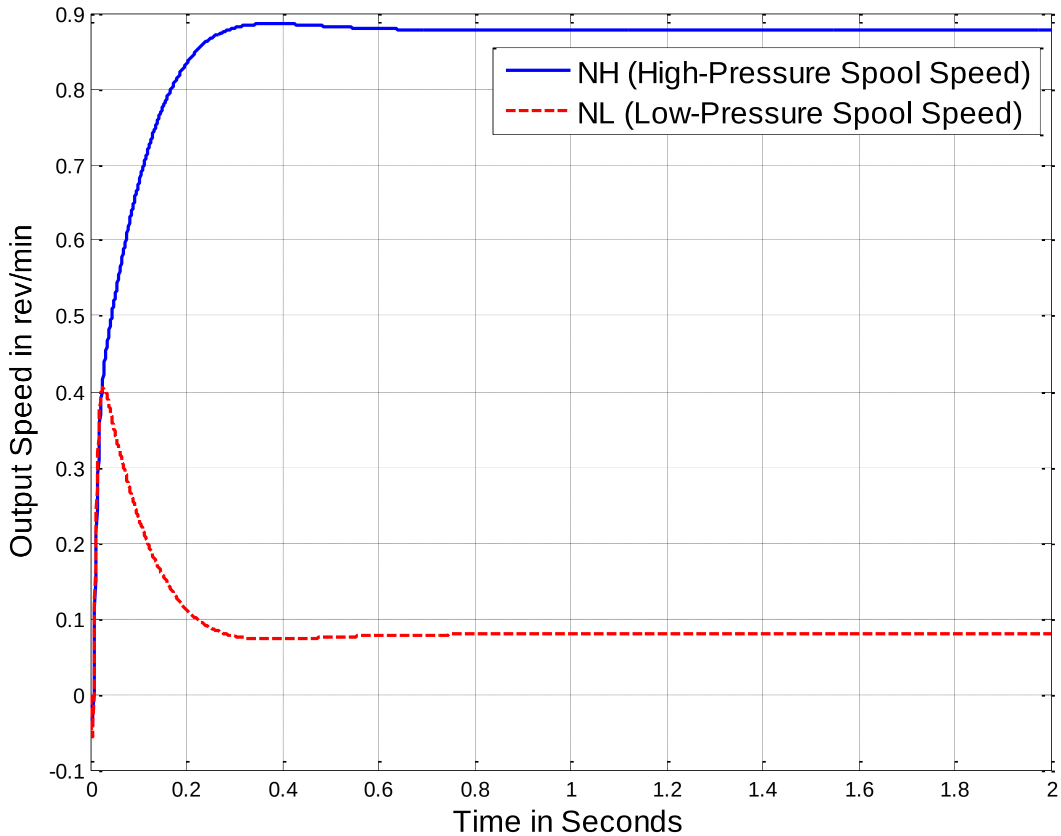


Figure 4.16, Closed Loop Response Following a Unit Step Change on $r_1(s)$

The closed loop response following a unit step change on the first reference input is shown in figure 4.16.It shows a quick stable response for both outputs. Rising and settling time are very short (less than half a second). It is also noticeable, that the interaction of the second output ΔN_L is successfully reduced to less than 10 percent. Moreover, both transient responses are acceptable. However, the response of the first output ΔN_H, attracts a steady state error, that could not be eliminated, unless reserving an integrator action.

On the other hand, the closed loop response following a unit step change on second reference input is shown in figure 4.17.

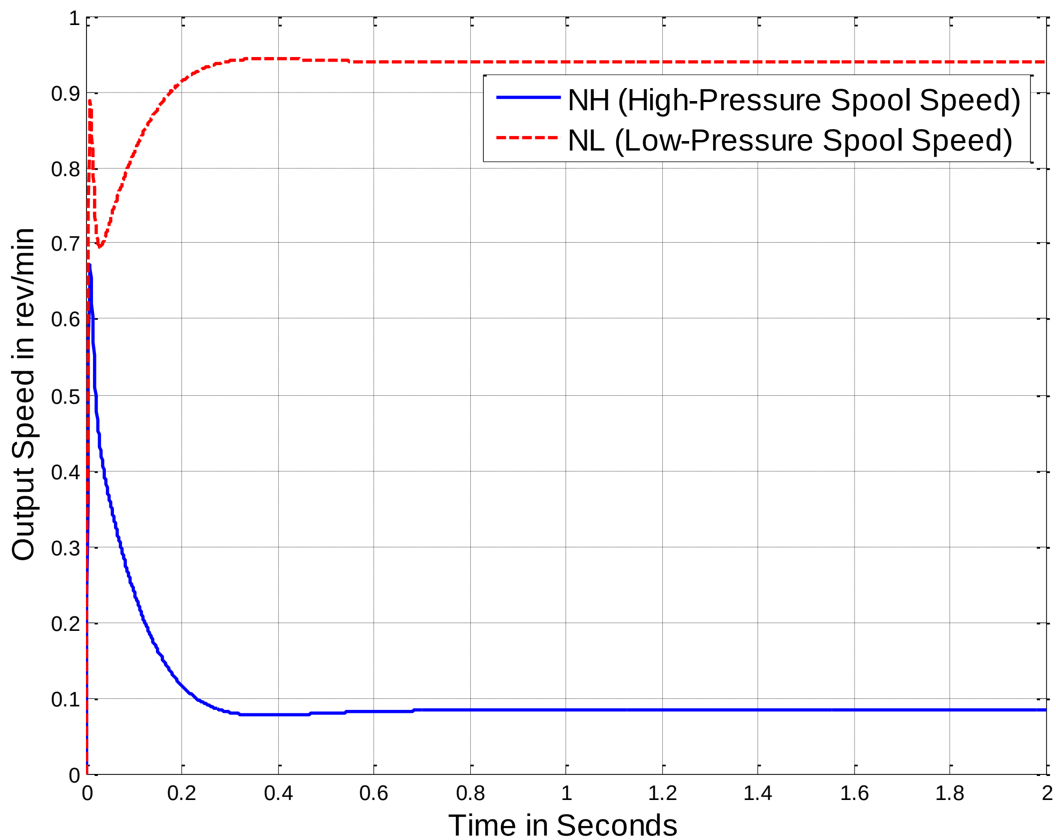


Figure 4.17, Closed Loop Response Following a Unit Step Change on $r_2(s)$

The response is stable for both outputs, but a steady state error also is clearly occurring for the second output ΔN_L. Meanwhile, the first output ΔN_H interaction is also successfully minimized to less than 10 percent. However, its transient response shows a large overshoot in zero time before settling down quickly, in less than half a second.

Both responses shown in figures 4.16 and 4.17, are similar to those achieved by Mueller, as reviewed by McMorran (1970). Although Mueller used a low order observer, which would result in a simpler compensator than the full order observer, the closed loop responses also recovered from steady state errors that could not be eliminated without an integrator action.

4.4 Comparison Study

Each technique for designing a multivariable controller has its unique difficulties in applying such approaches. There are advantage, disadvantage and trade-offs between the strategies employed. A detailed comparison study between these methodologies follows, concerning four important factors. The first one is the difficulties of application and the practicality of the resulting controllers. Second factor to be compared, is of course the closed loop response following a unit step change on each input independently. Disturbance rejection analysis is a vital factor in aircraft jet engine performance, so that a comparison for the closed loop responses following a unit step change disturbing each output independently is studied. The last factor is the energy consumed by the controller itself, in order to validate the least effort controller.

Least Effort Control is considered a British methodology, which requires designing a controller for the system transfer function matrix in the Laplace domain. However, the transfer function matrix should be approximated in order to meet requirements of equation 3.23. The guidance for the approximation is that approximately the same transient and steady state values as the original transfer function, must be achieved. This would not necessarily guarantee the effectiveness of the resulting controller applied to the original system, especially for complicated systems with more than two inputs/outputs. However, there are many approximations to assess and no definitive conclusion may be drawn.

