Thinking Strategies Used While Engaged in Solving the Tower of Hanoi, the River-crossing and Find the Pattern Puzzles

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Abstract

This study investigates thinking strategies of 100 students belonging to five age groups, namely, third graders, eighth graders, undergraduate sophomores and graduates. This study hypothesizes that the younger groups will be able to solve puzzles with more ease, and that the more complex and advanced the thinking strategies are, the harder it will be to solve the puzzles. The participants are asked to solve three puzzles, Tower of Hanoi, River crossing puzzle (Farmer and animals, Missionaries and Cannibals) and Find the pattern (months of the year). The participants are then asked to explain what thinking strategies they used to help them during the problem-solving process. The findings revealed eight thinking strategies- the use of mental imagery, reading and thinking aloud, trial-and-error or repetition, planning ahead, imitation or modeling, accessing prior knowledge, gaining hints and clues from surroundings, and the use of physical cues or gestures. Results show that students used a variety, sometimes a combination of thinking strategies.

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Chapter One- Introduction

The purpose of this study was to investigate the thinking process involved while solving puzzles in five different age groups. The setting was a private American school in Dubai and an American university in Sharjah, United Arab Emirates. This study was based on the assumption that individuals belonging to younger age groups will solve puzzles with ease because their thinking processes will be basic and simple as opposed to adults who will tend to use advanced and complex thinking processes.

This chapter introduces the puzzles and explains the rationale of this research. A literature review relevant to the study is also presented. Chapter Three covers the methodology used for the investigation. Chapter Four provides the results of the study followed by Chapter Five providing an analysis of the results. The dissertation ends with the limitations and conclusion in Chapters Six.

1.1 Puzzles

People have been solving puzzles for centuries. People solve them to pass time, improve their critical thinking skills, and some solve them simply for enjoyment. Puzzles involve finding a solution to a problem, some puzzles are timed, others require more than one player, and some even have rewards. People are fascinated with puzzles. Television aired game shows of a puzzle or problem-solving natures have gained a lot of popularity over the past few years such as “Who Wants to be a Millionaire?” , “Are You Smarter Than a Fifth Grader?” and “Jeopardy.” Contests are held in schools and cities where audiences gather to watch people compete in playing these mind challenging games such as chess.

For this research, three well-known puzzles will be used, namely, the Tower of Hanoi, the River-crossing puzzle and Find the Pattern.

Tower of Hanoi

The first puzzle given to all the groups is called the Tower of Hanoi. It is a mathematical puzzle, sometimes also known as the Towers of Brahma. There is dispute about who actually invented this puzzle. Initially it was believed the Tower of Hanoi was invented by the French mathematician, Edouard Lucas in 1883. However, in recent years, experts believe that it was an invention of the famous pioneer of disc-based puzzles, Harrison Heath. The Tower of Hanoi problem consists of three vertical pegs, and a minimum of three discs piled on the first peg (see Fig 1.1). These discs are arranged in the order of size, with the largest disc being placed at the bottom and the smallest on top. The player has to move all the discs from the first peg to the last peg in the same order. Only one disc can be moved at a time and a disc of larger size cannot be placed on a disc of smaller size. Fig. 1.2 shows some legal moves to solve the three discs problem.

River-Crossing Puzzle

According to Peterson, (2003) puzzles that require one or more individuals to transport their belongings across a river have been around for centuries. The version of this puzzle which will be used for Group 1 (See Fig. 1.3) involves a farmer who has to transport cabbage, a sheep and a wolf to the other side. However, in the absence of the farmer the wolf eats the sheep and the sheep eats the cabbage. Therefore, the animals and cabbage have to be transported to the other side without getting eaten. The solution to the problem is to move the sheep across first, then move the cabbage; bring the sheep back and transport the wolf, and then finally come back for the sheep. This puzzle dates back to the eighth century and the writings of Alcuin of York, who was a poet, educator and a cleric. (Peterson, ¶ 5, 2003).

The river-crossing puzzle has many variations dating back to different centuries. For Groups 2-5, the nineteenth century version will be used. It includes three missionaries and three cannibals (See Fig. 1.4) who have to cross to the other side of the river on a boat that can only carry two passengers at one time. Peterson (2003) explains that in this version, the cannibals must never be allowed to outnumber missionaries on either bank. It takes a minimum of nine trips to get everyone across the river. First a missionary and a cannibal cross; the missionary returns; two cannibals cross; one cannibal returns; two missionaries cross; one missionary and one cannibal return; two missionaries cross; one cannibal returns; and the remaining two cannibals cross (Peterson, ¶ 12, 2003).

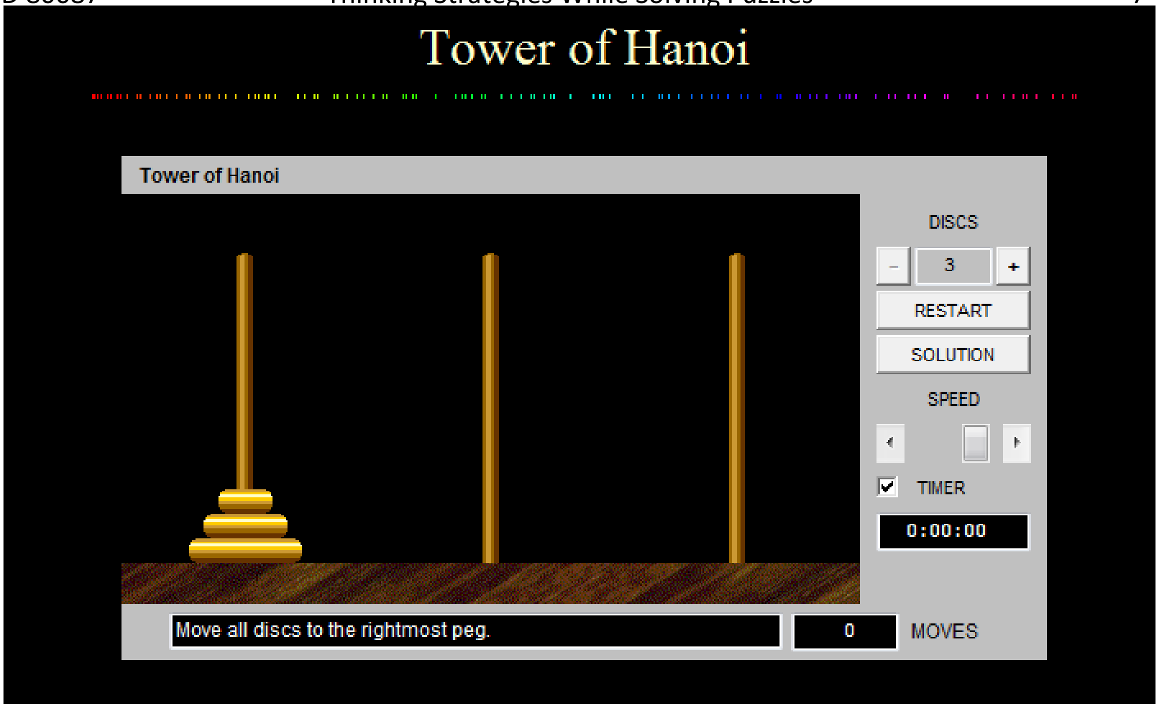


Fig. 1.1. Screenshot of Tower of Hanoi with three discs (from www. Mazeworks.com).

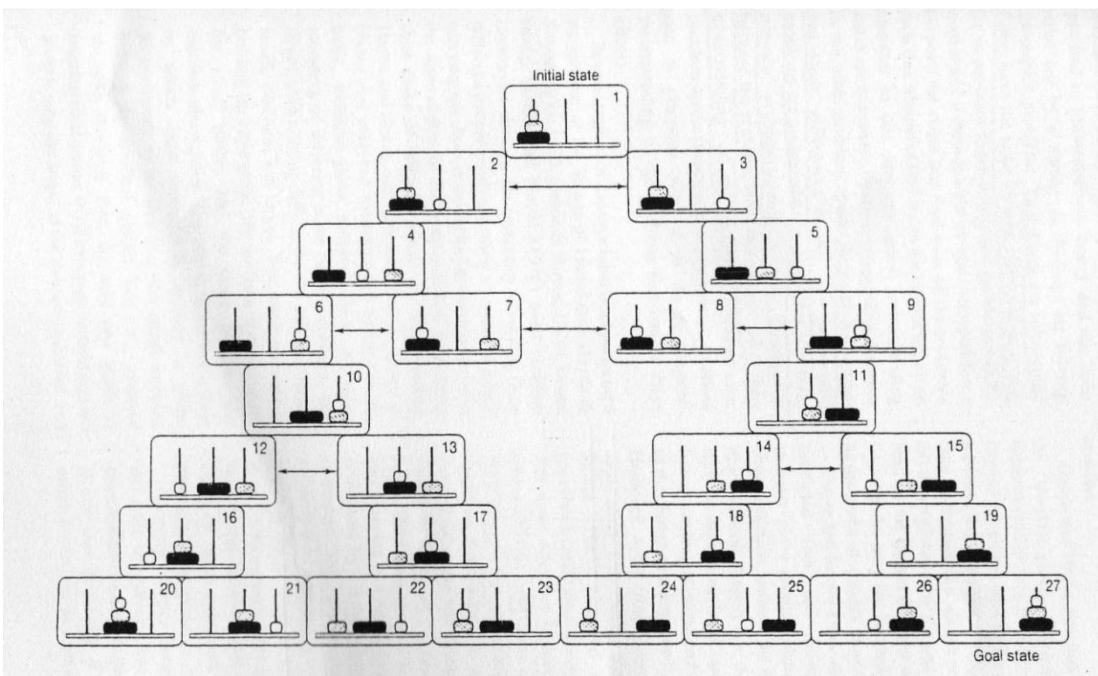
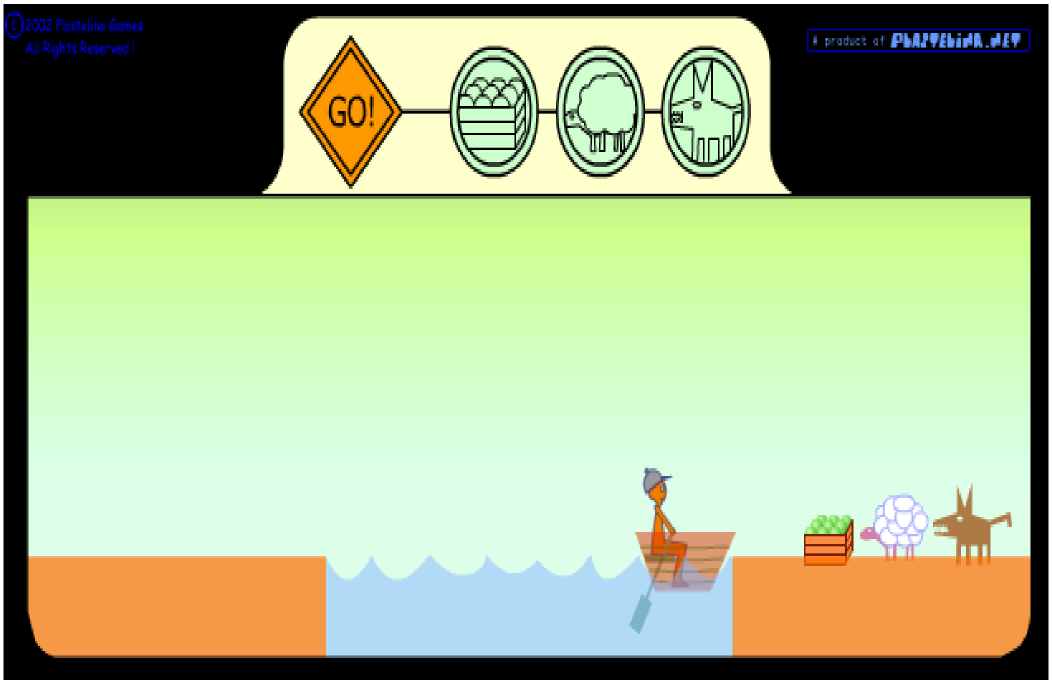


Fig. 1.2. Legal states to move Tower of Hanoi with three discs (Eyesenck et al., 1999).



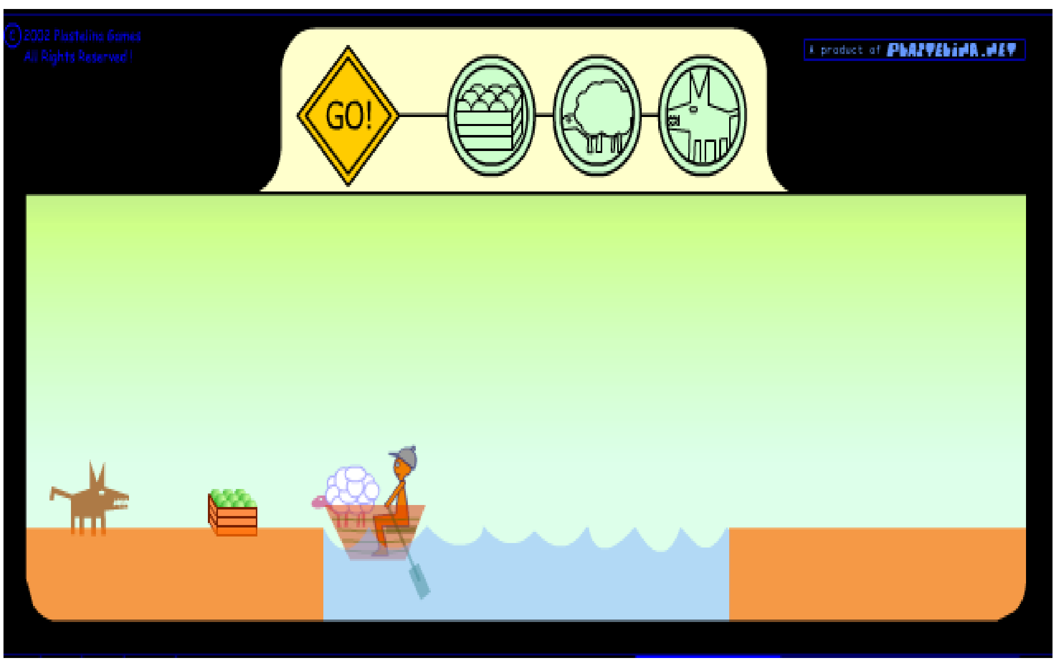


Fig. 1.3 Snapshot of River-Crossing Puzzle (Famer, cabbage and animals) for Group 1 (from www.plastelina.net).

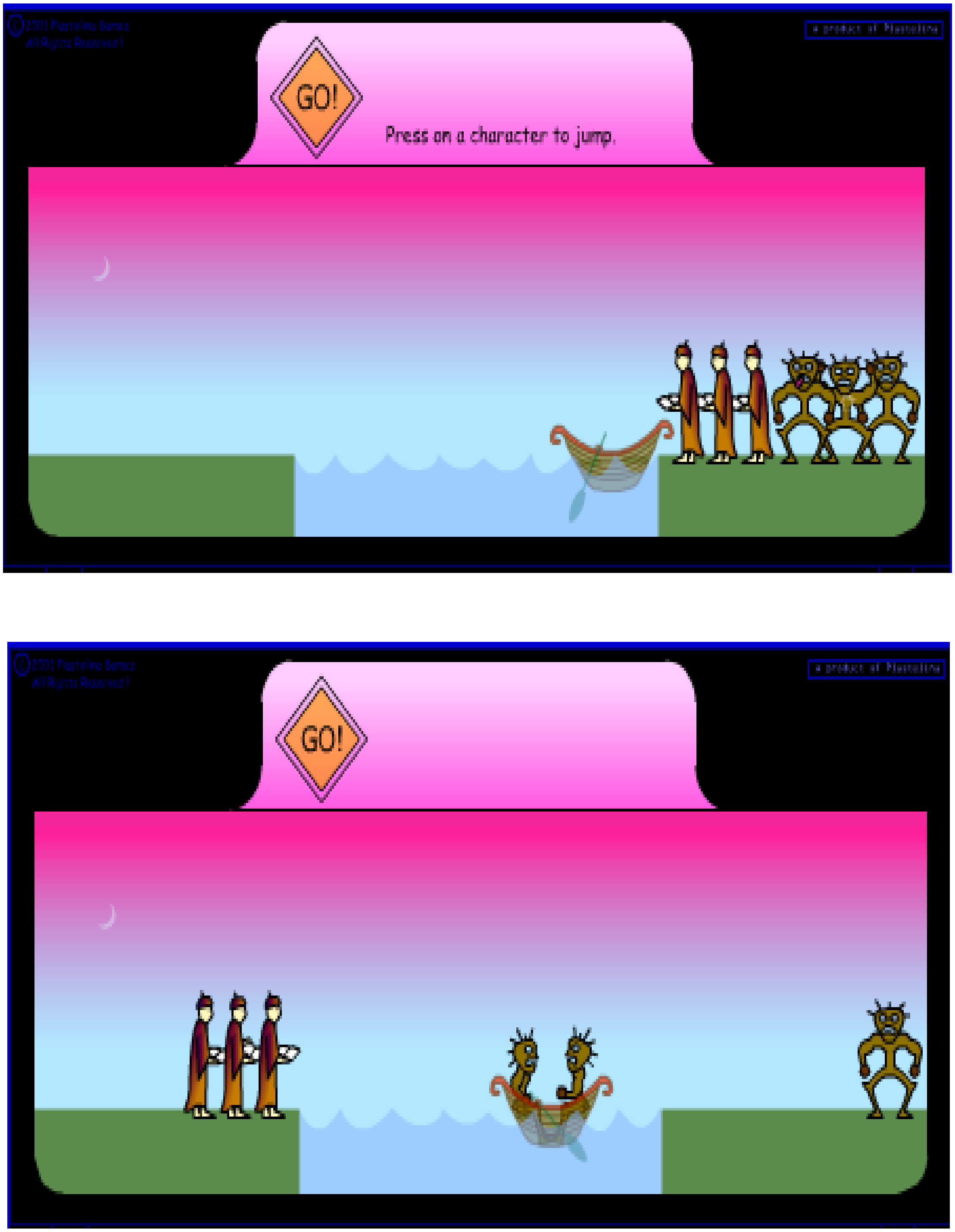


Fig. 1.4. Snapshot of River-Crossing Puzzle (Missionaries and Cannibals) for Groups 2-5 (from www.gizdic.com).