The Inverse Nyquist Array on the other hand, is also considered a classical British methodology. Its main difficulty lies on achieving the diagonal dominance, in order to reduce the system coupling, before the application of single-input single-output design techniques. However, there is no clear systematic approach to find that special pre-compensator that would induce diagonal dominance, nor is there a method enabling a "best" pre-compensator to be selected.

Optimal control is considered an "American School" methodology, which requires a system model in state space form. Although the advantages of this form regarding system feature analysis, such as Controllability and Observability, the optimal controller is considered a synthesis technique, not a real design method. In other words, there is no guidance for improving the closed loop response except altering the weighting matrices in the performance index in some arbitrary manner. Moreover, constructing an observer to estimate non-measurable states adds complexity to the control system design and does not inspire confidence.

The resulting controller, by least effort control, is clearly the simplest one. According to equations 4.6 and 4.7, the feed forward and the feedback are simple gains only. Meanwhile, the Inverse Nyquist Array method results in first and second order compensators, according to equation 4.11, with unity decoupling feedback loops. By way of contrast with those achieved by

British School, the optimal control resulted in a very complicated overall compensator including an observer and state feedback controller. This is obviously seen after transforming the compensator into the Laplace domain which results in a compensator of the 15th order according to equation 4.23, with very large coefficients values and unity decoupled feedback loops. One may argue that using a low order observer would simplify the overall compensator; however it is still much higher in degree and more complicated than the British ones.

Closed loop responses following a unit step changes on each reference input separately, are compared, according to each controller, as follows in figures 4.18 and 4.19.

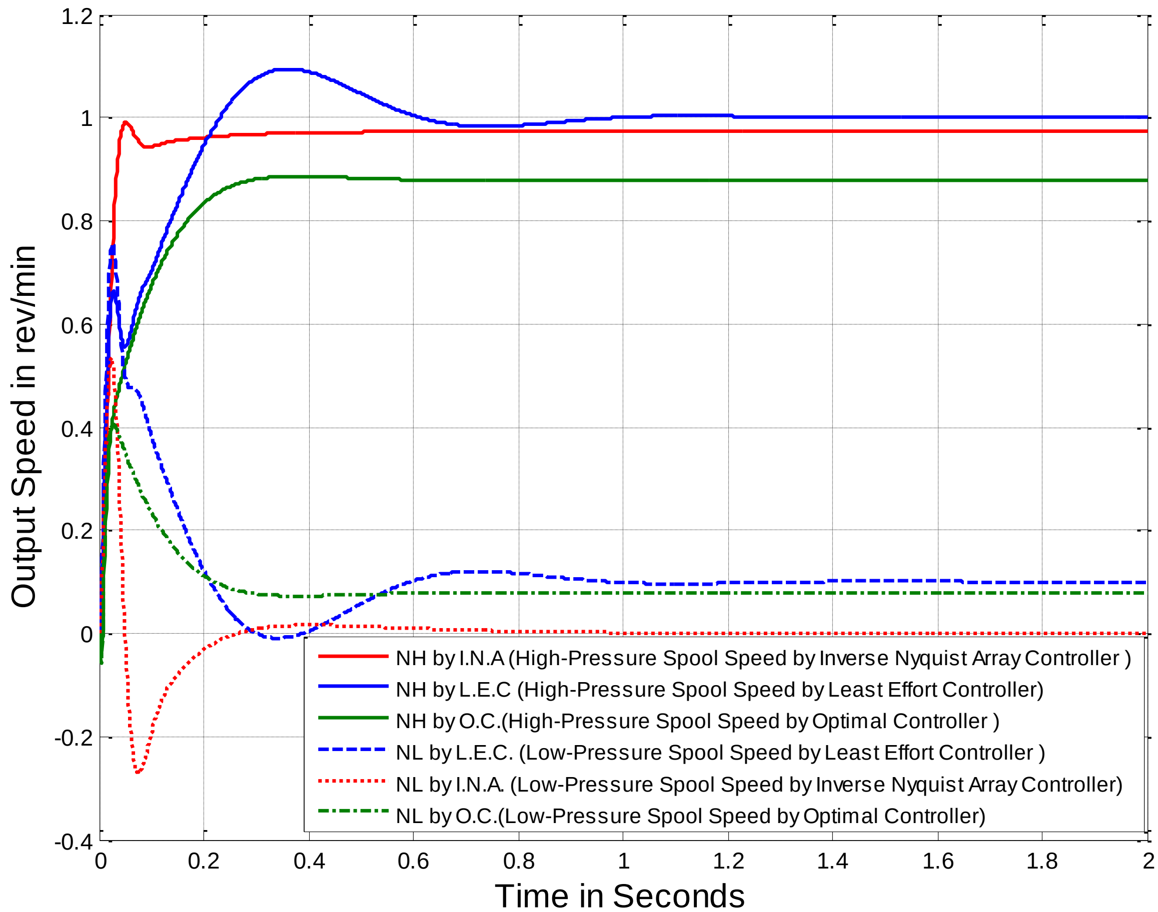


Figure 4.18, Closed Loop Response Following a Unit Step Change on $r_1(s)$ Compared Between Least Effort, Inverse Nyquist Array and Optimal Controllers

From figure 4.18, it is clearly seen that Least Effort Control results in the best first output ΔN_H response compared to the others, which suffer from steady state errors. Meanwhile, the interaction of the second output ΔN_L is completely removed by the Inverse Nyquist Array controller, compared with almost 10 percent, by Least Effort and Optimal Controllers. Moreover, Inverse Nyquist Array shows an acceptable transient response for ΔN_L, compared with the large overshoots that settle down quickly, by Least Effort and Optimal controllers.