Find the Pattern

The months of the year and number pattern puzzle are very common and are often found on riddle websites or used in critical thinking lessons. The first alphabet of the first six or seven months is shown to the player who is asked to complete the remaining blanks.

J,F, M, A, M, J, __, __, __, __, __, __.

Fig. 1.6. Find the Pattern puzzle.

1.2 Rationale

Initially I was interested in critical thinking because there is hardly any research done on studying critical thinking through puzzles or other methodologies that does not involve formal assessment. As I researched critical thinking, my research evolved and I was attracted to what people were thinking while they were solving problems.

I wanted to know whether they were using specific thinking strategies while engaged in problem-solving and whether the strategies differ from problem to problem or from one individual to another. Therefore, I also wanted to research metacognitive awareness of individuals, how aware are these individuals about how to think or what helps them think.

What inspired me to choose puzzles to assess and investigate the application of the thinking strategies was a critical thinking presentation where I was given a simple puzzle that I was unable to solve. I was also told that second graders were able to solve this with ease in a very short period of time. This got me thinking about thinking strategies and problem solving once again. Further research led me to *The Theory of Learning by Doing* by Anzai and Simon (1979) which describes the learning strategies of a student while solving The Tower of Hanoi problem. I thought it would be interesting to increase the number of subjects and to see whether age plays the role of a significant factor while engaged in problem solving. Therefore, I combined the two ideas, and decided to study learning or thinking strategies while solving puzzles. Puzzles are fun to solve for most people of every age group and people attempt solving them for various reasons such as the inherent challenge, hobbies, and boredom or to pass times. Puzzles can be found anywhere from television game shows to internet websites to magazines and newspapers along with crosswords and Sudoku. The reason I chose puzzles instead of ordinary problems is because they would be less threatening and it would be easy for subjects to volunteer because puzzles seem like fun games instead of an experimental test or an investigation, which I found very true during the data collection stage.

The puzzles were chosen because they were easy to access. They are very different from each other; therefore, the possibility of applying a greater number of thinking strategies is higher. They would all capture the interest of the subjects and among the other choices that were available to me; the first two puzzles of choice had more literature written about them.

While deciding on the age groups of the students, I made sure to include only those subjects that are enrolled in an academic program or are part of a mode of studying. While deciding the age groups, Piaget's stages of development were kept in mind. Subjects had to have organized, logical thought, and be able to engage in concrete if not abstract problem-solving. All subjects also had to know how to use a computer and speak fluent or an adequate amount of English to be able to explain their thinking processes or strategies.

The age difference between the subjects had to be gradual but also should not be a very far apart so a difference (if found) could be seen in their styles. Considering all these, the age groups were decided upon. Third graders were chosen, because they are between the ages of eight and ten, and belong to the concrete operational stage. Third graders are the senior most class in the primary department (of the school chosen for data collection). Eighth graders are the senior most class of their department (of the school chosen for data collection), as from ninth grade the school follows GCE O-Levels. Thus, eleventh graders are the senior most class of this level as twelfth graders belong to A-Level, AS-Level or

SAT department. The next group was undergraduates and graduates. Undergraduate sophomores were specifically chosen because an age gap had to be distinct between high school students and university students. The age gap would not be sufficient if freshmen were chosen and would be too large if seniors were chosen.

1.3 Research Questions

This study aims to investigate critical thinking through the following research questions:

- What thinking strategies do the students use while solving problems?
- Do these thinking strategies evolve with age?
- What is their metacognitive awareness?
- Are they aware of using thinking strategies, and if yes, can they specify these strategies.

Chapter Two- Literature Review

2.1 Thinking

According to Ashman and Conway (1997) thinking skills consist of a broad range of skills, namely, metacognition, critical thinking, creative thinking, cognitive processes such as problem solving and decision making, core thinking skills such as representation and summarizing, and understanding the role of content knowledge (as cited in Moseley Ellit, Gregson, Higgins, 2005, p.371).

Smith (2002) defines a thinking skill as a teachable, partially proceduralized, mental activity that reaches beyond normal cognitive capacities and can be exercised at will (as cited in Moseley, et al., 2005, p.369). Furthermore, the teaching of thinking skills is now a major international enterprise that draws not only upon the discipline of psychology but also of philosophy. (Hamers & Overtoom, 1997, Costa, 2001, Sutcliffe, 2003, as cited in Moseley et al., p.369).

As Gilhooly (1995) explains, thinking can either be sharply directed towards a specific goal such as to find an answer to a problem or undirected such as daydreaming about a topic, where thoughts wander from one subjects to another effortlessly. Critical Thinking has been defined by scholars in various ways:

Beyer (1987) describes thinking as “a mental process in which something is turned over in the mind in order to make sense out of experience” (Beyer, 1987, as cited in Geertsen, 2003, p.1), whereas, critical thinking is the process of determining the authenticity, accuracy and worth of information or knowledge claims (1985). Presseisen (1987) defines thinking as “the mental manipulation of sensory input to formulate thoughts, reason about, or judge something” (Presseisen, 1987, as cited in Geertsen, 2003, p.1) and critical thinking as reflective and reasonable thinking that is focused on deciding what to believe or do (Presseisen as cited in Cotton, p.3). According to Costa (1985), “thinking is the receiving of external stimuli through the senses followed by internal processing” (Costa, 1985 as cited in Geertsen, 2003, p.1).

2.2 Metacognition

In one of my first undergraduate literature class, the professor used to walk around the class, checking the students' books, losing his temper when he saw that in most of the books passages were not underlined, the pages did not have scribbles or notes along the sides. According to him, in order to read critically, and analyze the story it was important to underline the relevant references and write down any ideas one sees valuable. I always liked to keep my books as clean as possible because there was a greater chance that they would be sold at a better price the following semester. However, in order to not get picked on in class, I made an exception. I realized that not only did these new strategies improve my reading skills but also my knowledge and understanding of literary analysis.

People use different types of strategies while reading or solving problems to enhance their performance. Some people underline sentences they believe include important or relevant information. Others underline random sentences while reading, whether they are relevant or not. Some people read aloud or reread text they find hard to understand. Some people draw mental images or maps to create an image of a problem they are trying to solve. Other people draw a rough sketch to help visualize the problem. Some people need their environment or surroundings to be very quiet, while others need some background noise while studying. People use different methods to enhance their thinking. However, some are conscious of using these strategies, whereas others use them subconsciously.

Although metacognitive skills are also considered to be of crucial importance in mathematical performance, particularly problem-solving (Garofalo & Lester, 1985), there has been minimal research done on the topic. Garofalo and Lester list three reasons why very few studies have been conducted on the effect that metacognitive knowledge and beliefs have on performance or studies investigating the nature or development of tasks such as monitoring, assessing and strategy-selecting behavior:

> "First, covert behavior of any type is extremely difficult to observe and analyze. In fact, many psychologists believe that people have no direct access to their mental processes,

> thereby making self-reports highly suspect as a source of data. This suspicion is especially pronounced in the case of metacognitive processes. Second, re-searchers who accept self-reports as legitimate data recognize that asking a person to verbalize information while performing a task may affect cognitive processes if the verbalized information would not otherwise be attended to. Third, phenomena linked with metacognition, metamemory, and the like have been regarded by many psychologists as being too ill-defined to be the objects of scientific investigation" (Garofalo & Lester, 1985, p.166).

2.3 Problem-solving

George Polya's (2004) "How to Solve it" provides a four step heuristics approach to a problem-solving solution. The first step involves *understanding the problem*. Questions to be asked when first addressed with the problem could be-what is the unknown? What is the data? What are the conditions? The second step is *devising a plan*. This involves finding the connection between the data and the unknown. So problem solvers would ask themselves whether they have seen the puzzles before, or seen in a different form or context, whether they know of a related problem or a theorem that could be helpful. This step asks problem solvers to access prior knowledge and find something that relates to the problem they are being faced with. Once something is found, they can relate to it and decide if any of the methods or strategies used to solve the initial problem can be used for the current one and eventually obtain a plan of the solution to the problem. The third step is *carrying out the plan* or implementing the decision. While carrying out the plan of the solution, each step has to be checked to see firstly, whether it is correct, and secondly, whether it can be proven to be correct. The last step is *looking back*. This requires examining the solution obtained, checking the results and the arguments (Polya, 2004).

Schoenfeld (1983, as cited in Artzt & Armour-Thomas, p.139) devised a model for analyzing problem-solving moves that was derived from Polya's four steps heuristic process. Schoenfeld's model integrated findings from research on problem-solving by information-processing theorists within Polya’s four step structure. The model described problem- solving in five episodes,

- reading
- analysis
- exploration
- planning/implementation and
- verification

Schoenfeld’s findings (1983) showed that experts returned several times to different heuristic episodes. For example, in one case, an expert engaged in the following sequence of heuristics:

read→analyze→plan/implement→verify→analyze→explore→plan/implement→verify.

Whereas, the sequence of heuristics for a novice were simply read→explore (Artzt et.al, 1992, p.139).

Garofalo and Lester (1985) developed a framework similar to Polya’s but more broadly defined. They identified a metacognitive framework constructed of four categories of activities or problem-solving behaviors. The first is *orientation* which “includes comprehension strategies, analysis of information and conditions, assessment of familiarity with a task, initial and subsequent representation, and assessment of problem difficulty and chance of success” (Pugalee, 2004, p.29). The second category is *organization* which includes “identification of goals and subgoals, (Pugalee, 2004, p.29). *Execution* is the third category and includes performance of actions, monitoring progress and trade-off decisions. *Verification*, the final category, includes “evaluation of decisions and results of executed plans” (Pugalee, 2004, p.29). Artzt and Armour-Thomas modified and filled the limitations of Schoenfeld’s study and their findings revealed eight problem-solving episodes, listed below,

- read
- understand
- analyze
- explore
- plan
- implement
- verify, and
- watch and listen.

The Gestalt psychologists' theory on problem-solving states that problem-solving behavior can be both reproductive and productive. However, the associationists believe that problem-solving is productive. Reproductive problem-solving involves reusing previous experiences and hindering successful solutions whereas, productive problem-solving is characterized by insight into the structure of the problem and by productive restructuring of the problem. The insight occurs suddenly and is accompanied by what they call an "ah-ha" experience (Eyesenck et. al, 1995, p. 359).

Eyesenck and Keane (1995) explain problem-space theory, using a labyrinth as an analogy. Just like alternative paths in a labyrinth arise to junctions that make one use different strategies to find the way out, problem-solving behavior can be characterized as a set of states, beginning from an initial state (standing outside the maze), involving many intermediate states (moving through the maze), and ending with a goal state (being at the center of the maze). In the labyrinth actions are performed or "operators applied" such as turn left or turn right. During problem-solving, the application of these operators' results in a move from one state to another, where at any given state there might be several different operators that apply and each of these will further generate numerous alternative states. Therefore, there are various possible states and paths through this space only some of which will lead to the goal state (Eyesenck et al., 2995).

Newell and Simon (1972) further state that people use similar "knowledge states" while solving problems. They begin at the initial knowledge state and search through a space of alternative mental states until they reach a goal knowledge state. The movement from one knowledge state to the next is done by the application of mental operators. Just like the various alternative paths in a problem, people use various strategies or heuristic methods to move from the initial state to the goal state efficiently. It is important to have knowledge of these movements, because the subjects' conception of a problem and the knowledge they bring to it, which is the strategies they use, make important contributions to the observed problem-solving behavior. Thus, this behavior plays a great role in this study (Newell & Simon, 1972, as cited in Eysenck et al., 1995, p.362).

2.4 Imagery

A colleague asked me to give directions to a store. The first thing I did was create a mental map of my daily driving routine (pass the first traffic light, turn right at the second and take the first right after the petrol station). I then drew a map for her but I would not have been able to explain the directions to her without using mental imagery, even though I drive by the store every day.

Images may or may not arise in an individual's mind while thinking. Images can appear involuntarily in the form of dreams during sleeping or daydreaming. However, people can create an image voluntarily as well. Tuan (1975) states that unlike a percept, which is sustained by the information in the environment we see before us, an image appears when the environmental stimuli do not appear to justify it. Images which we can voluntarily call upon from the past that are sometimes "so vivid that they are like pictures projected on a screen to which one can turn to examine the details. These are known as eidetic images. [However,] they decline in sharpness and frequency with age and verbal education" (Tuan, 1975, p208).

The use of imagery is inevitable while teaching some lessons. For example, I recently introduced three dimensional solid figures to second graders for a math lesson. It would be nearly impossible to teach concepts like vertices, edges and faces without the help of images and models. Students also made three-dimensional models using jelly beans and toothpicks where the toothpicks were edges and the jelly beans were the vertices. Two days after the students were introduced to this concept, a student was having trouble solving a problem, where students were given the number of vertices, edges and faces and they had to name the solid figure. I asked the student to picture the model that she made two days ago, using the jelly beans and the toothpicks. Without further help or explanation, the student was able to answer the problem. She created a mental image of the pyramid model she had made.

Caban et al. (1978) investigated a procedure designed to evoke and enhance the use of mental imagery in spelling instruction among eighth graders. Their findings supported their hypothesis which stated that spelling words can be learned and retained better by using a mental imagery practice and instruction method rather than by using other spelling practice and instruction methods. Similar results were also found by second grade teachers and myself. Teachers introduce new vocabulary words every two weeks. Initially, students were shown flash cards with the words printed on them. Students were given the definitions and were asked to put these into sentences to ensure they can use the words in the right context. However, the classrooms then got equipped with computers and projectors. So the teachers started creating PowerPoint Presentations with pictures, video and audio clips in their lessons. Teachers noticed that not only did the children enjoy these presentations, but it also made the teacher's job easier to explain the meaning of these words to the students. Teachers have also noticed that by showing a visual image along with the word, students remember the meanings of the word better than without the use of images. For example, when I asked my students if they knew what a crane is, none of them knew or had heard of the word, but when I showed them a picture, almost all of them claimed that they had seen it around Dubai many times. The following week, during a review lesson, I hardly found a child who did not know what a crane was.

According to Tuan (1975), a mental map is a special type of image which is even less directly related to sensory experience. An image which is a mental map rather than a "picture" is obviously a construct. Tuan further states that no percept or image is a mere photograph of reality. A percept is not only the registering of current environmental stimuli but also an imaginative effort produced under the needs of the moment. Maps can be created in the mind without recourse to pen and paper.

When I was in my senior year at university, I came across my high school Physics teacher. I was so excited to see him and greeted him, but he did not remember my name.

He said to me, “You look familiar, I can see where you used to sit in class, but I do not remember your name.” In the Art of Memory, Yates (1966) writes that people interested in training their memory should first select places and then form mental images of the things that they want to remember. They should “store those images in the places, so that the order of the places will preserve the order of the things” (1966, as cited in Tuan, 1975, p.210). Images or information that is relevant or carries some importance to us get stored in our mind, and we can retrieve these images like data whenever we want subconsciously or not.

2.5 Think or read aloud

Independent work time in class is always exciting because children work in different ways. One common observation is children reading aloud. Although they use the same strategy, each one is unique in their own way. So even though most of them are reading aloud, their voice, tone and emphasis is very different and this gives me an idea about how the students makes sense of the text.

The analysis of think aloud protocols, the verbal reports, produced by subjects who express their thoughts while engaged in some activity, has allowed psychologists to explore domains of cognitive processing that were previously inaccessible. Through a powerful combination of task analysis, model building and thinking aloud, Newell and Simon (1972) were able to identify strategies that their subjects used in various activities such as playing chess, solving mathematical or logical problems. Furthermore, they developed models in the form of computer simulations that demonstrated how these strategies were employed in problem solving (Newell & Simon, 1972, as cited in Kucan & Beck, 1997, p.275).

Olshavsky (1977) also represents reading as problem solving and the usefulness of verbal protocols to identify strategies used by readers. Although Olshavsky applies Newell & Simon’s (1972) problem solving perspective to reading, her studies were different from prior studies because her subjects provided concurrent reports by stopping at predetermined places to verbalize their thoughts while reading the assigned short stories.

She “designed her investigation to take into account the constraints and abilities of the reader by using good and poor readers; the characteristics of the task environment, or reading situation, by using texts of greater and less complexity; and the reader's goal, or motivation, by matching subjects' expressed interests with the topics of the texts they were asked to read. Using a problem solving representation of reading, Olshavsky coded her subjects' protocols by identifying their interactions with the text as either problem identification strategies or problem solving strategies” (Kucan & Beck, p.276).

Anzai and Simon (1979) later used a similar approach in a detailed study where a subject was asked to think aloud while solving the Tower of Hanoi puzzle. Her protocols were recorded, and they were able to identify the strategies she used to learn while engaged in solving a problem. Later, Erisccon and Simon (1980) provided a detailed description of methodological considerations related to thinking aloud distinguishing between thinking aloud, retrospection and introspection (Kucan & Beck, 1997, p.275).

Harris and Hodges (1995) studied think alouds from a teaching point of view. According to them, a think aloud is a “metacognitive technique or strategy in which a teacher verbalizes thoughts aloud while reading a selection orally, thus modeling the process of comprehension” (Block & Israel, 2004, p. 154). Think alouds enable readers to stop periodically, and reflect on the thinking they do in order to understand a text. Then they can relate these literacy processes orally. Teachers use thinking aloud technique as a form of instructional practice to help the students verbalize their thoughts. They use it while reading, and thus bring thinking into the open so they can replicate it more effectively in the future (Oster, 2001, as cited in Block et al, 2004). Block (2004) and Oster (2001) also believe that thinking aloud is a form of metacognitive awareness and it not only does it significantly increase students’ scores on comprehension tests but it also adds to students’ self-assessments of their comprehension as well as enhancing students’ abilities to select thinking processes in order to overcome any comprehension challenges while they read (Block et al., 2004, p. 154).

2.6 Trial-and-error or repetition

During the making of a film, the director asks the actors to take numerous takes. They keep repeating their scenes till they get the perfect take.

Thorndike (Eyesenck et. al, 1995, p.357) placed hungry cats within a visible distance from food. A pole was fitted in the cage that when hit would open the door and allow the cats out of the cage. At first the cats walked around the cage, clawing at the sides. However, inevitably, at some point the cats would hit the pole and the door would open. The same procedure was repeated, and the cats behaved in a similar manner but gradually the cats learned that hitting the pole would open the door. Therefore, "new problems were initially solved by trial-and-error behavior and then accidental solutions were amalgamated into responses that were reproduced when the appropriate stimulus was presented" (Eyesenck et al., 1995, p.357).