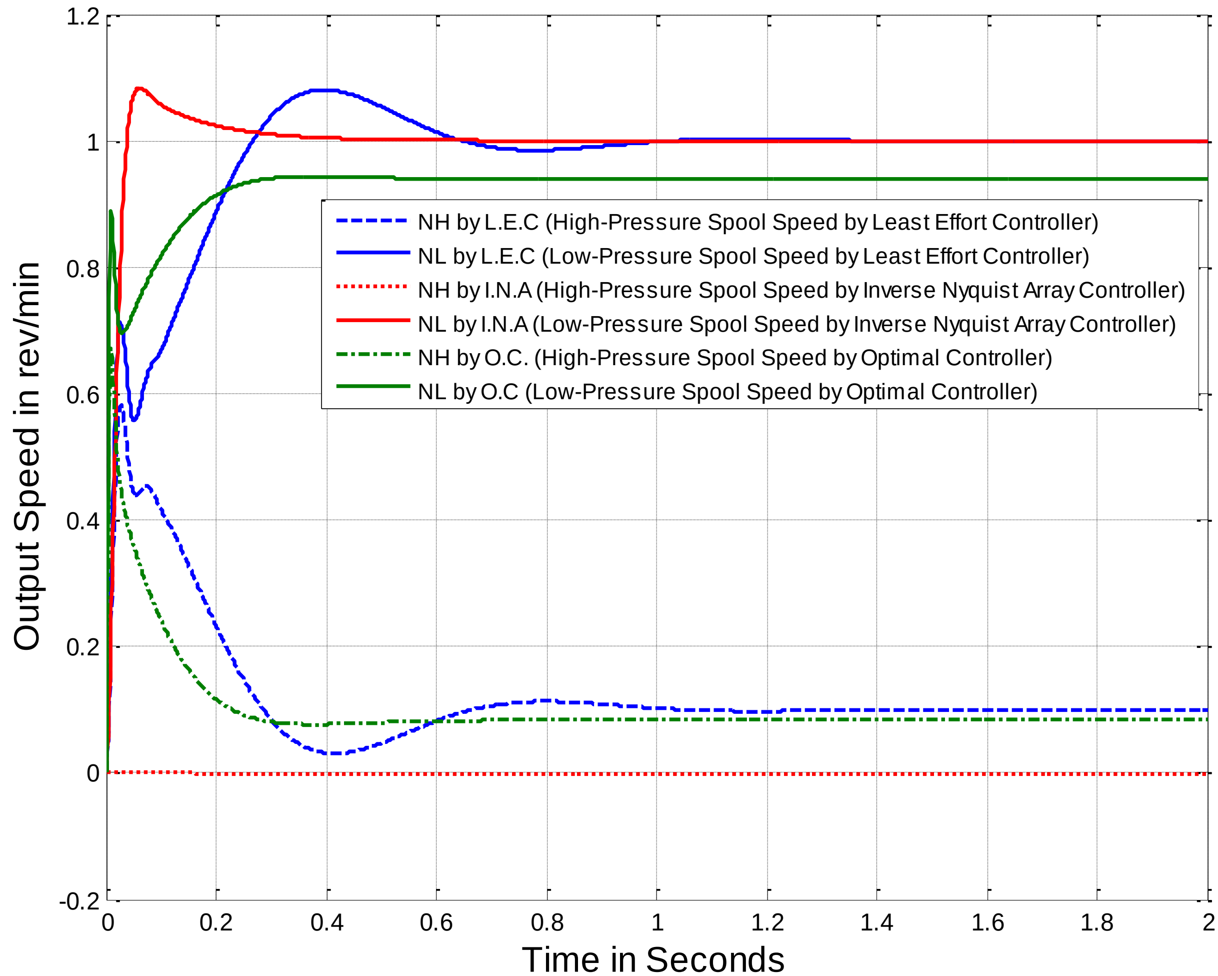


Figure 4.19, Closed Loop Response Following a Unit Step Change on $r_2(s)$ Compared Between Least Effort, Inverse Nyquist Array and Optimal Controllers

In a similar manner, from figure 4.19, it is shown that the second output ΔN_L responses are comparable between Least Effort Control and Inverse Nyquist Array. The effect of using proportional plus integral action by the Inverse Nyquist Array controller, clearly results in eliminating the steady state error while speeding up the response. However, the Least Effort Control response is comparable since the settling time is less than a second. On the other output ΔN_H responses, Inverse Nyquist Array controller results in the least interaction and best transient compared with others. Optimal control presents the worst response because of its steady state error.

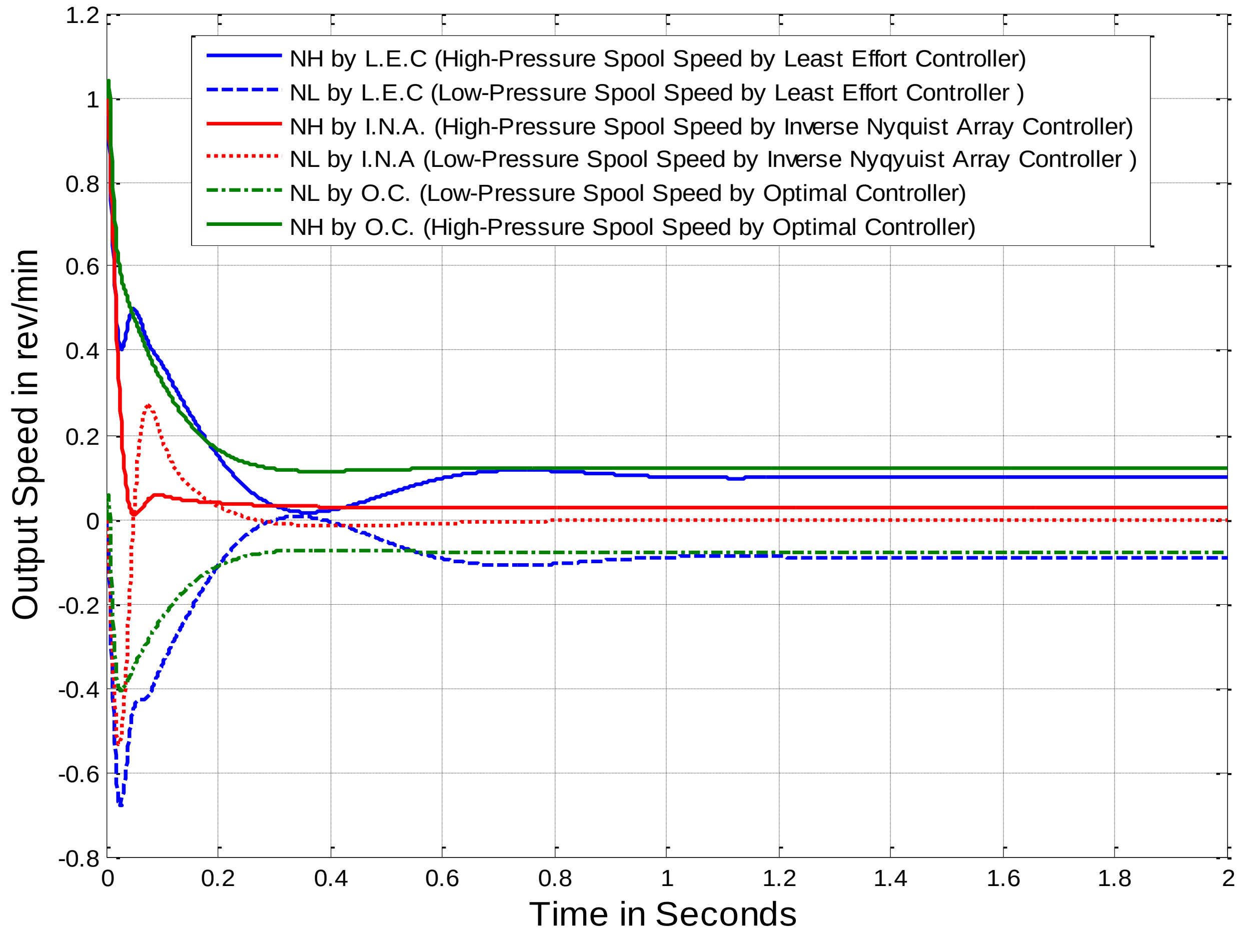


Figure 4.20, Closed Loop Response Following a Unit Step Change on $\delta_1(t)$ Compared Between Least Effort, Inverse Nyquist Array and Optimal Controllers

Despite the fact that disturbances occur randomly and do not last for long period of time ; the closed loop responses following a unit step disturbance change on each output separately, are employed for comparison and analysis of different controllers under disturbance suppression conditions.

As shown in figure 4.20, it is clearly seen that Inverse Nyquist Array controller results in the best disturbance recovery responses for both outputs. It recover about 95 percent of disturbance on ΔN_H in less than 0.1 second, with a diminished interaction of ΔN_L. However, Least Effort and Optimal Controllers result in comparable responses, where both of them recover 90 percent of disturbance on ΔN_H in less than a second with limited interaction of ΔN_L(about 10 percent).