Similar to Thorndike's study, Jackson, Carter and Taristano (2001) studied the behavior of Portia *fimbriata*, a genus of web-invading araneophagic jumping spiders. An *island* surrounded by an *atoll* was placed in a plastic water-filled rectangular tray. A plastic tube extended below the tray. The spider was introduced into the bottom of the tube and prodded up onto the island with a plunger. The spider then had two potential escape tactics; it could either leap or swim. Their findings revealed that the spiders used trial-and-error to solve escape from the island and when were put into the same situation again, repeated the action that allowed them to escape on the first trial successfully (Jackson et al., 2001).

2.7 Physical Cues or Gestures

A common observation during reading class is seeing the children read with their fingers following the sentences they read. Children also trace the words (often with their index fingers) as a method to learn and practice new spelling words.

At the beginning of the year, during the review weeks, I observed some very interesting gestures being used by second graders while doing mathematical problems such as addition or subtraction. For example, if the problem was, 18-5 =____, the students would put one hand on their head (eighteen is the larger number and so I put eighteen in my head) and then show five fingers on the other hand (five on my hand) and then count back. The students were taught this technique in kindergarten and first grade. However, I noticed that throughout the year, some children could not solve the math problems without using those physical actions. Most could do it mentally with practice, but others depended a lot on these gestures.

The gestures that they were using are called deictic gestures, used in concrete or abstract pointing. These gestures are context dependent. Deictic gestures are used during the early phases of language development to combine with speech in order to name objects, indicating a developmentally early correspondence between word and gestures (Goldin-Meadow, 1998). At about ten years of age abstract pointing develops to indicate abstract or non-present referent. Therefore, deictic gestures, coupled with deictic utterances, can play an important role during classroom interactions because they establish a distinction between figure (topic) and ground (Hanks, 1992, Roth, 2001, p.370).

Roth (2001) explains that gestures are not merely ancillary but also central features of communication. Gestures can have both narrative functions, such as iconic gestures, and grounding functions, such as deictic gestures. These connect the gestural and verbal narrative to the pictorial background (Roth, 2001, p.366).

Rendon (1980) describes the four characteristics of gestures:

- They begin from a resting position, move away from this position and then return to the initial resting position.
- They have a peak structure or a stroke, which can be recognized as a moment of the ascend movement to denote the function of meaning of a movement.
- The stroke phase is preceded by a preparation phase and then succeeded by a recovery phase n which the hand and arm move back to their resting position.
- They are often symmetrical (Rendon, 1980, in Roth, 2001, p.369).

Beats or batons are gestures that are void of propositional or topical content yet lend a temporal or emphatic structure to communication. Beats can be simple, non-pictorial gestures that can include the up and down flick of hand, or the tapping motions used to emphasize certain utterances. Beats also function as interactive gestures, which serve to regulate the coordination of speaking turns, to seek or request a response, or to acknowledge understanding (Efron, 1972, Freedman, 1977, Bevelas et al., 1955).

Goldin-Meadow's research group did extensive work on gesturing and learning. Their findings suggest that

> gesture is indeed a window to the mind. Gesture-speech mismatches indicate readiness for learning a concept. During the learning process, gesture serves as a window into what a learner is thinking particularly if she or he uses dual strategies…gesture might contribute to the learning process not only by providing a unique view of the learner's thought, useful to experiments and communication partners alike, but perhaps by stimulating those thoughts as well. (Goldin-Meadow, 1997, in Roth, 2001, p.375)

After studying past studies done on gestures, Roth and Welzel (2001) found three main results-

- Gesturing in the presence of the objects and events that are the content of students' expressions allows students to construct complex explanations by lowering the cognitive load.
- Gestures provide a medium on which the development of scientific discourse can piggyback.
- Gestures provide the material that "glues" layers of perceptually accessible entities and abstract concepts (Roth & Welzel, 2001, in Roth, 2001, p.377).

2.8 Prior Knowledge

When I went to watch the page-turner-adapted-into-a-movie, the Da Vinci Code, I could keep track of the cultural literacy references inferred in the movie since I had recently read the book. The book explains the references in vivid details as opposed to the movie. Having a background in art also helped. However, a friend who accompanied me seemed lost with all the unexplained references.

Dochy, Segers and Buehl define prior knowledge as “the whole of a person’s actual knowledge that: (a) is available before a certain learning task, (b) is structured in schemata, (c) is declarative and procedural, (d) is partly explicit and partly tacit, (e) and is dynamic in nature and stored in the knowledge base” (Dochy et al., 1999, p.146).

Dochy et al. (1999) created six categories to assess prior knowledge-

- multiple choice tests
- combination of open and close questions or completion tests
- association tests
- recognition and matching tests
- free recall, and
- experimenter judgment and self-estimation (Dochy et al., 1999, p.147).

Research by Byrnes and Guthrie (1992) showed that prior knowledge can have an indirect effect on performance through the clarity of study materials (Dochy et al., 1999, p.151).

Dochy et al. (1999) disagree with the assumption that prior knowledge is reasonably complete and correct, of reasonable amount, accessibility, availability and structure because students often tend to have misconceptions (Dochy et al., 1999, p.152). Complete dependence on prior knowledge is not favored because students have often been seen to resist altering their views even when they are presented with plausible evidence that provides a more adequate account of a phenomenon. Therefore, prior knowledge can hinder the learning, problem-solving or thinking process (Cohen, 1981; Hynd & Alvermann, 1989; Lipson, 1982, in Dochy e al., 1999, p.152). Pintrichet, Marx, and Boyle (1993) also believe that at the same time, students with little or prior knowledge of an area will lack the necessary knowledge frameworks. Thus, they will face difficulties in the structuring and judging of the validity of new information or material (Pintrichet et al., 1993, in Dochy et al., 1999, p.154).

Further investigations also proved that prior knowledge is an important factor in contributing to the explanation of post-test variance. Therefore, prior knowledge is a factor which is used to explain the subjects' performance on post-test measures (Bloom, 1976; Dochy, 1992; Tobias, 1994 in Dochy et al., 1999, p. 154).

Analyzing the relationship between age, intelligence, metacognitions, prior knowledge and performace, Korkel (1987) also found that prior knowledge played an important role in performance which is explained in Fig. 2.1. (Dochy et al., p. 158). The extensive research on prior knowledge done by Dochy et al. also supports Korkel's studies. They conclude that there is a strong relationship between prior knowledge and performance. The majority of the studies (91.5%) they reviewed reported positive effects of prior knowledge on performance (Dochy et al., 1999, p. 171). They also state that

> prior knowledge is indeed an effective aid for learning new knowledge. This result supports the current practice of activating prior knowledge at the beginning of a learning process. Such practice is very explicit in the applications of powerful learning environments based on constructivism (De Corte, 1990). In problem based learning and the problem method, for example, activating prior knowledge is an explicit phase. [However,] the students' reflection on their prior knowledge is facilitating learning. Likewise, students' reflection on what knowledge is important for the learning process probably enhances learning. (Dochy et al., 1999, p. 173)

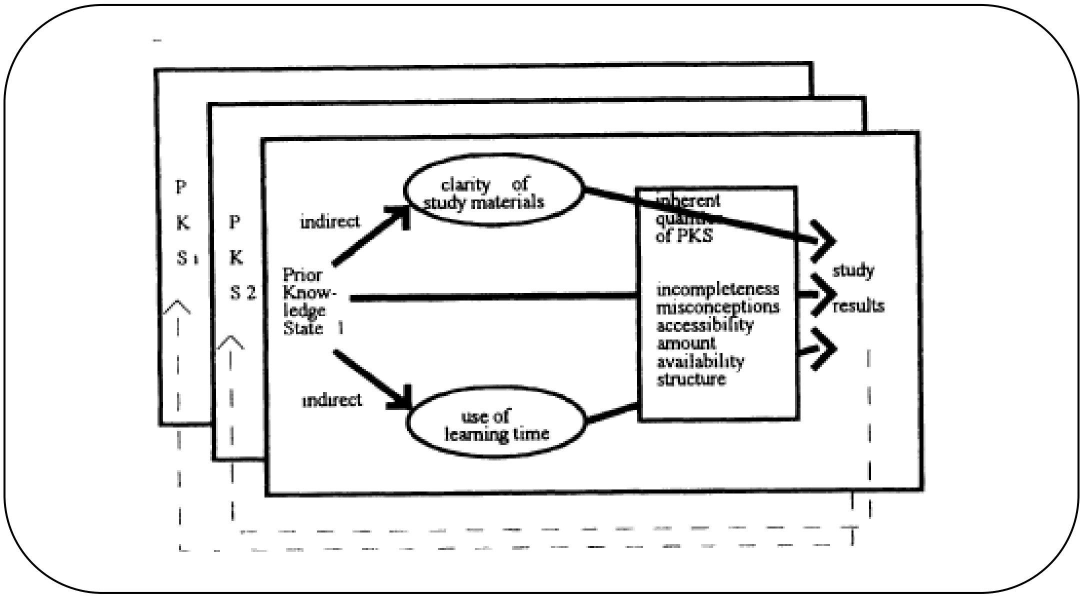


Fig.2.1. Interaction effect involving inherent qualities and the facilitating effect (Dochy et. al, 1999).

2.9 Imitation or modeling

"Put your right hand up," said the teacher. I raised my hand. "The other right hand," responded the teacher looking at me. I was confused. I had the same hand raised as hers then why was she asking me to raise the other right hand?

Imitation or the copying of an otherwise improbable response plays a critical role in the life of the developing child. Imitation is viewed differently by biologists and psychologists. From a biologist's point of view, learning from observing the behavior of others is related to adaptive advantages. However, from a psychologist's point of view, a form of imitation in which the imitated response is perceptually opaque to the observer is very interesting. According to R.W. Mitchell (2002), the ability of organisms to match cross-modally (specifically, kinesthetic- visual matching) allows them to solve the problem of correspondence. Furthermore, the ability to match one's (personally) unseen behavior to the similar behavior produced by a demonstrator results from experience (Mitchell, 2002, as cited in Zental, 2003, p.92).

Zbiek and Conner (2006) study mathematical modeling in learning. They write that there is a common perception in the schooling context that mathematical modeling tasks motivate students to engage in mathematics. They further contend that engagement in mathematical modeling activities support three different types of motivation. Firstly, confirmation that real-world situations appeal to the learners. Secondly, a further motivation to study or continue to study mathematics in general. For example, modeling may suggest to students that they need to study a variety of mathematics in order to address a range of real-world problems. Thirdly, to learn new mathematics emerges when a student modeler embraces a purpose that in the modeler's opinion of not sufficiently met by the mathematics that the modeler knows, and the modeler seeks understanding of the needed mathematics. Thus, the modeler is motivated to add new pieces of knowledge or new connection among the known pieces of knowledge (Zbeik & Conner, 2006, p.105-106). Zbiek and Conner (2006) conclude that mathematical modeling is important because it provides a venue on which students can learn curricular mathematics in various ways. Learning through modeling potentially involves a deeper understanding of the known curricular mathematics as well as the motivation to learn new curricular mathematics. They also suggest that with different mathematical modeling subprocesses lie opportunities for conceptual and procedural development for all modelers (Zbeik & Conner, 2006, p.110).

Chapter Three- Methodology

3.1 Participants

The participants were 100 students in a private school in Dubai and a private university in Sharjah in the UAE. Participants belonged to the five groups:

Third graders, aged 8-10

Eighth graders, aged 13-15

Eleventh graders, aged 15-17

Undergraduate sophomores, aged 18-21

Graduates, aged 23-29

Each group consisted of twenty students, ten females and ten males.

3.2 Instruments

Tower of Hanoi and the River-crossing puzzles were available online. Find the Pattern was available as a Microsoft Word document file. The questionnaire was also available online. (See Appendix)

3.3 Procedures

Before data collection, the Director General of the school and the Associate Dean of CAS in the university were asked for permission to carry out the research. After the permission was granted the data collection was done. With the exception of the first group, the other groups solved the puzzles and filled out the questionnaires independently. The first group of students were asked to solve the puzzles on a one-on-one basis by the researcher and then asked the questions verbally. With the help of the Associate Dean, emails with instructions, puzzle links and the questionnaire was sent to the student body.

Students were assured that their responses will remain confidential and their performance will not influence their grades.

The three puzzles are,

1) Tower of Hanoi

2) River-crossing Puzzle (Farmer and Animals for Group 1 and Missionaries and Cannibals for Groups2-5)

3) Find the Pattern- Months of the Year

<u>Group 1:</u> Third Graders

Third graders were asked to solve the puzzles on a one on one basis. The instructions were read out to each student and the results recorded. Before starting each puzzle, the students were asked if they had solved it before. None of the students had seen or solved it before. Students were told to think about how they are solving the puzzle as they will be required to explain this procedure.

<u>Puzzle 1</u>

The instructions of the first puzzle, Tower of Hanoi, were read to the subjects,

"The goal is to move all three discs from the left peg to the right one. Click and drag with the mouse to move a disc. Only one disc may be moved at a time. A disc can be placed either on an empty peg or on top of a larger disc. Try to move all the discs using the smallest number of moves possible."

When each student finishes, time is recorded. They are then asked questions about the solving procedure. The questions are:

1. Did you find the puzzle easy?
2. Can you tell me how you solved it?
3. How did you know which disc to move?

4. Why did you move the small disc to peg X?
5. When you moved the first disc, did you know how all the discs would be placed in the end?
6. What were you thinking of when you were solving the puzzle?
7. What helped you solve the puzzle?

The questions varied depending on each student. Questions were repeated or rephrased if the student did not understand.

Subjects were shown a demonstration of how to use the mouse to click on the disc they want to move and drag it to the peg they want it to be placed on.

Puzzle 2

The instructions for puzzle 2, the River-crossing Puzzle were read to the subjects,

"Please help the man in the boat to move the wolf, the sheep and the box of cabbage to the other side of the lake. Notice that, wolves eat sheep and sheep eat cabbages when no man is around."

After the students finish, the time is recorded and they are asked a series of questions to understand their thinking process.

1. Did you find the puzzle easy?
2. Did you understand the rule?
3. Can you explain the rule?
4. Can you tell me how you solved it?
5. How did you know you have to move the fox/sheep/cabbage first?
6. Why couldn't you leave the sheep and fox or sheep and cabbage alone?
7. What were you thinking of when you were solving the puzzle?
8. What helped you solve the puzzle?

The questions varied depending on each student. Questions were repeated or rephrased if the student did not understand.

Subjects were shown a demonstration on how to use the mouse to click on the cabbage, sheep or fox in order to move them and then click GO so they can be transported to the other side of the river. Once they have crossed the river, subject has to click on the cabbage or animal of choice again so that it can be dropped off on the other side.

Puzzle 3

For Find the Pattern puzzle, third graders were first shown an example of a similar puzzle, find the number pattern:

Problem: O, T, T, F, F, S, __, __, __, __.

Solution: O: **O**ne, T: **T**wo, T: **T**hree, F: **F**our, F: **F**ive, S: **S**ix, S: **S**even, E: **E**ight, N: **N**ine, T: **T**en.

The alphabets are the first letters of the numbers from one to ten.

Final Answer: O, T, T, F, F, S, S , E , N , T .

The instructions for puzzle 3 were read to the students,

"Carefully look at the alphabets, each one represents something, try to figure out what each alphabet stands for and then complete the blanks."

Once the student has finished, the time is recorded and the following questions are asked,

1. Did you find the puzzle easy?
2. Can you tell me how you solved it?
3. How did you know they are months?
4. What were you thinking of when you were solving the puzzle?
5. What helped you solve the puzzle?

The questions varied depending on each student. Questions were repeated or rephrased if the student did not understand.

<u>Groups 2-5</u>

These groups were sent emails with website links to puzzle 1 and 2, and an attachment to puzzle 2. Also attached is Questionnaire A (Appendix 1), which asks the students how they solved the puzzles. Students are also given links to an online stopwatch for puzzles 2 and 3. They return the completed questionnaire once finished.

The instructions for the puzzles are as follows,

Puzzle 1: “The goal is to move all the discs from the left peg to the right one. Click and drag with the mouse to move a disc. Only one disc may be moved at a time. A disc can be placed either on an empty peg or on top of a larger disc. Try to move all the discs using the smallest number of moves possible.”

Puzzle 2: “Please help the 3 cannibals and the 3 missionaries to move to the other side of the lake. Notice that, when there is on one side, more cannibals than missionaries, they eat them.”

Puzzle 3: “Carefully look at the alphabets, each one represents something, try to figure out what each alphabet stands for and then complete the blanks.”

The level of difficulty of Tower of Hanoi increases for each group.

Group	**No. of discs**
third graders	3
eighth graders	4
eleventh graders	5
undergraduate	6
Graduates	7

Chapter Four- Results

The purpose of this study was to investigate various thinking strategies that students use during problem-solving and whether the strategies change according to age.

After observing third graders and analyzing the questionnaires of Groups 2-5, the following eight thinking strategies were apparent.

1. Creating a mental image
2. Thinking or reading aloud
3. Planning ahead
4. Repetition or trial and error
5. Imitation or modeling
6. Accessing prior knowledge
7. Looking into surroundings for hints or clues
8. Physical cues

A summarized chart showing the results of each puzzle solved by all groups are shown below. Details charts can be found in the Appendix.

4.1 Tower of Hanoi

This was the only puzzle where each of the subjects in groups 2-5 was able to explain or mention the thinking strategy he or she used during the problem-solving process. (see Table 4.1)

Third Graders

Ninety percent of third graders solved the puzzle through repetition and trial error. Thirty percent of the students imitated the auto-solve demonstration. Fifty-five percent of the students were thinking aloud as they were solving the steps, and fifteen percent of the students claimed that they would see the picture in their mind before moving a disc. Forty-five percent of the students used physical cues to help them solve the puzzle. For example, one student moved the discs around with his finger before moving them on the computer. Third graders were the only group that used physical cues while solving the puzzle.

Eighth Graders

Sixty-five percent of eighth graders chose the planning ahead strategy, thirty percent used trial and error, fifteen percent used reading or thinking aloud, and only five percent used mental imagery as a thinking strategy to solve the puzzle.

Eleventh Graders

Forty-five percent of eleventh graders used trial and error, forty percent planned ahead; and ten percent used both mental imagery and reading or thinking aloud as a thinking strategy.