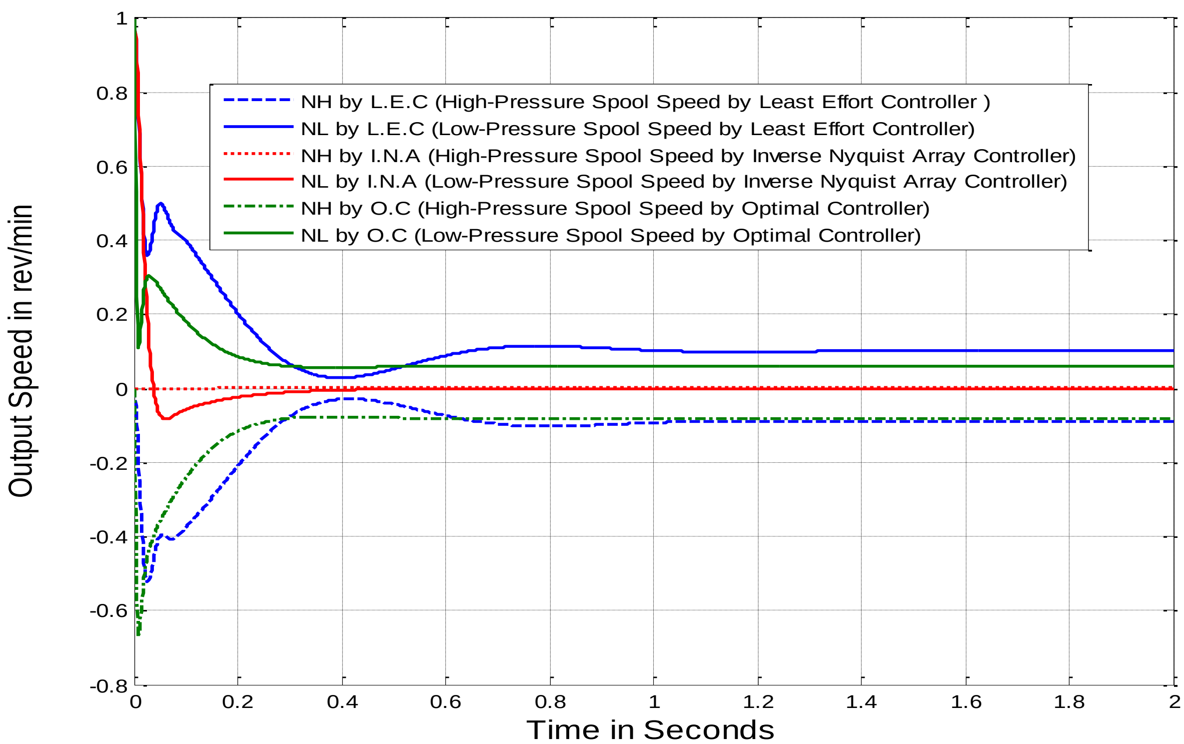


Figure 4.21, Closed Loop Response Following a Unit Step Change on $\delta_2(t)$ Compared Between Least Effort, Inverse Nyquist Array and Optimal Controllers

It is also clearly shown from figure 4.21; that Inverse Nyquist Array controller results in the best disturbance recovery response. It recovers 100 percent of disturbance on ΔN_L in less than half a second with no interaction of ΔN_H. Meanwhile, Optimal controller recovers about 95 percent of disturbance on ΔN_L in less than half a second with limited interaction (almost 10 percent) of ΔN_H . On the other hand, Least Effort Controller is still comparable with others; is that it recovers 90 percent of a step disturbance on ΔN_L with also limited interaction (almost 10 percent) of ΔN_H.

Although the Inverse Nyquist Array Controller results in the best disturbance recovery responses, which is refer to the use of the integrator, the Least Effort Controller still comparable without the resorting to integrator action.

Finally, the energy consumed by the controller could be computed according to (Whalley and Ebrahimi ,2006) by :

$$E(t) = \int_{t=0}^{t=600} (u_1^2(t) + u_2^2(t))dt \qquad 4.24$$

Simulation models could be constructed according to equation 4.24 as shown in the Appendix. Following a random disturbance on both outputs for a period of 10 minutes, the energy that is consumed by each controller could be computed as shown in figure 4.22.

It is clearly seen from figure 4.22 , that the Optimal Controller is the most expensive one , followed by Inverse Nyquist Array Controller . It is obviously demonstrated that Least Effort Controller consumes the least control energy.

This would reflect directly on improving fuel consumption of gas turbine engines, especially, that one of the controlled inputs is the demanded fuel flow rate into combustion chamber of turbo jet engine.

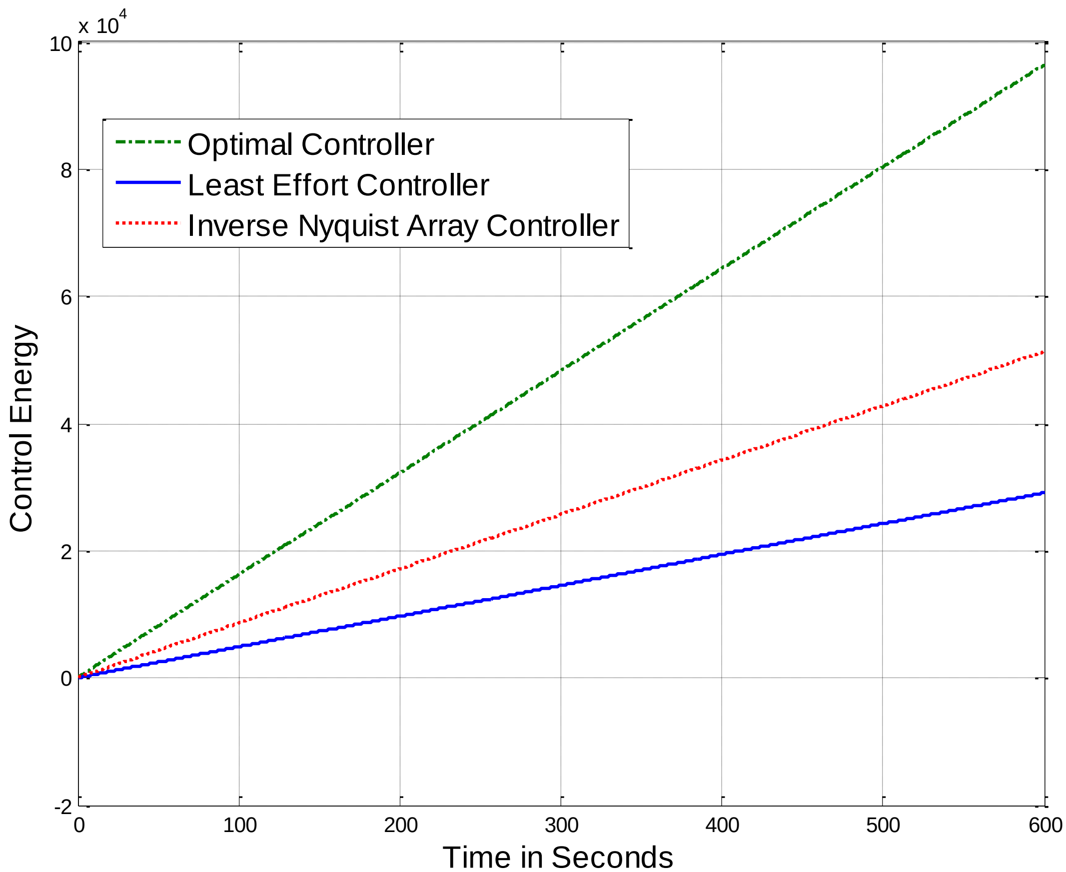


Figure 4.22, Energy Consumed by the Controller Compared Between Least Effort , Inverse Nyquist Array and Optimal Controllers

Chapter V

Conclusions and Recommendations

5.1 Conclusions

- The Least Effort Control (L.E.C) methodology generates a reliable controller. It meets the requirements and control objectives of jet engines. The controller's response is stable quick and well behaved. Despite the strong coupling of the twin-spool jet engine system, the controller succeeds in reducing the output interaction, as specified. This reflects the power of its design philosophy, which gives freedom to enhance the performance and the closed loop response of multivariable systems. The inner loop design provides a powerful tool to stabilize the system and improve the transient response. Meanwhile, the outer loop design is flexible and reduces the output interaction and enhances the disturbance recovery rates. The entire strategy is based upon minimum control effort.
- The Least Effort Controller satisfied all the research objectives. It proved that it is superior among other controllers, which are designed by Inverse Nyquist Array and Optimal Control methods. First, it is the simplest one, which makes it ideal for implementation purposes due to the low cost of application. Second, its performance is comparable with the others, even in presence of disturbances. Third, it proved that it is the least effort controller, which results in the improvement of the engine's fuel consumption and low operating costs.

- The only difficulty, which may face the Least Effort Control method in application among time-invariant systems; is the complexity of the system transfer function matrix. Approximations may be required to reduce the order of the transfer functions to fit with formulas. Moreover, systems with more than two inputs and two outputs complicate the optimization for control effort. However, these difficulties are easier to overcome compared with diagonal dominance achievement for Inverse Nyquist Array method, and the observer design and weighting matrices selection, for Optimal Control method.