Undergraduates

Sixty percent of undergraduate sophomores used the planning ahead strategy before moving the discs. Twenty percent of the students used mental imagery, and five percent of the students used thinking and reading aloud as well as trial and error to solve the puzzle.

Graduates

Seventy percent of graduates used thinking or reading aloud as a thinking strategy. Sixty-five percent used planning ahead, twenty-five percent used trial and error, and twenty percent used imitation as a thinking strategy to solve the puzzle. None of the graduates used mental imagery.

4.2 River-crossing puzzle

Third Graders

All of the third graders used trial and error to solve the river crossing puzzle. However, thirty percent of third grader also used planning ahead as well as thinking or reading aloud, and twenty percent used physical cues as help while solving the puzzles. Once again third graders were the only group that used physical cues to solve the puzzle.

Eighth Graders

Forty-five percent of eighth graders used trial and error to solve the puzzle, while twenty-five percent of the students planned ahead, and five percent used thinking or reading aloud. Thirty percent of the students did not know what thinking strategy they used.

Eleventh Graders

Fifty percent of eleventh graders used trial and error to solve the river crossing puzzle. Fifteen percent of the students used the help of mental imagery whereas five percent planned ahead. However, thirty-five percent of the students did not know what thinking strategy they used to solve the puzzle.

Undergraduates

Thirty-five percent of undergraduate sophomores used the planning ahead strategy before solving the puzzle. Thirty percent of the students chose trial and error; twenty percent used mental imagery whereas, fifteen percent did not know what thinking strategy they used to solve the puzzle.

Graduates

Sixty-five percent of graduates used trial and error as a thinking strategy to solve the puzzle. Thirty-five percent of the students created a mental image to help them solve the puzzle whereas twenty-five percent of the graduates used thinking or reading aloud and planning ahead to help them. Five percent of the graduates did not know what thinking strategy they used to solve the river-crossing puzzle.

(see Table 4.2)

4.3 Find the Pattern

Third Graders

Eighty percent of third graders used hint or clues from their surroundings to help them think of a solution to the Find the Pattern puzzle. Seventy percent of the students used thinking or reading aloud to help them. Forty percent created a mental image, whereas thirty percent used physical cues. All the students used imitation and accessed prior knowledge to help them solve the puzzle. As seen before, third graders were the only group that used physical cues.

Eighth Graders

Seventy percent of eighth graders did not know what strategy they used to help them think. Fifteen percent of the students used thinking or reading aloud whereas ten percent planned ahead, and five percent accessed prior knowledge or used hints and clues from the surroundings to help them think.

Eleventh Graders

Seventy percent of eleventh graders did not know what thinking strategy they used. Fifteen percent accessed prior knowledge; ten percent used thinking or reading aloud whereas five percent planned ahead and used trial and error.

Undergraduates

Fifty-five percent of undergraduate sophomores did not know what strategy they used to solve the puzzle. Twenty percent of the students planned ahead, ten percent of the students used thinking or reading aloud as well as accessing prior knowledge, whereas five percent used both trial-and-error as well as hints or clues from the surroundings as thinking strategies while solving the puzzle.

Graduates

Sixty percent of graduates used trial and error to solve the puzzle. Thirty-five percent of the students used the thinking or reading aloud strategy to solve the puzzle. Twenty-five percent of the graduates' accessed prior knowledge, twenty percent created a mental image, fifteen percent used hints or clues from their surroundings, and ten percent of the graduate students did not know what strategy they used.

(see Table 4.3)

4.4 Comprehensive Results

The results also showed that sixty-five percent of the third graders were able to solve the Find the Pattern puzzle correctly, (See Table 4.4) whereas only twenty-five percent of the eighth graders, thirty-five percent of the eleventh graders, twenty-five percent of the undergraduates, and thirty-five percent of the graduates solved the puzzle correctly.

For the Tower of Hanoi puzzle, the most common strategy applied was planning ahead, that an average of forty-seven percent of the subjects used. None of the subjects accessed prior knowledge, and used hints or clues from the surroundings. As for the River-crossing puzzle, an average of fifty-eight percent of all the students used trial and error or repetition to solve the puzzle. None of the subjects used imitation or modeling, accessing prior knowledge, and looking into surroundings for hints or clues to receive help to solve the puzzle. For the last puzzle, Find the Pattern, forty-one percent of the subjects were not aware of the strategy they used to solve the puzzle. It is also the only puzzle where subjects used all eight thinking strategies.

students	mental image	think/read aloud	plan ahead	trial and error	imitation or modeling	accessing prior knowledge	hints/clues from surroundings	physical cues	do not know
third graders	15	55	5	90	30	0	0	45	0
eighth graders	5	15	65	30	0	0	0	0	0
eleventh graders	10	10	40	45	0	0	0	0	0
undergraduates	20	5	60	5	0	0	0	0	0
Graduates	0	70	65	25	20	0	0	0	0

Table 4.1 Percentage of Tower of Hanoi.

Students	mental image	think/read aloud	plan ahead	trial and error	imitation or modeling	accessing prior knowledge	hints/clues from surroundings	physical cues	do not know
third graders	0	30	30	100	0	0	0	20	0
eighth graders	0	5	25	45	0	0	0	0	30
eleventh graders	10	0	5	50	0	0	0	0	35
undergraduates	20	5	35	30	0	0	0	0	15
Graduates	35	25	25	65	0	0	0	0	5

Table 4.2. Percentage of River-Crossing Puzzle

Students	**mental image**	**think/read aloud**	**plan ahead**	**trial and error**	**imitation or modeling**	**accessing prior knowledge**	**hints/clues from surroundings**	**physical cues**	**do not know**
third graders	40	70	0	50	100	100	85	30	0
eighth graders	0	15	10	0	0	5	5	0	70
eleventh graders	0	10	5	5	0	15	0	0	70
Undergraduates	0	10	20	5	0	10	5	0	55
Graduates	20	35	0	60	0	25	15	0	10

Table 4.3. Percentage of Find the Pattern.

students	**solved correctly**	**solved incorrectly**
third graders	65	35
eighth graders	25	75
eleventh graders	35	65
undergraduates	25	75
graduates	35	65

Table 4.4 Percentage of students who solved the Find the Pattern puzzle correctly and incorrectly.

Puzzle One- Tower of Hanoi		
thinking strategies	**most used by**	**Percentage**
mental imagery	Undergraduates	20
think/read aloud	Graduates	70
plan ahead	eighth graders, graduates	65
trial-and-error	third graders	90
imitation/modeling	third graders	30
accessing prior knowledge	-	0
hints or clues	-	0
physical cues	third graders	45
do not know	-	0

Table 4.5. Percentage value of thinking strategies in Tower of Hanoi.

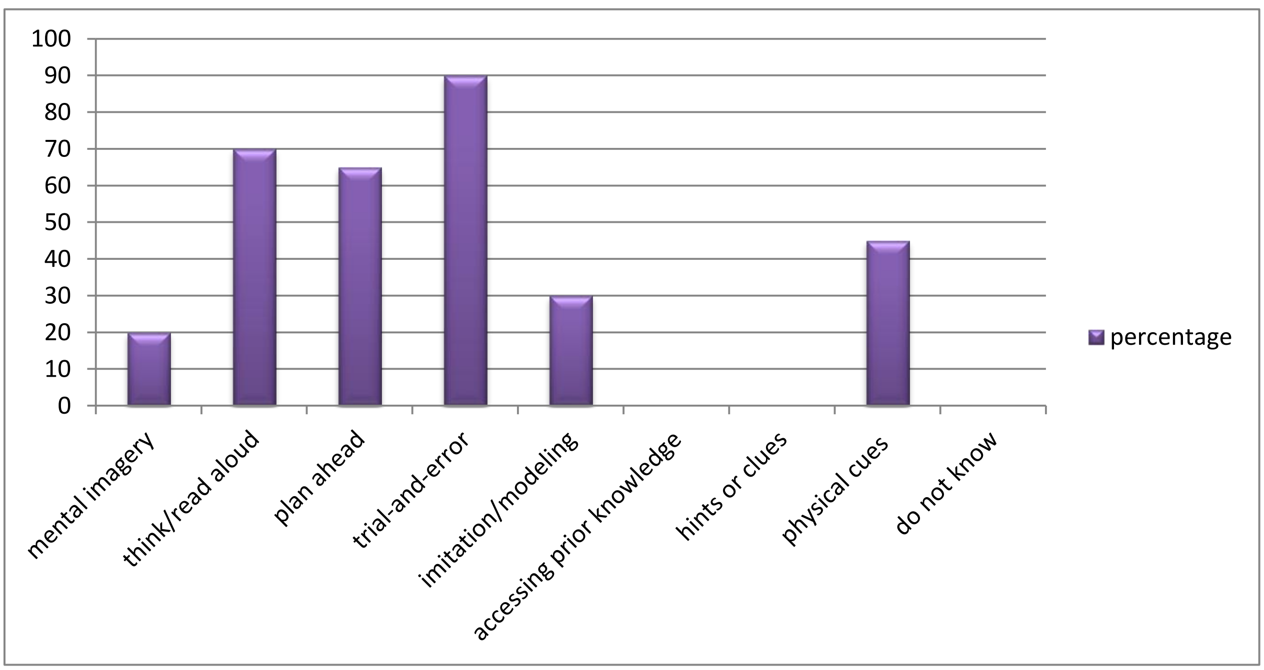


Fig 1.1. Thinking strategies for Tower of Hanoi puzzle.

Puzzle Two- River-crossing puzzle		
thinking strategies	**most used by**	**percentage**
mental imagery	graduates	35
think/read aloud	third graders	30
plan ahead	Undergraduates	35
trial-and-error	third graders	100
imitation/modeling	-	0
accessing prior knowledge	-	0
hints or clues	-	0
physical cues	third graders	20
do not know	eleventh graders	35

Table 4.6. Percentage of thinking strategies used for River-crossing puzzle.

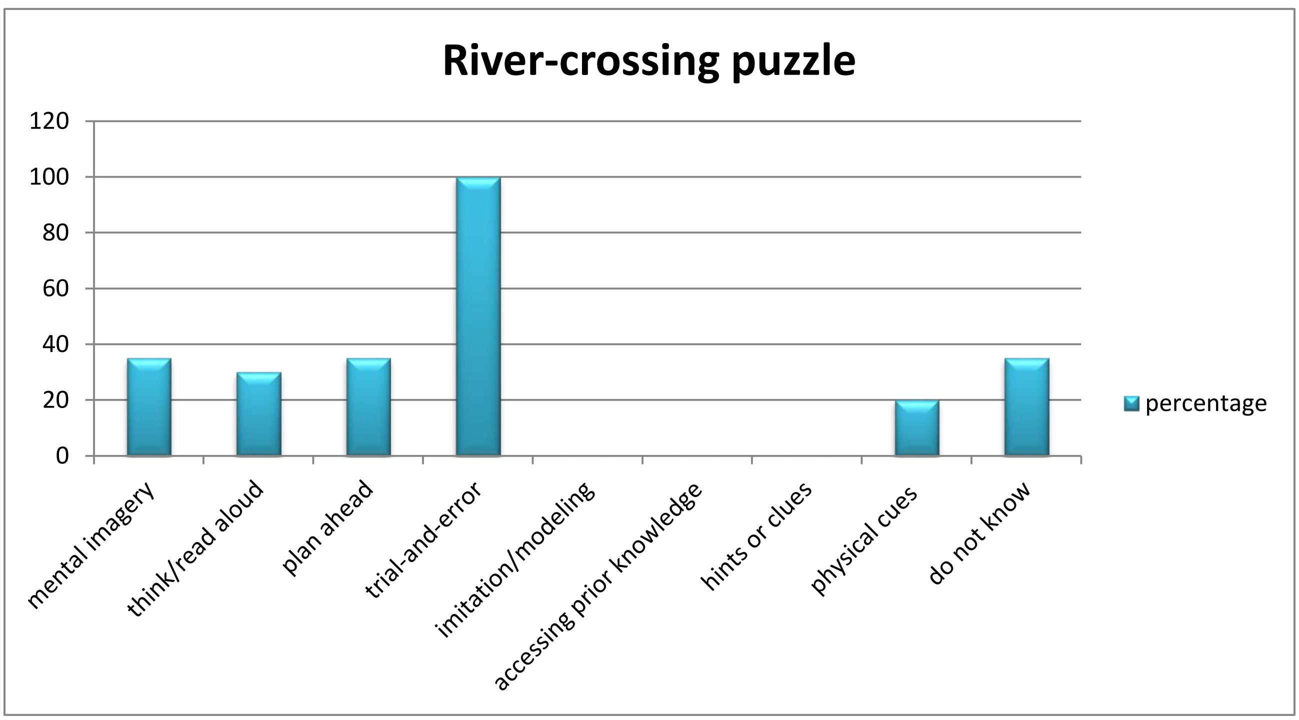


Fig 1.2. Thinking strategies for River-crossing puzzle.

	Puzzle Three- Find the Pattern	
thinking strategies	**most used by**	**Percentage**
mental imagery	third graders	40
think/read aloud	third graders	70
plan ahead	undergraduates	20
trial-and-error	graduates	60
imitation/modeling	third graders	100
accessing prior knowledge	third graders	100
hints or clues	third graders	85
physical cues	third graders	30
do not know	eighth graders, eleventh graders	70

Table 4.7. Percentage of thinking strategies used for Find the Pattern.

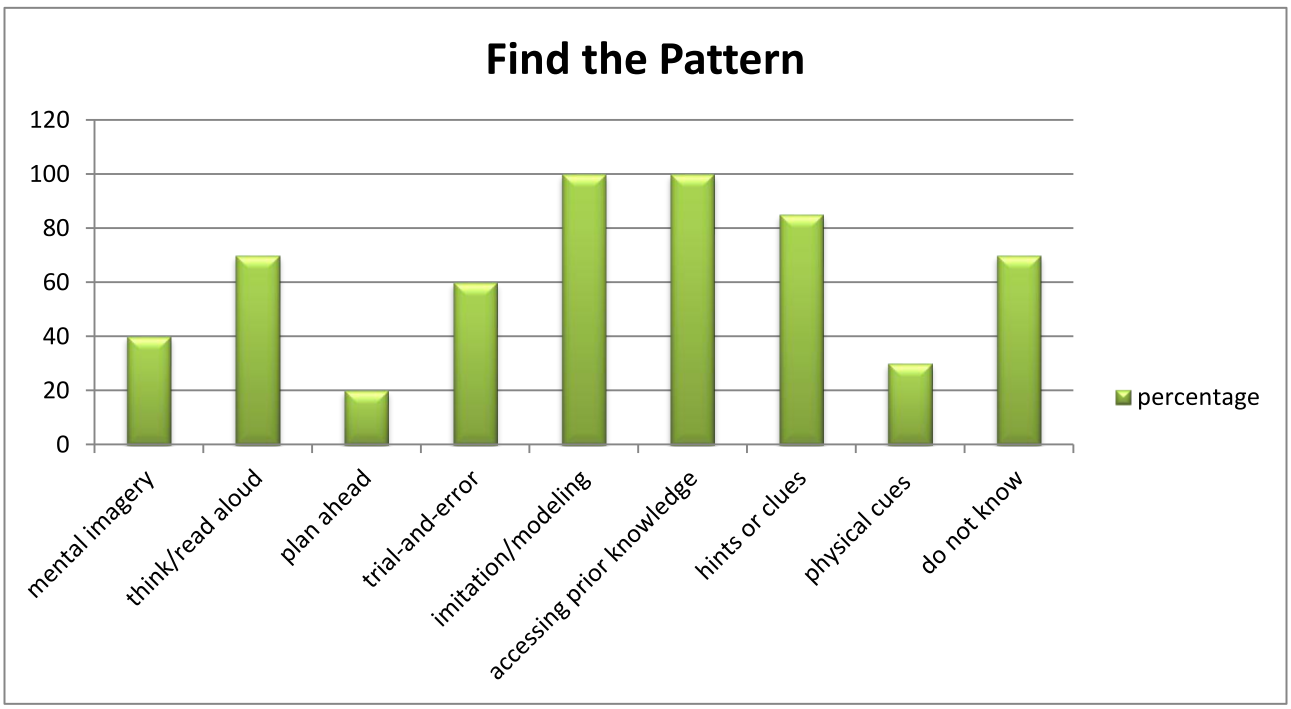


Fig. 4.3. Thinking strategies for Find the Pattern.

Chapter Five- Discussion of Results

As mentioned earlier, the aim of this study was to investigate thinking strategies used while solving puzzles. After studying the results, eight thinking strategies were found to be used by a hundred participants that solved the three puzzles, Tower of Hanoi, River-crossing puzzle, and Find the Pattern. This chapter will discuss the results that were found such as why certain strategies were more common among certain groups, or in combination with other strategies. The eight thinking strategies are listed in the order from being applied most frequently to being applied the least for all three puzzles

5.1 Trial-and-error or repeating

River-crossing puzzle

Thirty-five percent of undergraduate sophomores planned ahead before solving the puzzle. Thirty percent of the students chose trial and error; twenty percent used mental imagery whereas fifteen percent did not know what thinking strategy they used to solve the puzzle. Thomas' (1974) study also supports these findings. His study, that also looked at the Missionaries and Cannibals puzzle, suggested that "subjects made three or four major planning decisions in solving the problem, and, having made each of these decisions, carried out whole blocks of moves with increasing speed" (Eyesenck et. al, 1995, p.368).

Most of the subjects used a combination of planning and trial and error to solve the river-crossing puzzle. A total of fifty-eight percent of the students used trial and error and twenty-four percent of the students used planning ahead. Twenty-three percent of these subjects used a combination of both the strategies.

Other studies on this puzzle have looked at more complex versions of the problem. For example, Simon and Reed (1976) investigated a version that consisted of five missionaries and cannibals. Although it has more legal states and can be solved in eleven moves, they found that the subjects took an average of thirty moves to solve the puzzle. Their study suggested three main strategies, firstly, *balancing strategy*, where subjects tried to ensure that equal numbers of missionaries and cannibals remained on either side of the river. Secondly, *means-ends strategy*, where subjects tried to move more people to the goal side of the river. Finally, *anti-looping heuristic*- subjects avoided moves that reverse the immediately preceding move (Eyesenck et. al, 1995, p. 368). They maintained that they key to efficient solution of the problem rested on a strategy shift from the balancing strategy to the means-ends strategy.