5.2 Recommendations

- Least Effort Control is recommended to be applied in the future to more complicated systems, especially with more than two-inputs and two-outputs. Both approximation and optimization techniques developments are required.
- It is recommended that a design package for controls using Least Effort method, over the entire operating envelope of the jet engine is investigated and tested in comparison with existed controls.
- The theory of Least Effort Control is recommended to be extended, to include digital control theory. This is would pave the way for future implementations into gas turbines regulation and other applications.

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Appendix

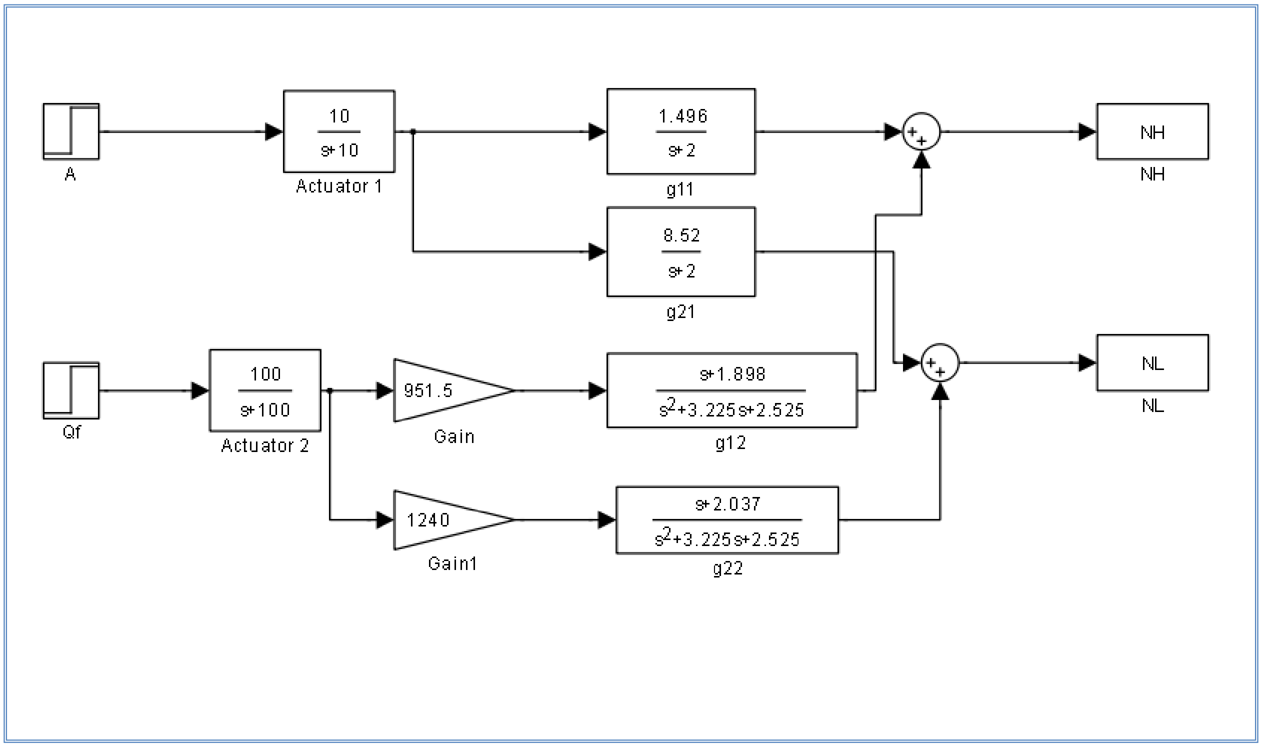

Figure A.1 Open Loop Simulation Model

1. **Disturbance Rejection Analysis for Least Effort Control Methodology:**

In many multi-body electro-mechanical systems , the only continuously perturbed signals entering the system , arise from changes in $\delta(s)$ so that with $r(s) = 0$, equation (2.6) becomes

$$y(s) = S(s)\delta(s) \qquad \text{A.1}$$

Where

$$S(s) = (I_m + G(s)(k(s) >< h(s) + PF))^{-1}$$

The sensitivity matrix $S(s)$ in equation (A.1) should be such that changes in $\delta(s)$result in acceptable changes in $y(s)$. These output excursions arise from relatively low frequency variations in $\delta(s)$ as discussed by Skogestad and Postlethwaite . Moreover , since , for physically realizable , electromechanical systems , $G(s)$ must be strictly proper , the closed-loop system provides no attenuation at high frequencies as $S(s) \rightarrow I_m$.

Consequently, the adjustment of the steady-state system disturbance suppression properties, by increasing f in equation (A.1) , would ensure enhanced attenuation following 'low frequency ' perturbations. Moreover , for increasing values of frequency , the singular values of $S(s)$ should be smooth, differentiable , and almost monotonic curves, providing a prudent policy of regulation is pursued.

The attenuation secured by the system lies between the lower and upper singular values of the $S(s)$matrix , as shown by Zames .

Specifically

$$\underline{\lambda}(S(s)) \leq \frac{\|y(s)\|_2}{\|\delta(s)\|_2} \leq \overline{\lambda}(S(s)) \qquad \text{A.2}$$

Where $\underline{\lambda}(S(s))$ and $\overline{\lambda}(S(s))$ are the smallest and largest singular value amplitudes of $S(s)$, respectively , and $\|\blacksquare\|_2$ denotes the Euclidean norm , $s = i\omega$, $0 \leq \omega \leq \infty$, of $y(s)$ and $\delta(s)$, respectively.

It is also apparent from equation (A.2) that providing the upper bound $\overline{\lambda}(S(s)) \leq 3dB, s = i\omega$, $0 \leq \omega \leq \infty$, the system will be stable with a maximum 'high' frequency resonant peak ≤ 1.4 , $\|\delta(s)\|_2 \neq 0$, once the closed-loop outputs are substantially decoupled.

The singular values of the low frequency sensitivity matrix, can be computed in the frequency domain for $f = 0$, from

$$\det[I_m\lambda^* - (I_m + G(s)(k(s) >< h(s)))^{-1}]_{s=i\omega} = 0$$

The steady-state singular values may be estimated from

$$\det[I_m\lambda^* - (I_m + G(0)(k(0) >< h(0)))^{-1}] = 0$$

as$G(0)(k(0) >< h(0))$ has m singular values of

$\gamma_1 = \gamma_2 = \cdots . \gamma_{m-1}and\gamma_m = \sum_{j=1}^{m} \overline{k}_j h_j$

Where $\bar{k} = G(0)k(0)$ A.3

Consequently , it is easy to show that the singular values , λ_j^*, $1 \le i \le m,$ of equation (A.2) are

$\lambda_1^* = \lambda_2^* = \cdots .. \lambda_{m-1}^* = 1$and

$\lambda_m^* = \frac{1}{1+\sum_{j=1}^{m} \bar{k}_j h_j(0)}$ A.4

Where $\bar{k}_j$ is the j^{th} element of $\bar{k}$.

$S(s) = (1-f)(I_m + G(s)k(s) >< h(s))^{-1}$, so that the low frequency singular value amplitudes of $S(s)$ are reduced to λ_j where

$\lambda_j = (1-f)\lambda_j^*,\ \ 1 \le j \le m.\ \ 0 < f < 1$ A.5

Consequently, the bound on the low frequency attenuation properties of the system becomes

$\underline{\lambda}(S(0)) \le \frac{\|y(0)\|_2}{\|\delta(0)\|_2} \le \bar{\lambda}(S(0))$ A.6

The estimate from equation (A.6) is conservative. An accurate measure of the low frequency disturbance attenuation properties of the system can be easily computed from the direct evaluation of equation $y(s) = S(s)\delta(s)$.

In this regards if

$\bar{s}\big(S(0)\big)$=largest element of $S(0)$

$\underline{s}\big(S(0)\big)$=smallest element of $S(0)$

Then

$$\underline{s}(S(0)) \le \frac{\|y(0)\|_2}{\|\delta(0)\|_2} \le \bar{s}(S(0))$$

Provides an improved , low frequency attenuation bound .