5.2 Planning ahead

Tower of Hanoi

This strategy was the most frequently applied strategy by forty-seven percent of the subjects. Along with eighth graders, graduates used planning as the most used thinking strategy to solve the puzzle. Sixty-five percent of graduates used planning and twenty-five percent used trial-and-error. Therefore, a large number of graduates took time and really thought through the steps. It is also important to note that the difficulty level for graduate students was the highest, consisting of seven discs, which is time consuming and confusing to keep track of. Perhaps, this is also the reason why none of the graduates made use of mental imagery; it would be too confusing and hard to remember all those images and keep track of the steps. The higher level of difficulty could also be the reason why twenty-five percent of the graduates used trial and error as a strategy. They started off with a plan but when they reached a state where they could not plan see other options and found themselves stuck or repeating their mistakes, they got frustrated and then without planning their steps opted for trial-and-error.

Although Polya's framework is popular among mathematicians, "How to Solve it" also proposes a step by step process on solving a word puzzle, where the problem is to rearrange the letters contained in a given words into one word. Therefore, the questions and answers to follow the four steps would be-

What is the unknown? A word.

What are the data? *x* number of words that are to be rearranged.

What is the condition? The desired word has *x* number of letters, and *x* number of words in the sentence.

The next step would involve drawing a figure, therefore, to make out *x* number of blank spaces followed by attempts to come up with a plan to solve the problem, such as, separating the vowels from the consonants. If the proposed plan is not working, then the next step would be to try to solve some related problems first, perhaps of a simplified form and later come back to the initial one and use the same strategies.

Although these strategies might be used by undergraduates, graduates and even some high school students, it is highly unlikely for third graders to follow the proposed steps. However, most third graders were still able to solve the Months Pattern puzzle successfully as compared to the rest of the groups.

5.3 Thinking or reading aloud

Thirty-one percent of the subjects used this strategy to solve the Tower of Hanoi puzzle, twenty-eight percent applied it to the River-crossing puzzle and thirteen percent of the subject chose this strategy to solve the Find the Pattern puzzle. Third graders described the steps that they would follow. For example, stating which peg the disc is being moved to. Studies by Anzai and Simon (1979) and by Newell and Simon (1972, as cited in Kucan & Beck, 1997, p.275) also show that verbal protocols reveal further thinking processes being used by the subjects.

5.4 Creating a mental image

Tower of Hanoi

This strategy was used by ten percent of the subjects. Fifteen percent of the third graders, five percent of the eighth graders, ten percent of the eleventh graders, and twenty percent of the undergraduates used mental imagery to help them solve the puzzle. Some subjects would picture the outcome of a movement of a disc. Therefore, they would make predictions and plan with the help of creating a mental plan or map. The subjects voluntarily called upon these eidetic images to help them find the solution to a problem. Graduates are the only group who did not use this strategy to solve the puzzle. This result was expected as Tuan (1975) suggested that the use of eidetic images declines in sharpness as well as frequency with age and verbal education. “Pseudo-sensory images give children time to appraise their surroundings world in their own way. With adults the role of imagery in thinking and learning is clear. Some people claim dependency on images when they think, others on words” (Tuan, 1975, p.208) which can be seen in the case of the graduate students. “Scientists may be visualizers or verbalizers; some make use of both images and words when they cogitate, and a few claim to use neither” (Tuan, 1975, p.208).

5.5 Imitating or modeling

Tower of Hanoi

Thirty percent of third graders and twenty percent of graduates used the imitation or modeling strategy for the Tower of Hanoi puzzle. Third graders were encouraged to solve the puzzle without using the auto-solve as a model. However, when they gave up or would not understand, then they were shown the auto-solve model. However, graduates seem to get frustrated with themselves when they were unable to solve the puzzle or when it was taking too long. As mentioned earlier, the level of difficulty for this level is high and so the problem-solving process can be time consuming as well as confusing. Therefore, the twenty percent of the graduates who used the auto-solve option probably did it at a point where they could not keep track.

Find the Pattern

All third graders were shown a sample of the Find the Pattern puzzle before they solved the months’ version. Therefore, they all had the opportunity to use the modeling strategy. This is also the puzzle that they performed better in as compared to the other groups.

Therefore, it is be suggested that learning through imitating or using modeling as a thinking strategy during problem-solving enhances performance.

5.6 Accessing prior knowledge

Find the Pattern

Third graders were the only group that were shown a sample of Find the Pattern puzzle, where numbers were used instead of months. Thus, they were also the only group that was provided with necessary information to ensure accessibility and availability of prior knowledge. Therefore, when results reveal that their performance was the highest in this puzzle, it can be suggested that the use of prior knowledge enhanced the performance of the third graders. These results are also supported by numerous studies done by Korkel (1987) and Dochy et al. (1999) (mentioned in Chapter Two- Literature Review).

5.7 Using hints or clues from the surroundings

Find the Pattern

Even though the other groups of students are also exposed to the calendar daily, for example, writing the date, making appointments or reading the newspaper, they were still unable to use this knowledge of it. Instead, they tried to think of very complex ideas to solve the puzzle. For example, this is a graduate student's explanation of solving this puzzle,

"I assigned to the A a value of joint (i.e. OOO A OOO A OOO A OOO) and then changed the order of the other three letters –JFM / MJF / FMJ –. Notice that each letter takes each position only once (J- - / - J - / - - J)" (Gr 14).

5.8 Using physical cues or gestures

Four percent of the subjects chose this strategy and as seen earlier, it is also the only one that is only used by third graders. It is very interesting that third graders were also the only group that used physical cues and gestures while solving all three puzzles. For example, a third grader (3rd 14) put his index finger to the computer screen and imagined moving the discs in the Tower of Hanoi puzzle, before actually move them using a mouse. The same student also used a lot hand and arm movement during the explanation of how he solved and the puzzle.

Another third grader (3rd 7) wrote the first three alphabets with her index finger in the air, and then solved the puzzle successfully. When asked why she moved her finger, she was unaware of doing so. Also, she could not explain how she knew the solution; she said that she just knew it. While the latter was using deictic gestures like the second graders when they were performing addition or subtraction problems, the former seems to be using iconic gestures or representational gestures (Kendon, 1988, as cited in Roth, 2001,p.370). According to McNeill (1985), these include those hand or arm movements that bear a perceptual relation with concrete entities and events. They draw their communicative strength from being perceptually similar to the phenomenon that is being talked about. Therefore, iconic gestures have a transparent relationship to the idea that they cover, particularly within a narrative event in which they depict concrete objects and events (Roth, 2001, p. 370).

A study by Goldin-Meadow (2004) showed that it is not only the students who use gestures and physical cues during problem-solving. A gesture that a child produces in a problem-solving situation could provide insight into the way that child represents the problem. She also found that during one-on-one mathematics tutorial sessions, forty percent of the problem solving strategies that teachers conveyed to the students were in the form of gestures, and the use of gestures was equally frequent in the classrooms as well. Flevares and Perry (2001) also found that math teachers used from five to seven nonspoken representations of various mathematical ideas per minute, which is almost one representation every ten seconds. The use of gestures was by far the most frequent nonspoken form of nonspoken representation for all the teachers. Among the other forms were pictures, objects and writing. Furthermore, experimental studies also suggest that a lesson accompanied by gestures is more effective than when the same lesson is not accompanied by gestures (Church, Aymna-Nolley, &Mahootian, 2004; Perry, Berch &Sirleton, 1995; Valenzeno, Alibali, & Klatzky, 2003, in Goldin-Meadow, 2004, p.319, Flaherty, 1975).

5.9 Other significant findings

Sixty percent of the graduates, seventy percent of the undergraduate sophomores and, sixty-five percent of eleventh graders, seventy percent of eighth graders and thirty-five percent of third graders solved the Find the Pattern puzzle incorrectly. As compared to the older students, only a small percent of the third graders solved it incorrectly. Most of the students', who got the puzzle wrong, solved it as a pattern. It is suggested that most adults experienced a case of what Willingham describes as familiarity or recollection. It "is the knowledge of having seen or otherwise experienced some stimulus before, but having little information associated with it in your memory" (Willingham, 2003, p.39).

These findings also support Ohlsson's (1992, as cited in Eyesenck, 1995, p. 373) Insight Theory, which states that insight occurs in the context of an impasse, which is unmerited. The thinker has the knowledge to solve the problem, but for some reason cannot use it. According to the Insight Theory, an impasse occurs because the initial representation of the problem is a bad memory probe for retrieving the operators needed to solve the problem, just like the students felt when they first saw the puzzle. This impasse is broken when the representation of the problem is changed (is reinterpreted, re-represented, or restructured) thus forming a new memory probe that allows the retrieval of the relevant operators. As Ohlsson suggested, this re-representation occurred through various ways.

Most third graders used *elaboration*, the first method of re-representation, where they get to the solution by "adding new information about the problem from inference or the environment" (Eyesenck et.al, 1995, p.373) such as hints. Eighty-five percent of third graders looked around their classroom walls, boards, books for hints and clues to solve the puzzle. They are also the only group that used this strategy. Besides the third graders, only one eight grader, one undergraduate sophomore, and three graduates used this strategy to solve the puzzle.

The second method *constraint relaxation* and third, *re-encoding* can be quite similar. The former is changing some of the constraints or the goal and the latter is changing aspects of the problem representation through re-categorization or deleting some information (Eyesenck, 1995, p. 373). Most of the subjects change their strategy, or representation of the problem. Instead of looking at it, one subject read it aloud, or reread it, or looked at it from a distance (appendix). Although the main goal remains the same, that is, to find the solution to the puzzle, the subgoal has changed. Initially it was to complete the pattern of random alphabets, but now after re-reading the puzzle, the goal changed to finding out what each alphabet can represent. This is when the impasse is broken, and "a full or partial insight may occur. A full insight occurs if the retrieved operators bridge the gap between the impasse state and the goal state" (Ohlsson, 1992, as cited in Eyesenck et., 1995, p.373).

Willingham's familiarity factor explained in the article "Why students think they understand when they do not" justifies why most of the adults could not solve the Months Pattern puzzle. It also seems like the familiarity factor is the reason why third graders performed better in solving the third puzzle. Third graders are taught the months of the year as part of the "Calendar" lesson in their Mathematics curriculum, as well as "Proper Nouns" in their Language Arts curriculum. Therefore, when presented with the problem, they were familiar with it, and just needed to "refresh" their memories, or as Ohlsson (1992, as cited in Eyesenck, 1995, p. 373) puts it, need to break the impasse. Third graders also had the advantage of being shown a similar example. Hence, not only were the third graders exposed to it every morning, where the day and the date are written on their classroom boards, but they were also taught this material. Furthermore, students also write down the day and date in their books regularly.

Whereas, when the older students looked at the puzzle, a random alphabet pattern is the first thing that they thought of, "completing the pattern" looked familiar to them and they completed it without realizing it was wrong. Willingham explains that "an insidious effect of familiarity is that it can give you the feeling that you know something when you really do not" (Willingham, 2003, p.39). The rest of the adults realized that their solution to the puzzle is wrong, but could not break the impasse and solve the puzzle as did the third graders.

Metcalfe (1986a, as cited in Eyseck et. al, 1995, p. 372) describes problems that are not solved by an incremental accumulation of information from memory, but by a sudden illumination, which is best described as "insight." In Metcalfe's study, subjects were asked for their metacognitions –their assessments of their feeling-of-knowing a solution or feelings-of-closeness to a solution –on insight problems and trivia questions that were unable to answer. Her findings revealed that even though people had reasonably accurate metacognitions for the memory/trivia questions, they had no predictive metacognitions for the insight problems.

The findings of this study also support Metcalfe's. After analyzing the questionnaires, a lot of the students are unable to explain how they solved a puzzle, and their answers reveal that the answer or the solution just came to them, an "ah-ha" experience. Below are a few examples from the questionnaires (for detailed questionnaires please refer to the appendix).

(8th, 9) As soon as I looked at it I knew they were the months.

(11th, 3) First I kept trying to make a pattern, repeating the same pattern then mixing it up by adding another letter, then looking at the number of each letter, but then when I read the hint that each one represents something then I knew that it must be names of something and then I just got it, it's the calendar.

(11th, 8) I just knew it was the months in order.

(11th, 10) As soon as I read that each letter represents something, I looked at the letter, my brain automatically associated the J with January as it was Followed by F (February).

(11th, 11) First I tried to repeat all the same letters but there were some blanks left, so when I was saying the letters out loud to myself, it hit me, they were the months.

(Sop, 18) As soon as I read the alphabets loudly, I knew they are the twelve months.

Seventeen percent of the subjects could not solve the River-crossing puzzle. The subjects attempted to solve the puzzle, but gave up, even when they were close to the solution. Thomas (1974) showed that during states 5 and 8 (see Fig. 1.5) subjects took longer and produced more errors than other points. Thomas claimed that at state 5, “the difficulty lies in the many alternative moves that are possible at this point (five in number). Only two of these moves are illegal, and of the remaining three legal moves, only one is really helpful” (Eyesenck et. al, 1995, p. 369). Similarly, at state 8, subjects often are misled because they need to move away from the goal state in order to achieve the goal. Referring to Fig. 1.5, it can be seen that “going from state 8 to state 9, one enters a state that seems further away rather than closer to the goal. At this point, subjects typically think that they have reached a blind alley and start to backtrack” (Eyesenck et.al, 1995, p. 368).

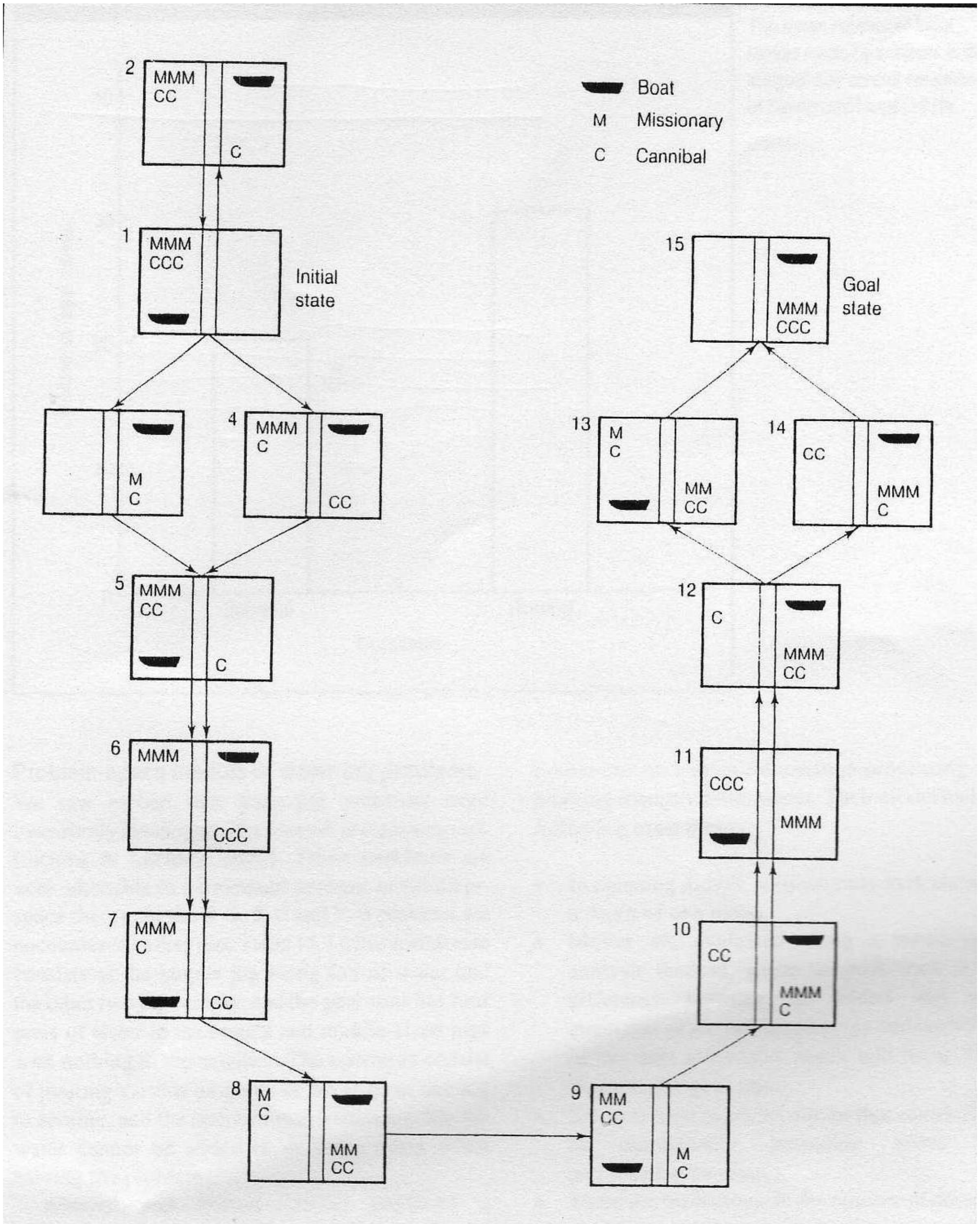


Fig. 1.5 shows the search space intervening between the initial state and the goal state of the missionaries and cannibals version of the river-crossing puzzle (Eyesenck et al., 1999).

students	mental image	think/read aloud	plan ahead	trial and error	imitation	accessing prior knowledge	hints/clues from surroundings	physical cues	do not know
third graders	3	11	1	18	6	0	0	9	0
eighth graders	1	3	13	6	0	0	0	0	0
eleventh graders	2	2	8	9	0	0	0	0	0
undergraduates	4	1	12	1	0	0	0	0	0
graduates	0	14	13	5	4	0	0	0	0
Total	10	31	47	39	10	0	0	9	0

Table 5.1. Thinking strategies used for Tower of Hanoi.

students	mental image	think/read aloud	plan ahead	trial and error	imitation	accessing prior knowledge	hints/clues from surroundings	physical cues	do not know
third graders	0	6	6	20	0	0	0	4	0
eighth graders	0	1	5	9	0	0	0	0	6
eleventh graders	2	0	1	10	0	0	0	0	7
undergraduates	4	1	7	6	0	0	0	0	3
graduates	7	5	5	13	0	0	0	0	1
total	13	13	24	58	0	0	0	4	17

Table 5.2. Thinking strategies used for River-crossing puzzle.

students	mental image	think/read aloud	plan ahead	trial and error	imitation	accessing prior knowledge	hints/clues from surroundings	physical cues	do not know
third graders	8	14	0	10	20	20	17	6	0
eighth graders	0	3	2	0	0	1	1	0	14
eleventh graders	0	2	1	1	0	3	0	0	14
undergraduates	0	2	4	1	0	2	1	0	11
graduates	4	7	0	12	0	5	3	0	2
total	12	28	7	24	20	31	22	6	41

Table5.3. Thinking strategies used for Find the Pattern.

5.10 Limitations and suggestions for future research

As any other research this study has its limitations. However, for this study the limitations are also suggestions for future research. Due to shortage of time and availability of participants, it was extremely difficult to observe each participant individually and so that was only done for the third graders. For future research, the results would be more robust if the subjects were asked what thinking strategies they use before and after solving the puzzles. Their responses could then be compared to see whether they used what strategies they thought they would use, or usually use. It would also be interesting to have a control group that was given some awareness about thinking strategies and perhaps shown a list of various strategies that are used. The results of these could then be compared to the results of a group that was not given any information of thinking strategies. Results would show whether prior knowledge plays a role in metacognitive awareness as this study shows evidence that a lot of the subjects were unaware of the thinking strategies that they were using. However, this would be a very large project and would need a team of researchers to work with such a large number of subjects per group.

Chapter Six- Conclusion

Trial-and-and error was used the most for all three puzzles. The next frequently used strategy is planning ahead followed closely by thinking or reading aloud. The rest of the strategies were used by less than fifty percent of the students. Using physical cues or gestures was a strategy that was used the least. It is also the only used by third graders. The strategy used most by third graders was trial and error for the first two puzzles and accessing prior knowledge as well as imitation or modeling for the third puzzle. Out of the three puzzles, the most exceptional results are for the third puzzle, Find the Pattern. Results showed that the older subjects struggled to solve the Find the Pattern puzzle as opposed to the third graders. Their explanations suggest that their thinking was very complex and they were unable to think past their complex thinking. Adults were also very easily frustrated and a larger number of the older participants gave up on solving the puzzle whereas none of the younger participants (third graders) gave up. The findings of the study confirm the hypothesis that third graders would solve the puzzles with much ease as compared to their older counterparts.

The study investigated various thinking strategies used by subjects in five age groups and found the use of eight thinking strategies while engaged in the problem solving process. These strategies are, (i) trial-and-error, (ii) planning ahead, (iii) thinking or reading aloud, (iv) creating a mental image, (v) accessing prior knowledge, (vi) using hints or clues from the surroundings, (vii) imitating or repeating and (viii) using physical cues or gestures. Results also revealed that subjects often used a combination of these strategies. Trial-and-error was used with planning ahead by twenty-three percent of the subjects for the River-crossing puzzle. This study also revealed that the subjects were unaware of their metacognitive processes while solving the puzzles. Seventeen percent of the subjects did not know their thinking strategies for the River-crossing puzzle and forty-one percent did not know their thinking strategies for Find the Pattern. Furthermore, the strategies that the subjects said they applied did not match up with their explanations.

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Puzzles:

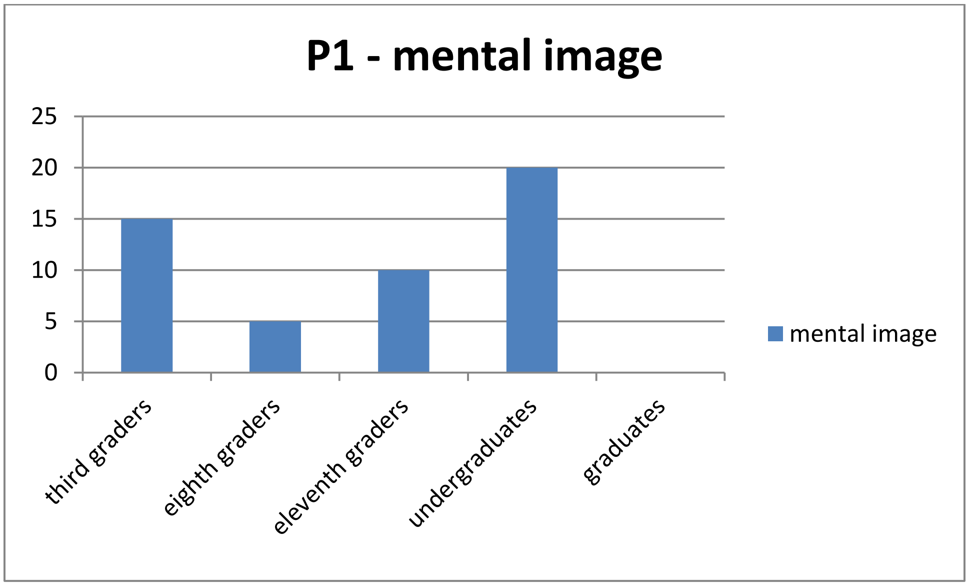
Farmer, animals and cabbage. www.plastelina.net/examples/games/gamer1.html

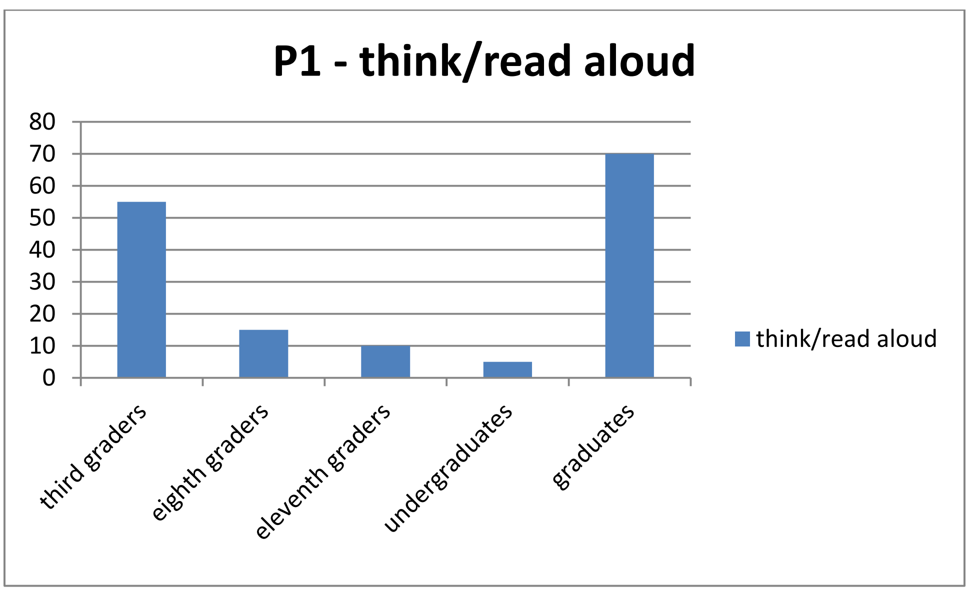
Missionaries and Cannibals. www.gizdic.com/freegames/gamespages/cannibals_and_missionaries.htm

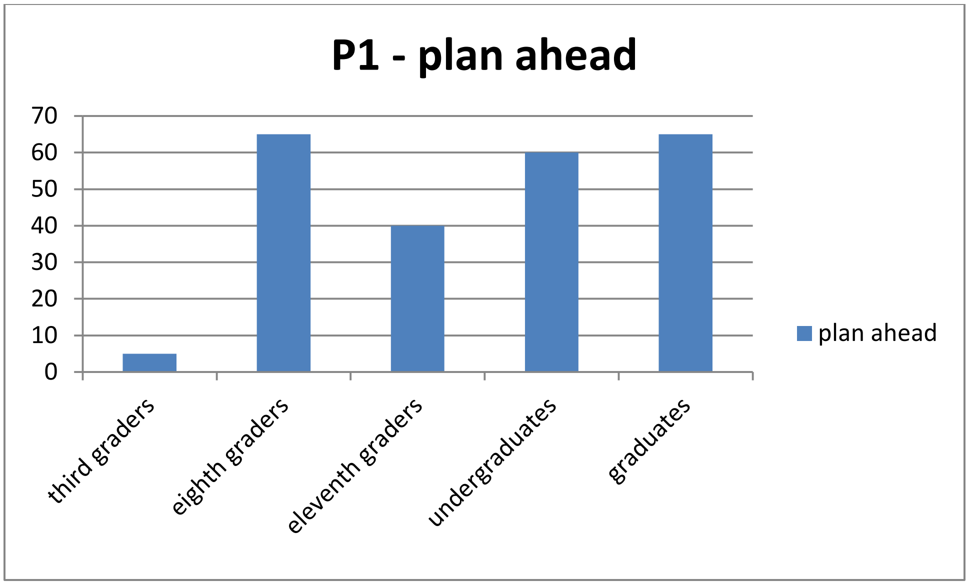
Tower of Hanoi. www.mazeworks.com/hanoi/index.htm

Appendix

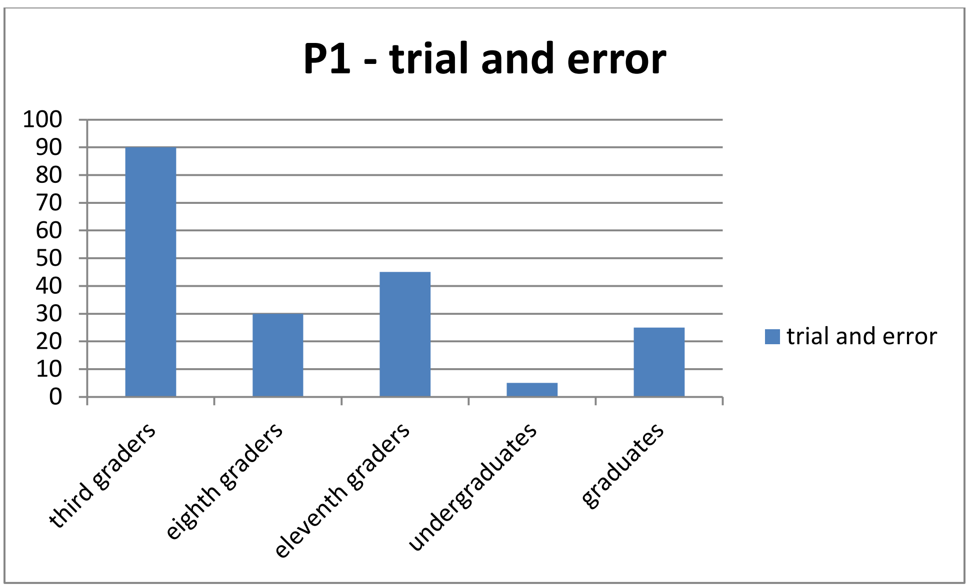
<u>Puzzle 1: Tower of Hanoi</u>



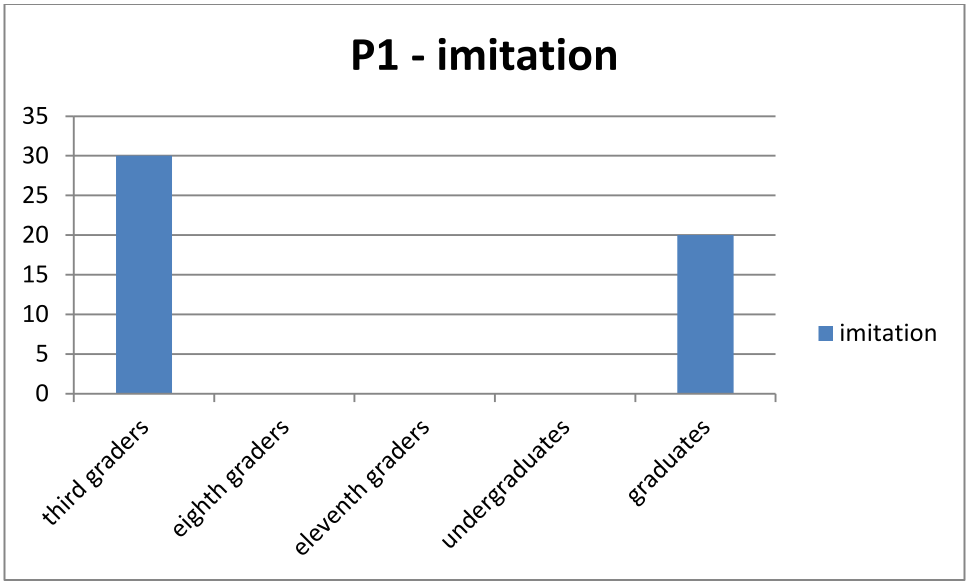


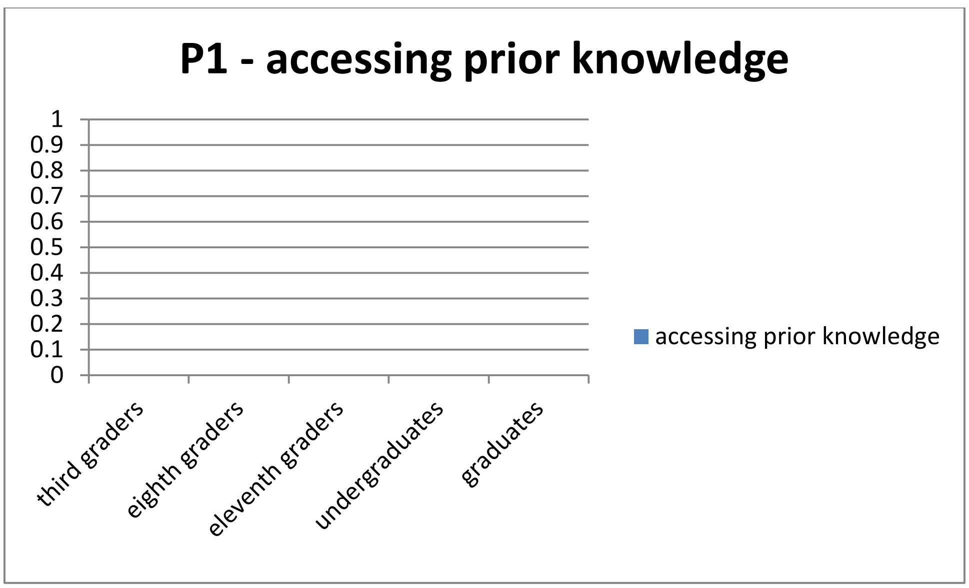


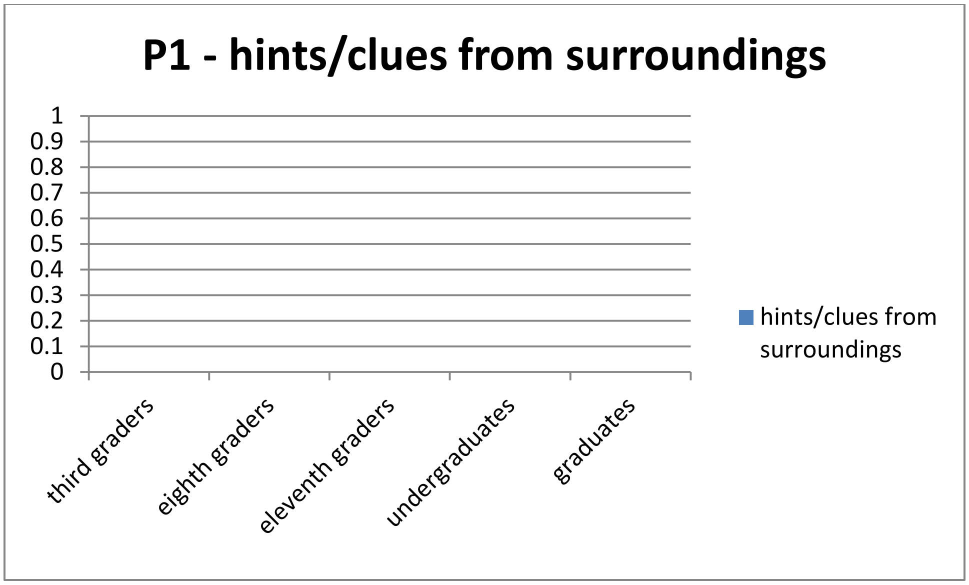
P1 - plan ahead
70
60
50
40
30
20
10
0
third graders
eighth graders
eleventh graders
undergraduates
graduates
plan ahead

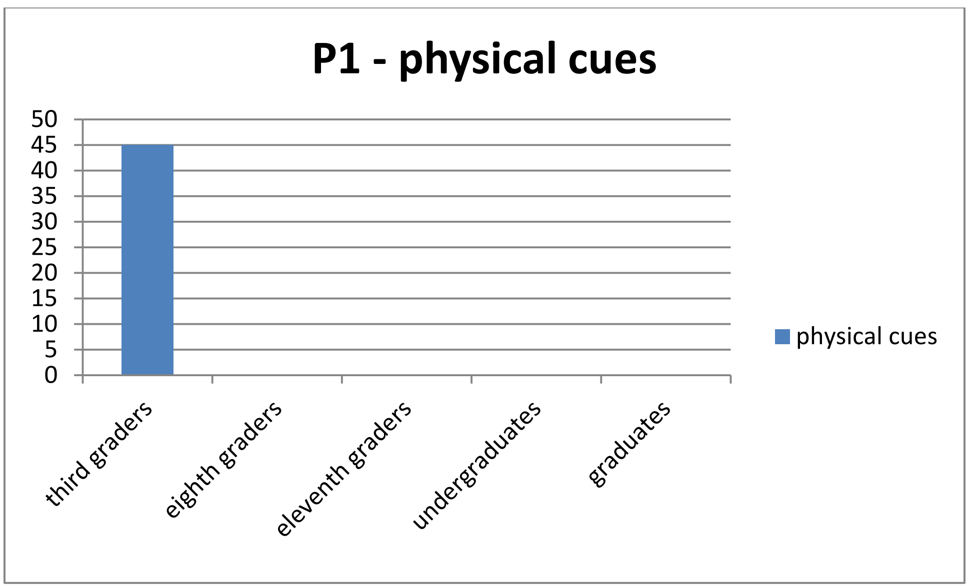


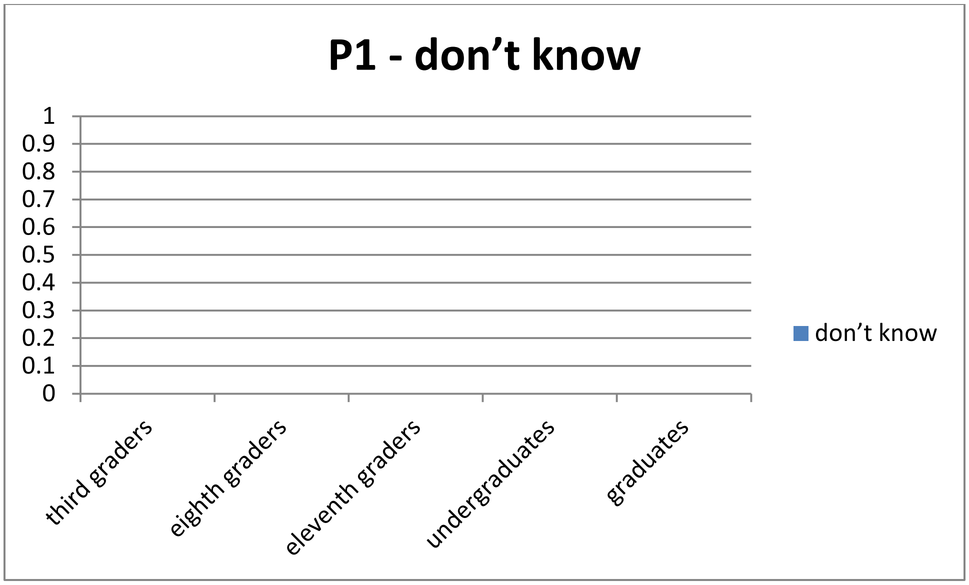
P1 - trial and error
100
90
80
70
60
50
40
30
20
10
0
third graders
eighth graders
eleventh graders
undergraduates
graduates
trial and error