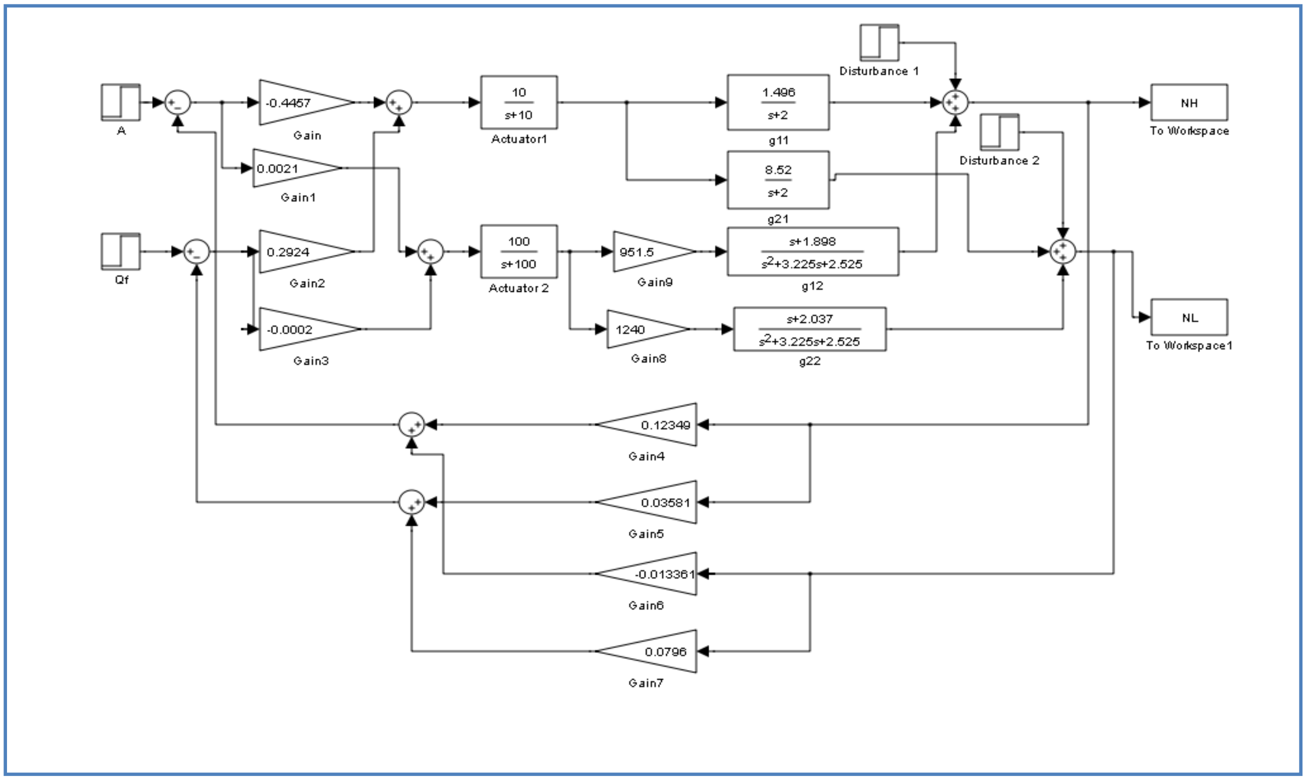

Figure A.2 Closed Loop by Least Effort Controller with $f = 0.1$ Simulation Model

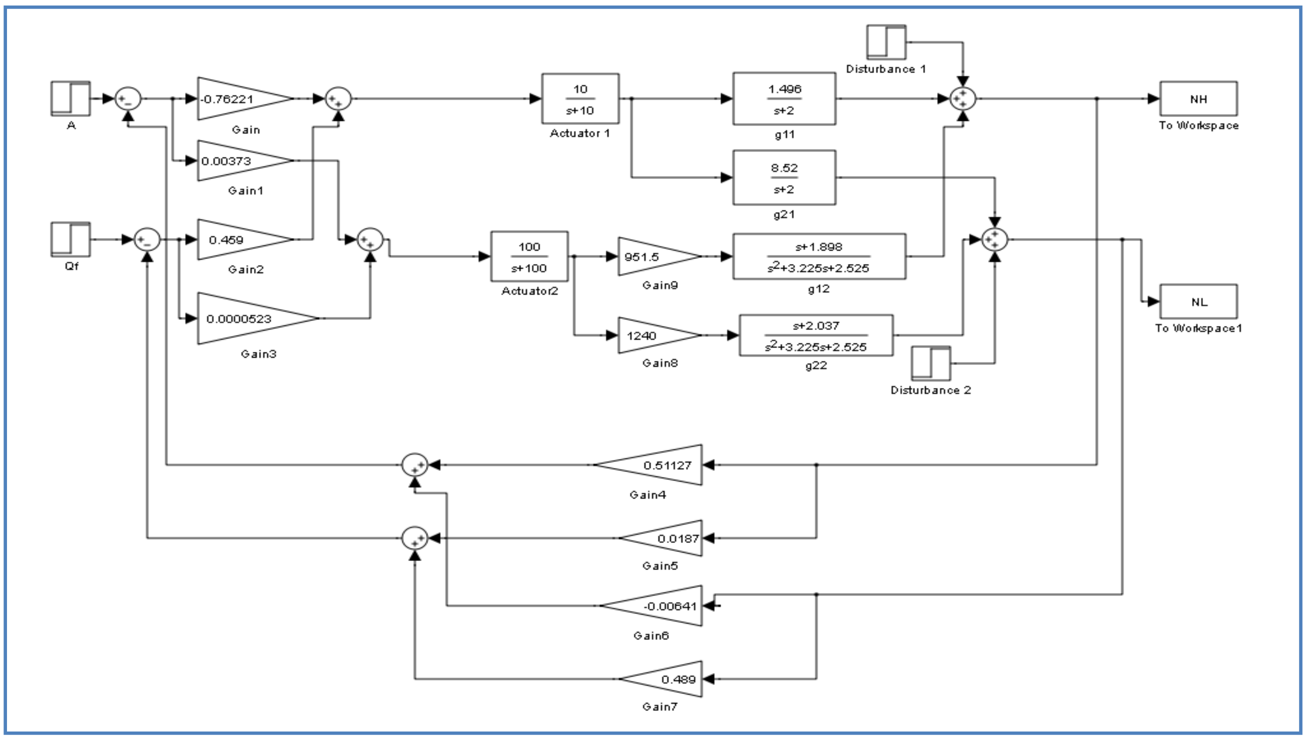

Figure A.3 Closed Loop by Least Effort Controller with $f = 0.5$ Simulation Model