P1 - imitation
35
30
25
20
15
10
5
0
third graders
eighth graders
eleventh graders
undergraduates
graduates
imitation

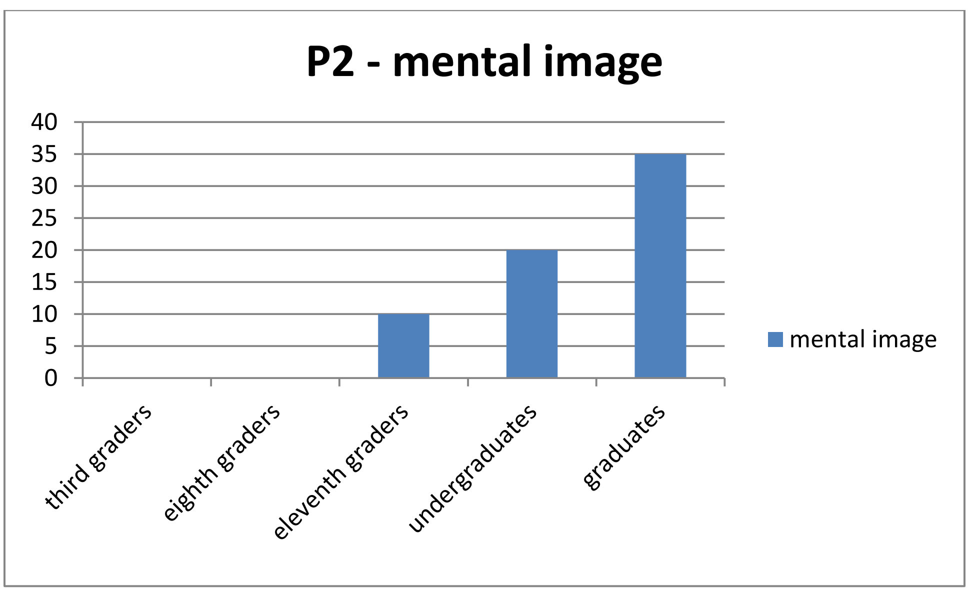

P1 - accessing prior knowledge
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
third graders
eighth graders
eleventh graders
undergraduates
graduates
accessing prior knowledge

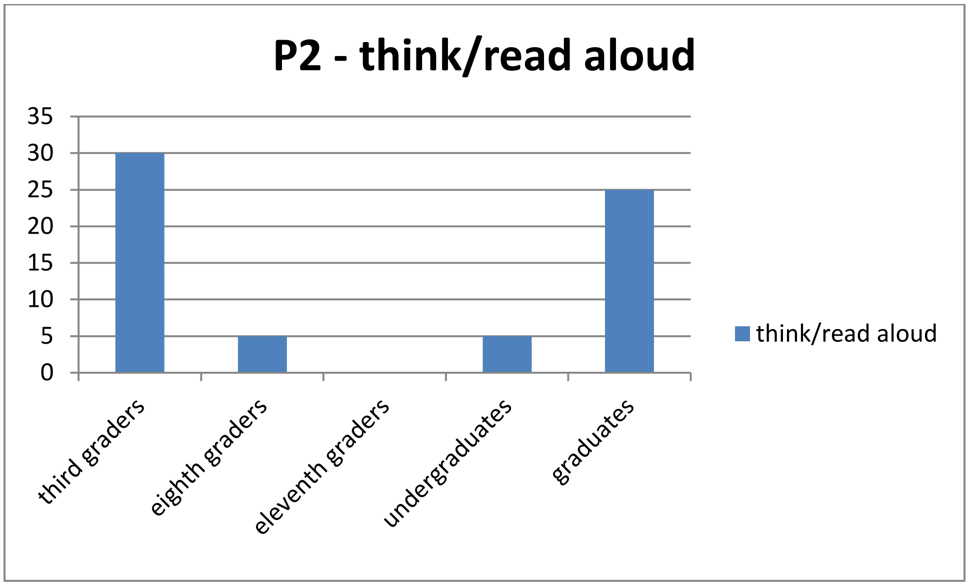

P1 - hints/clues from surroundings
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
third graders
eighth graders
eleventh graders
undergraduates
graduates
hints/clues from surroundings

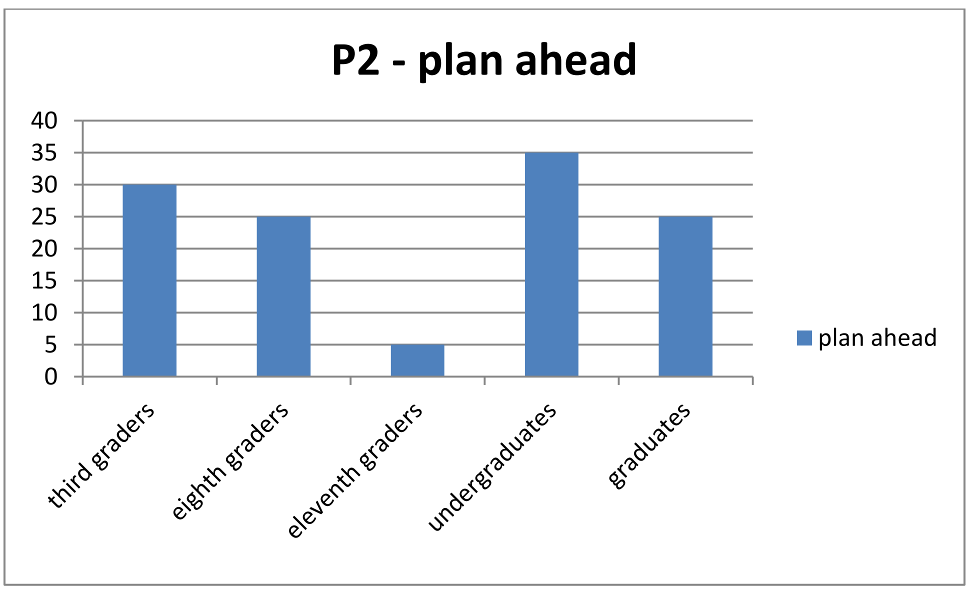

P1 - physical cues
50
45
40
35
30
25
20
15
10
5
0
third graders
eighth graders
eleventh graders
undergraduates
graduates
physical cues

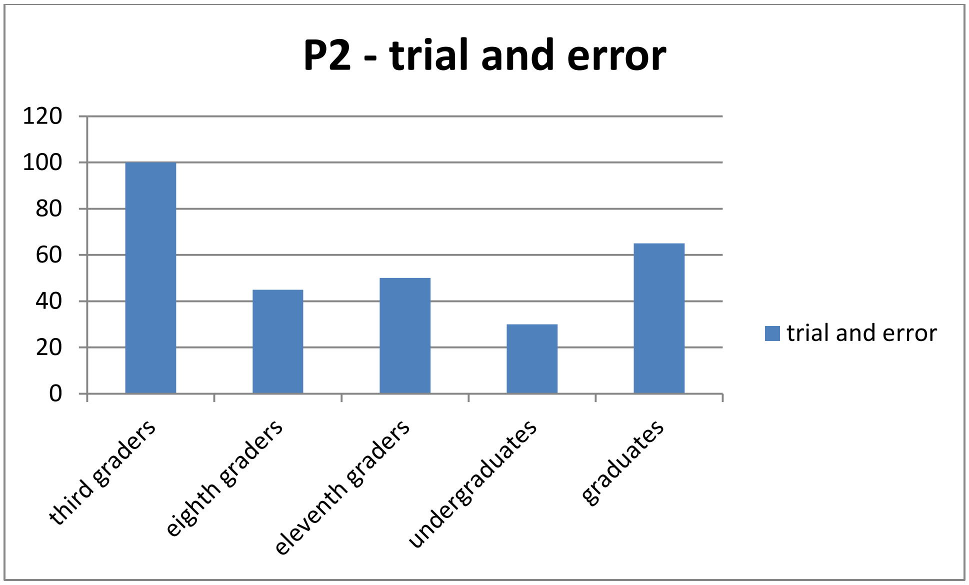


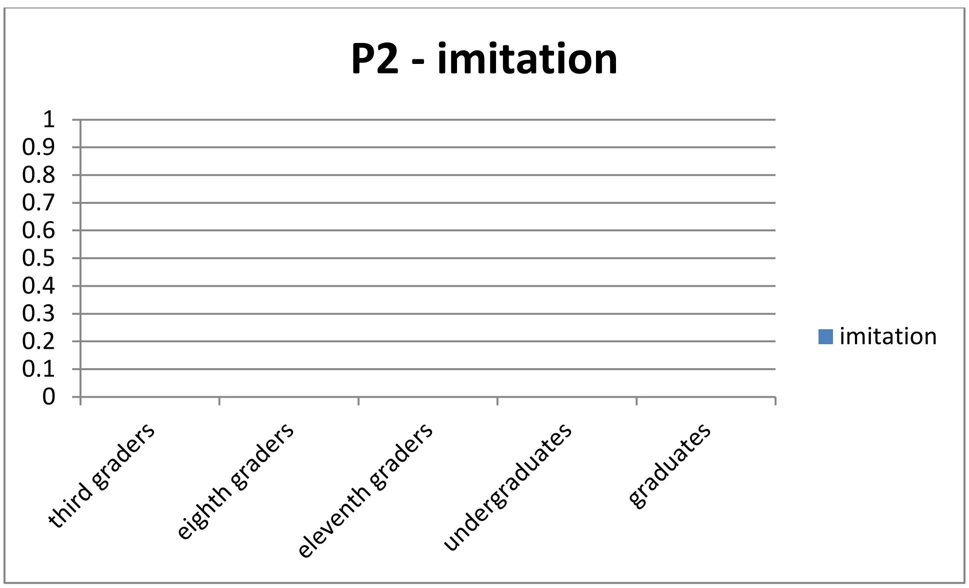
Puzzle Two: River-crossing puzzle

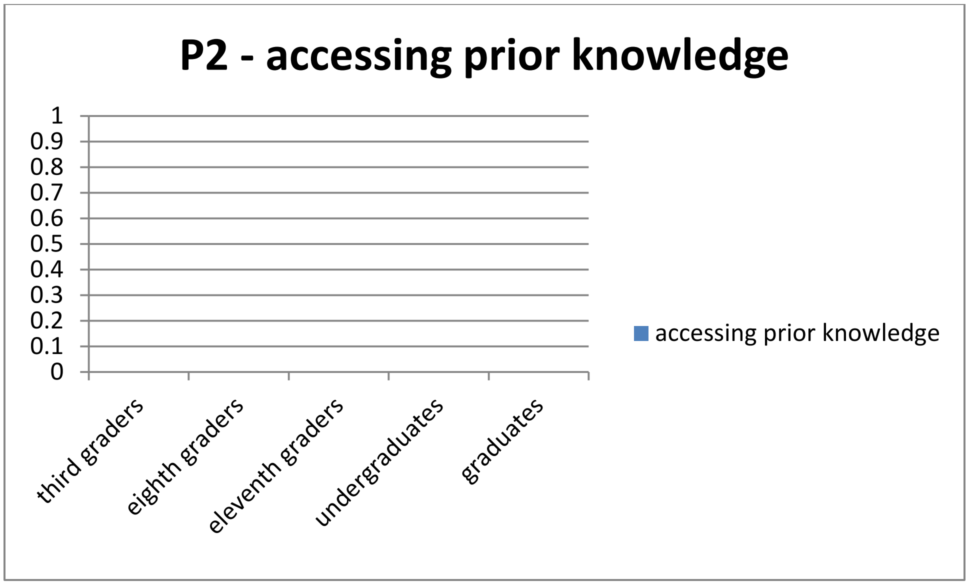


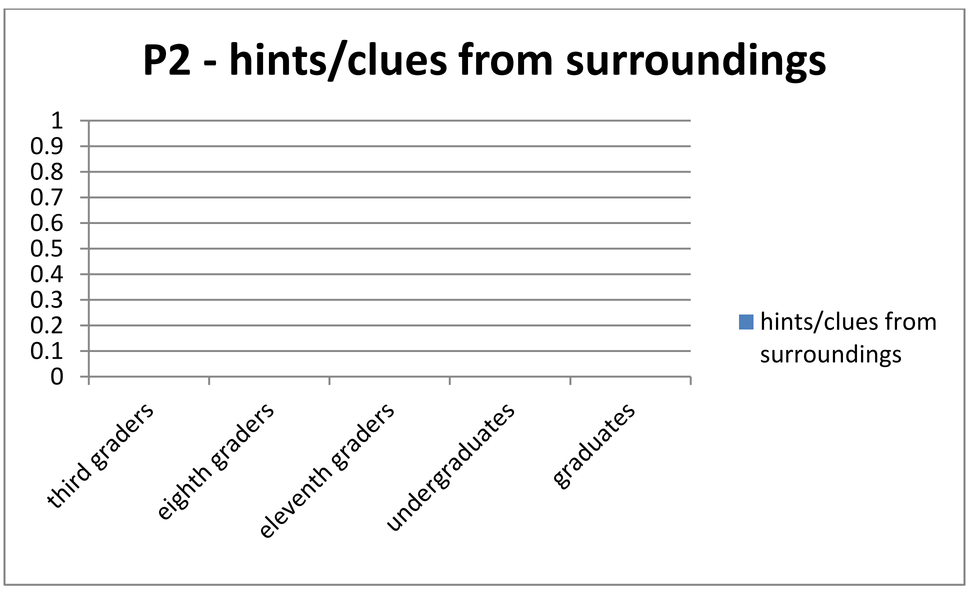

P2 - think/read aloud
35
30
25
20
15
10
5
0
third graders
eighth graders
eleventh graders
undergraduates
graduates
think/read aloud

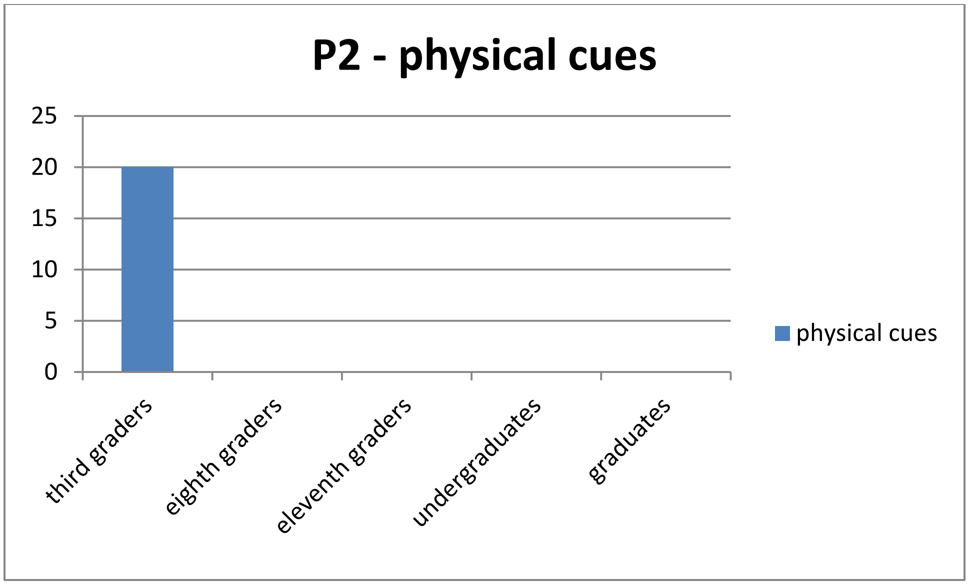

P2 - plan ahead
40
35
30
25
20
15
10
5
0
third graders
eighth graders
eleventh graders
undergraduates
graduates
plan ahead

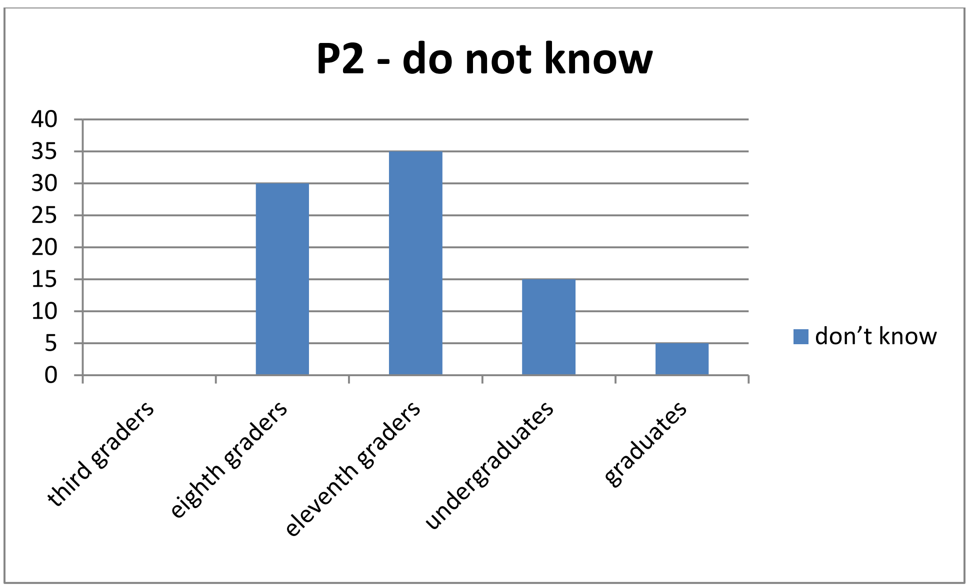

P2 - trial and error
120
100
80
60
40
20
0
third graders
eighth graders
eleventh graders
undergraduates
graduates
trial and error