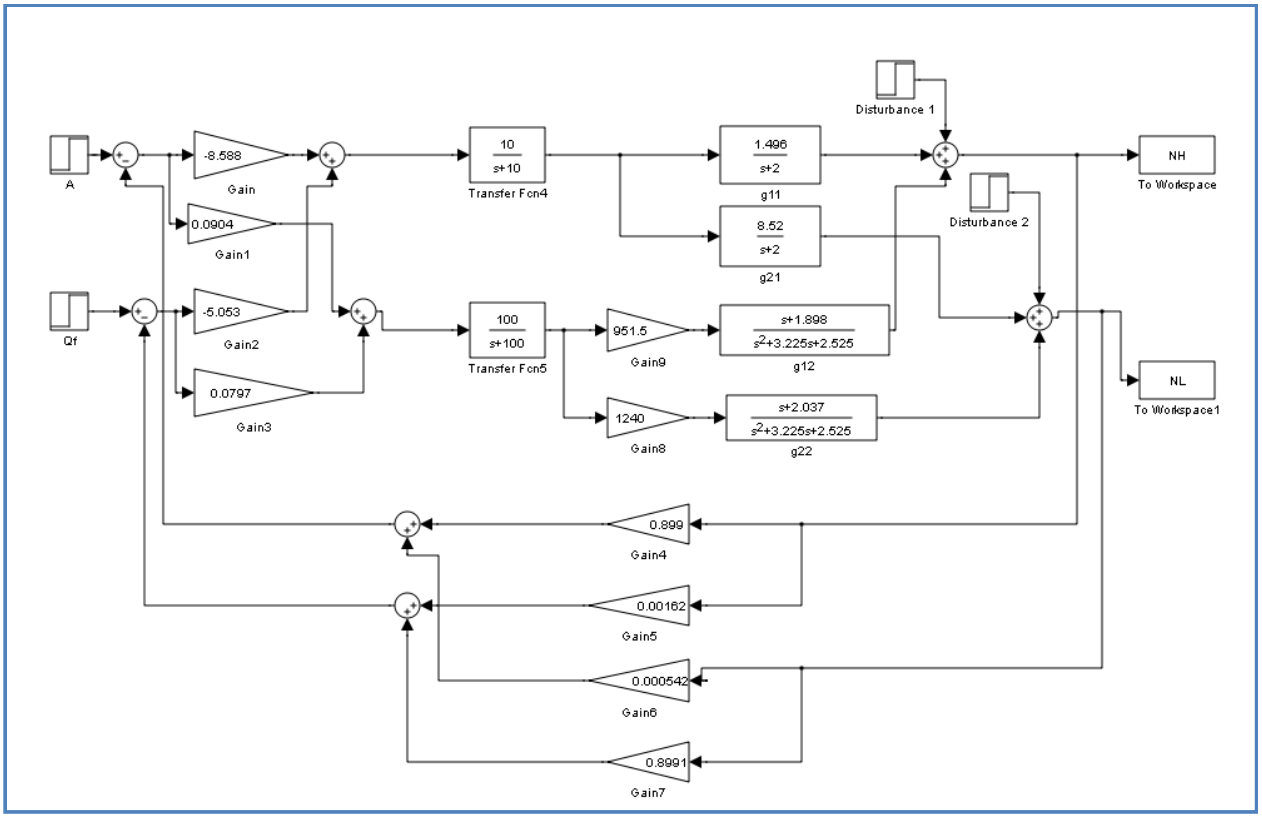

Figure A.4 Closed Loop by Least Effort Controller with $f = 0.9$ Simulation Model

2. Inverse Nyquist Array methodology m.file :

```
G=tf({14.96  [95150 95150*1.898];85.2  [124000 124000*2.037]},{[1 12 20]
[1.0000  103.2250  325.0250  252.5000];[1 12 20]  [1.0000  103.2250
325.0250  252.5000]}); % Transfer Function Matrix
k1=[1000 0;0 1];% first precompensator
k2=tf({[-1.57 -1.57*20.6] [1.205 1.205*11.6];[1.08 1.08*146.3] [-0.189 -
0.189*101.4]},{[1 158.5] [1 158.5];[1 158.5] [1 158.5]});%2nd Compensator
Q=inv(G*k1*k2);% Inversed Overall Transfer Function Matrix
gershband(Q);% creates Figures 4.11 and 4.12

function gershband(a,b,c,d,e)
%GERSHBAND - Finds the Gershorin Bands of a nxn LTI MIMO SYS model
% The use of the Gershorin Bands along the Nyquist plot is helpful for
% finding the coupling grade of a MIMO system.
%
% Syntax: gershband(SYS) - computes the Gershgorin bands of SYS
%         gershband(SYS,'v') - computes the Gershgorin bands and the
%                              Nyquist array of SYS
% Inputs:
%    SYS - LTI MIMO system, either in State Space or Transfer Function
%    representation.
```

```
%
% Example:
%    g11=tf(2,[1 3 2]);
%    g12=tf(0.1,[1 1]);
%    g21=tf(0.1,[1 2 1]);
%    g22=tf(6,[1 5 6]);
%    G=[g11 g12; g21 g22];
%    gershband(G);
%
% Other m-files required: sym2tf, ss2sym
% Subfunctions: center, radio
% See also: rga
%
% Author: Oskar Vivero Osornio
% email: oskar.vivero@gmail.com
% Created: February 2006;
% Last revision: 11-May-2006;

% May be distributed freely for non-commercial use,
% but please leave the above info unchanged, for
% credit and feedback purposes

%------------- BEGIN CODE --------------
%--------- Determines Syntax -----------
ni=nargin;

switch ni
    case 1
        %Transfer Function Syntax
        switch class(a)
            case 'tf'
                %Numeric Transfer Function Syntax
                g=a;

            case 'sym'
                %Symbolic Transfer Function Syntax
                g=sym2tf(a);
        end
        e=0;
    case 2
        %Transfer Function Syntax with Nyquist Array
        switch class(a)
            case 'tf'
                %Numeric Transfer Function Syntax
                g=a;

            case 'sym'
                %Symbolic Transfer Function Syntax
                g=sym2tf(a);
        end
        e=1;

    case 4
        %State Space Syntax
        g=ss2sym(a,b,c,d);
```

```
        g=sym2tf(g);
        e=0;
    case 5
        %State Space Syntax
        g=ss2sym(a,b,c,d);
        g=sym2tf(g);
        e=1;
end
%---------------------------------------
[n,m]=size(g);
w=logspace(-1,6,200);
q=0:(pi/50):(2*pi);

for i=1:n
    for j=1:m
        if i==j
            figure(i)
            nyquist(g(i,i));
            grid on
            title(['Nyquist Diagram of G(',num2str(i),',',num2str(j),')'])
            for iest=1:n
                for jest=1:m
                    if iest~=jest
                        hold on
                        C=center(g(i,j),w);
                        R=radio(g(iest,jest),w);
                        for k=1:length(C)

plot((R(k)*cos(q))+real(C(k)),(R(k)*sin(q))+imag(C(k)),'g-')
                        end
                        hold off
                    end
                end
            end
        end
    end
end

if e==1
    figure(n+1)
    nyquist(g);
    grid on
end
%------------ Subfunction --------------
function C = center(g,w)
g=tf2sym(g);
C=subs(g,complex(0,w));

function R = radio(g,w)
g=tf2sym(g);
R=abs(subs(g,complex(0,w)));
%------------- END OF CODE --------------
```

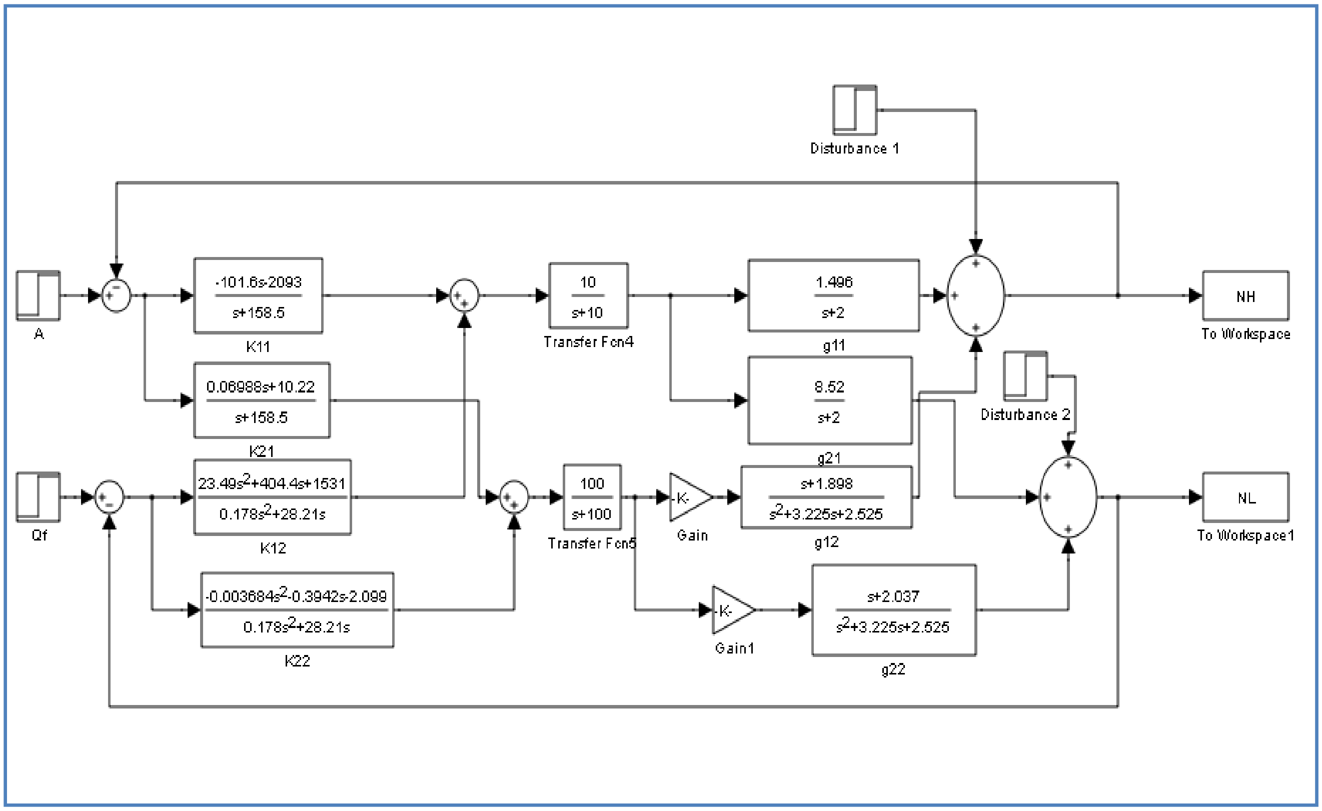

Figure A.5 Closed Loop by Inverse Nyquist Array Controller Simulation Model

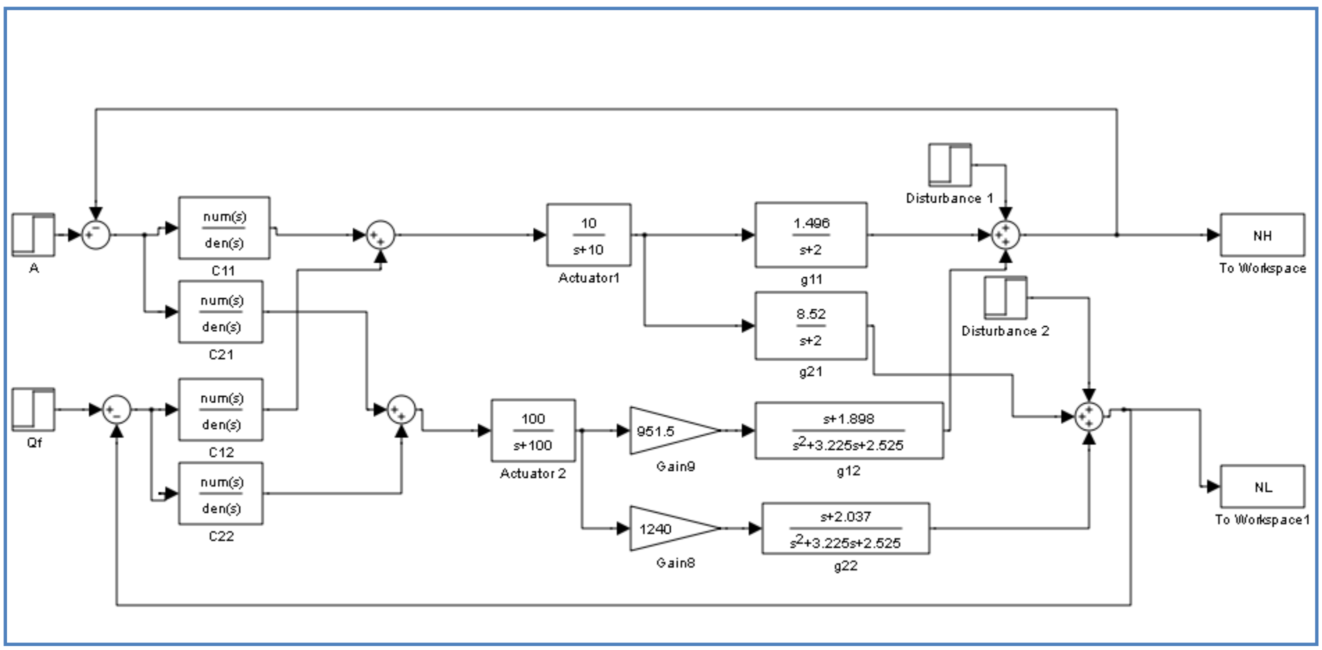

Figure A.6 Closed Loop by Optimal Controller Simulation Model

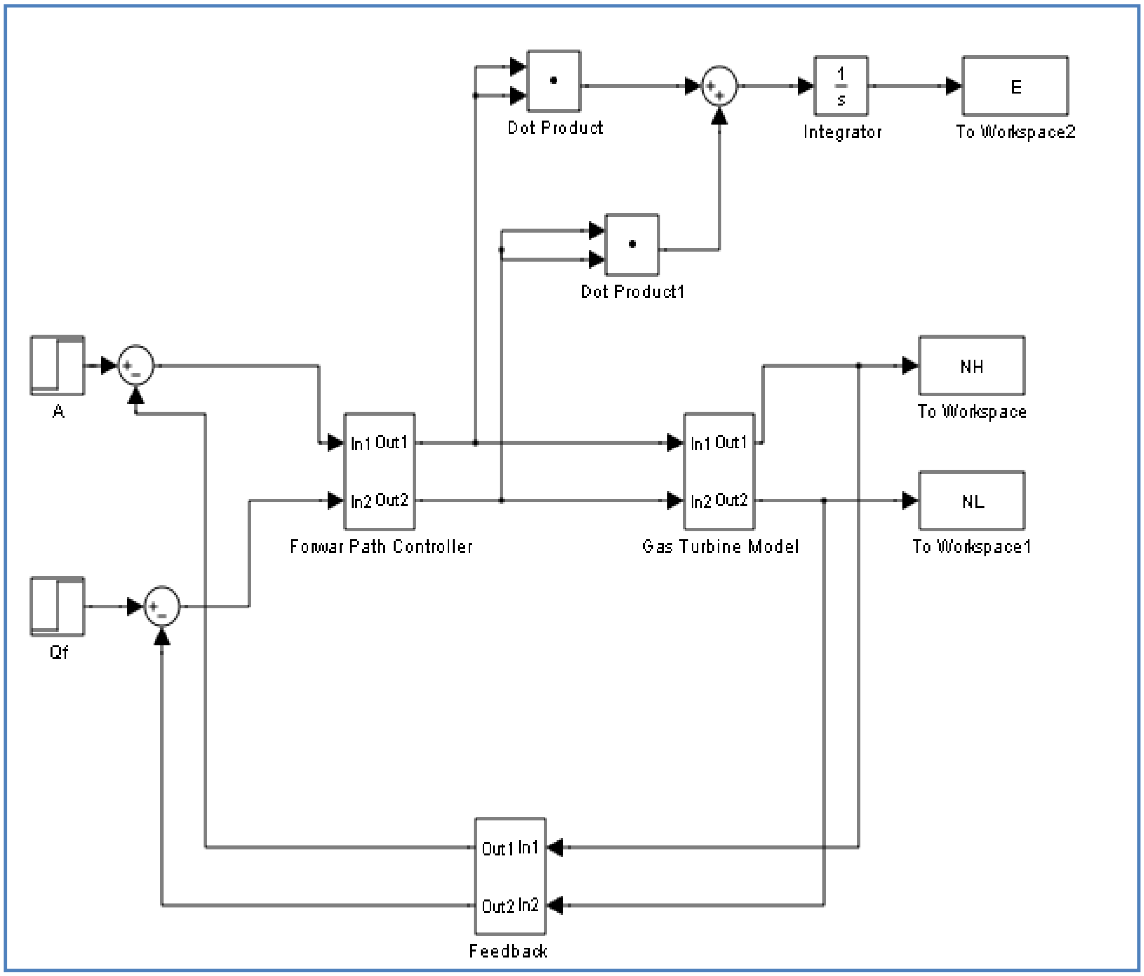

Figure A.7 Control Effort Simulation Model