P2 - imitation
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
third graders
eighth graders
eleventh graders
undergraduates
graduates
imitation

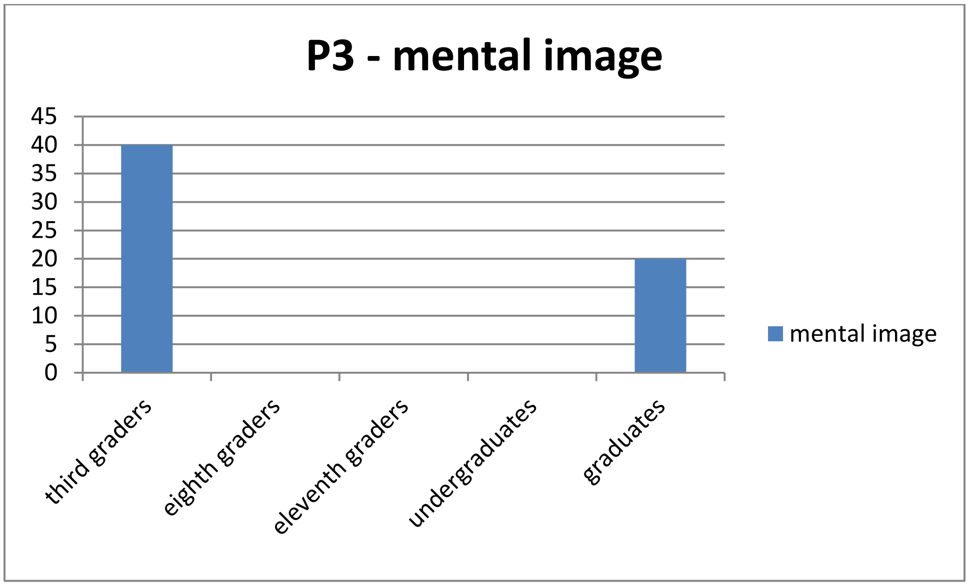

P2 - accessing prior knowledge
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
third graders
eighth graders
eleventh graders
undergraduates
graduates
accessing prior knowledge

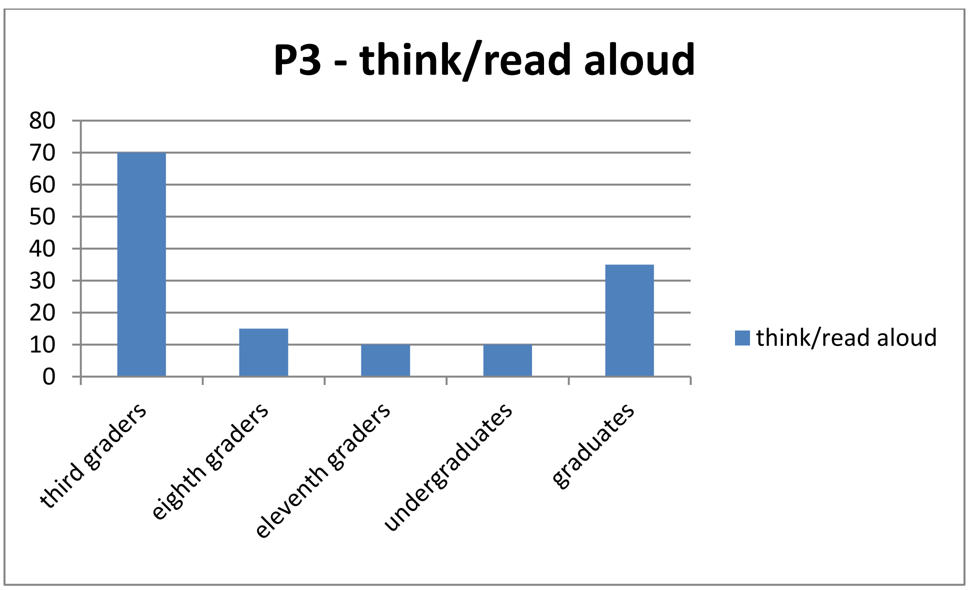

P2 - hints/clues from surroundings
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
third graders
eighth graders
eleventh graders
undergraduates
graduates
hints/clues from surroundings

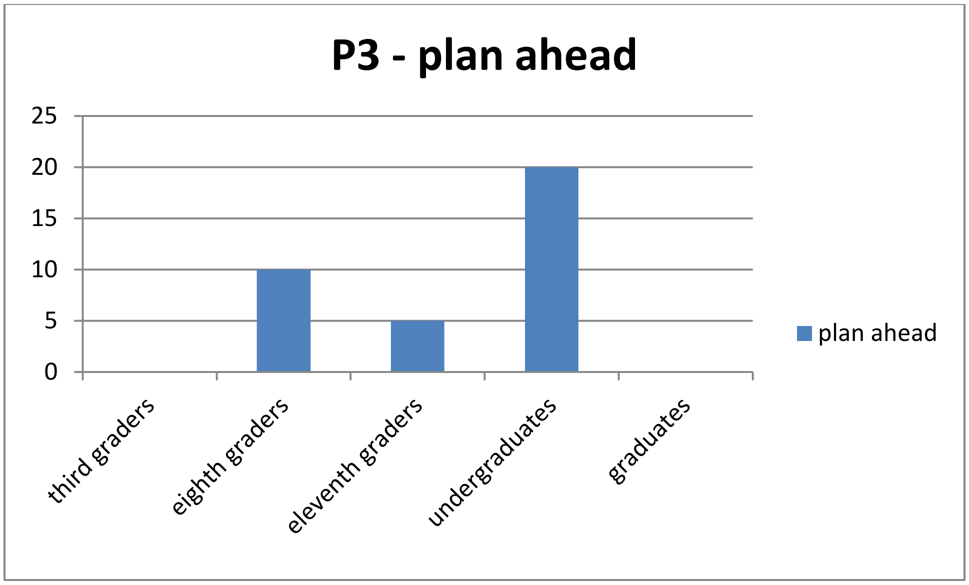

P2 - physical cues
25
20
15
10
5
0
third graders
eighth graders
eleventh graders
undergraduates
graduates
physical cues

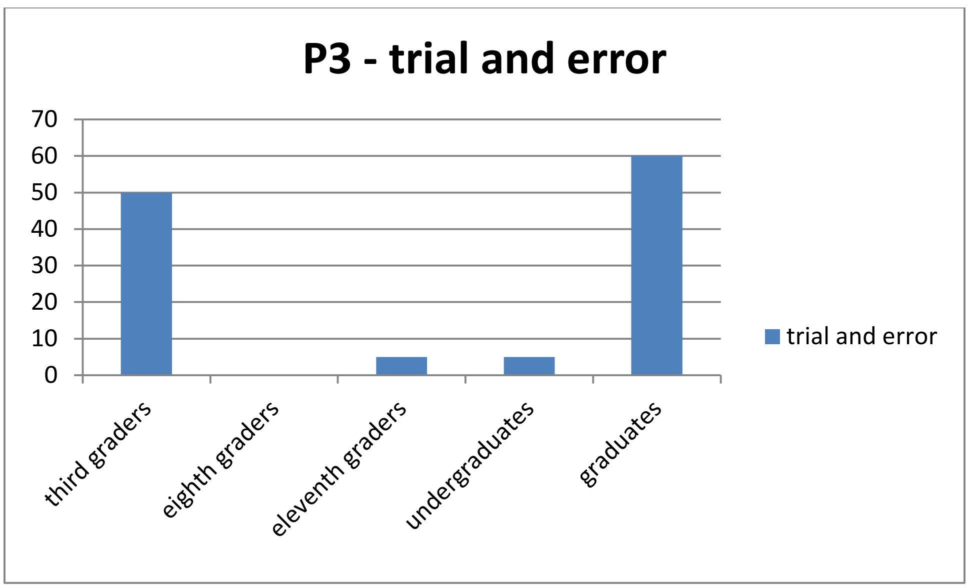

P2 - do not know
40
35
30
25
20
15
10
5
0
third graders
eighth graders
eleventh graders
undergraduates
graduates
don’t know

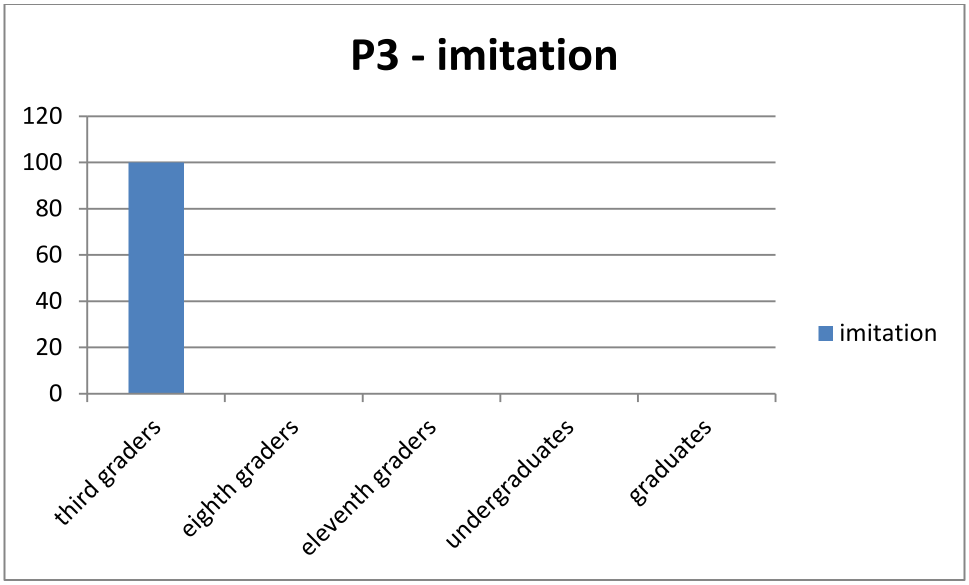
<u>Puzzle Three- Find the Pattern</u>

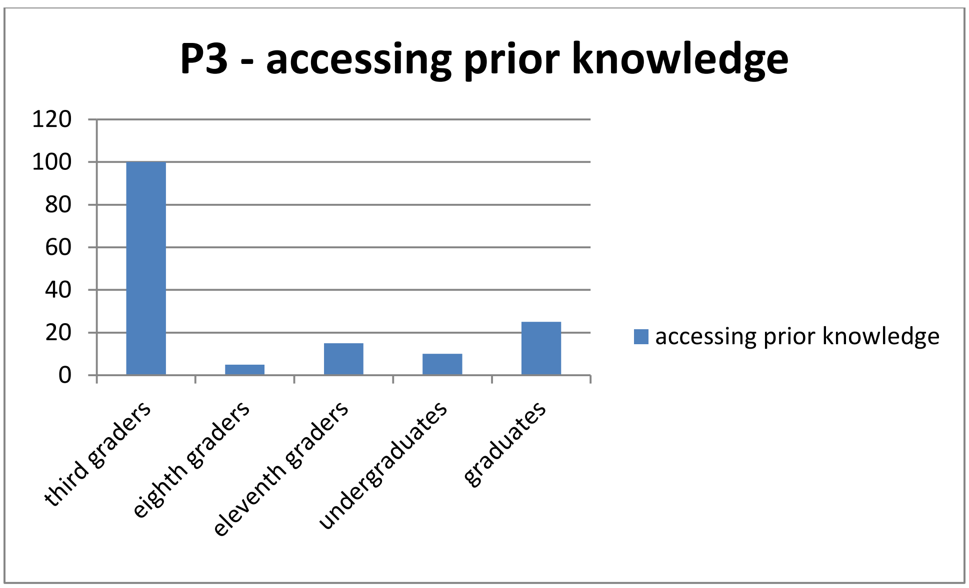


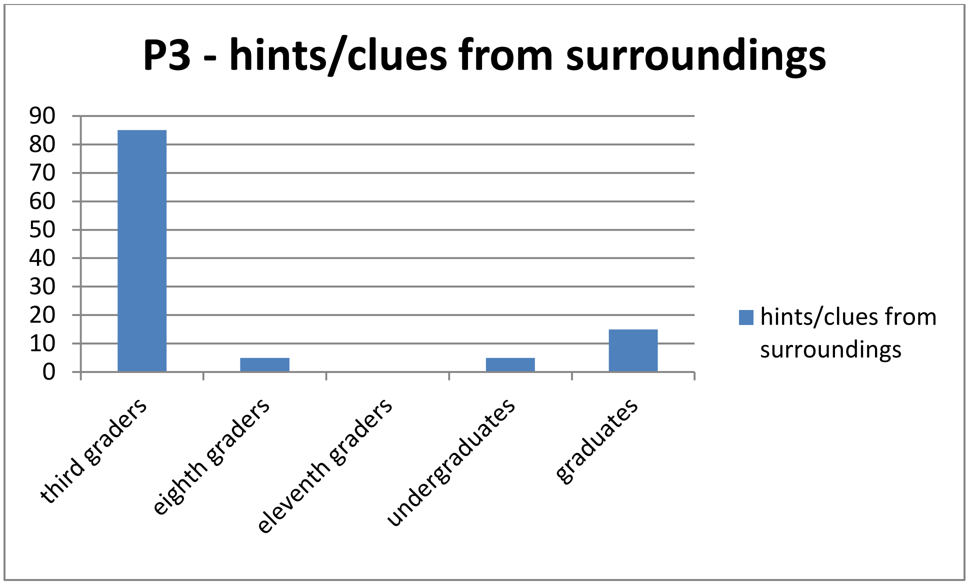


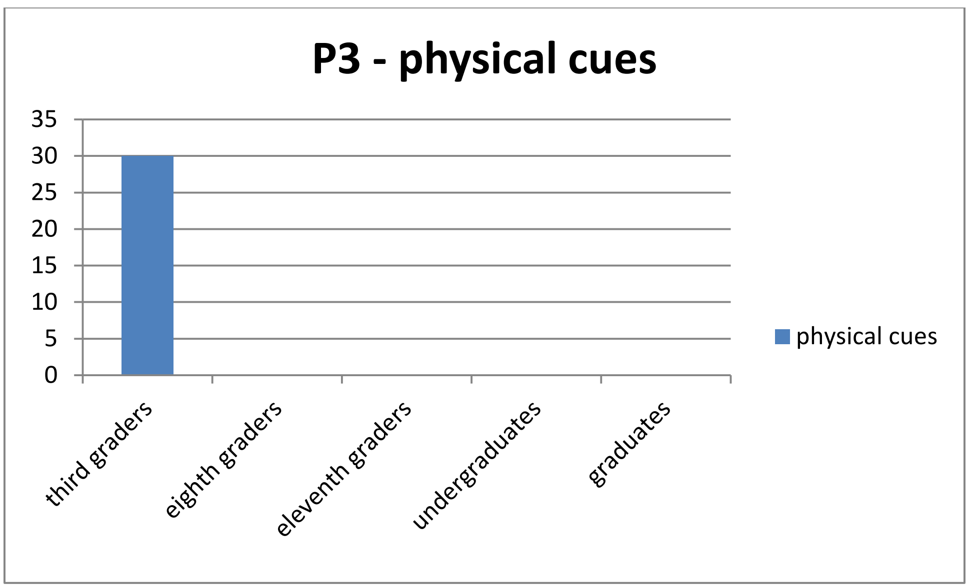

P3 - plan ahead
25
20
15
10
5
0
plan ahead
third graders
eighth graders
eleventh graders
undergraduates
graduates

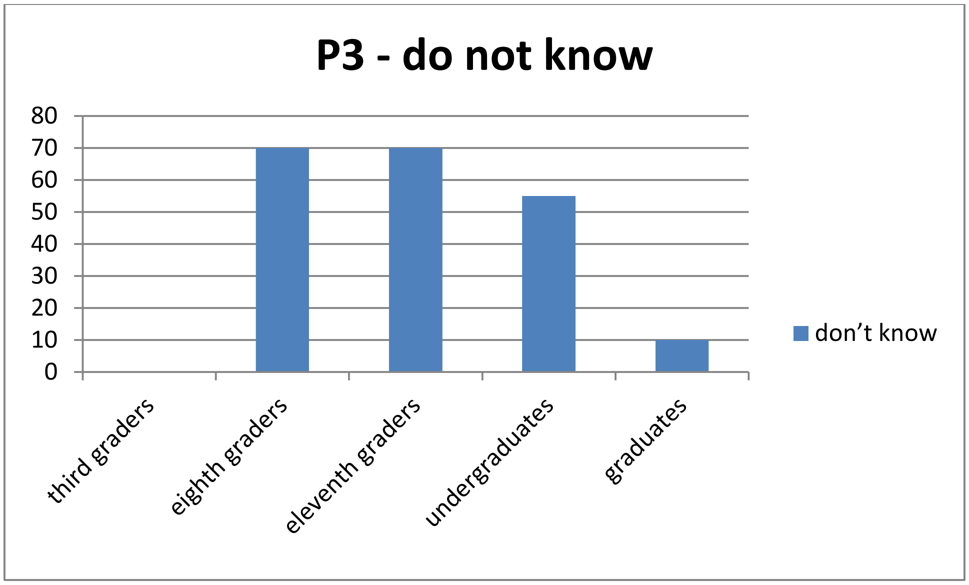

P3 - trial and error
70
60
50
40
30
20
10
0
trial and error
third graders
eighth graders
eleventh graders
undergraduates
graduates


P3 - imitation
120
100
80
60
40
20
0
third graders
eighth graders
eleventh graders
undergraduates
graduates
imitation


P3 - accessing prior knowledge
120
100
80
60
40
20
0
third graders
eighth graders
eleventh graders
undergraduates
graduates
accessing prior knowledge

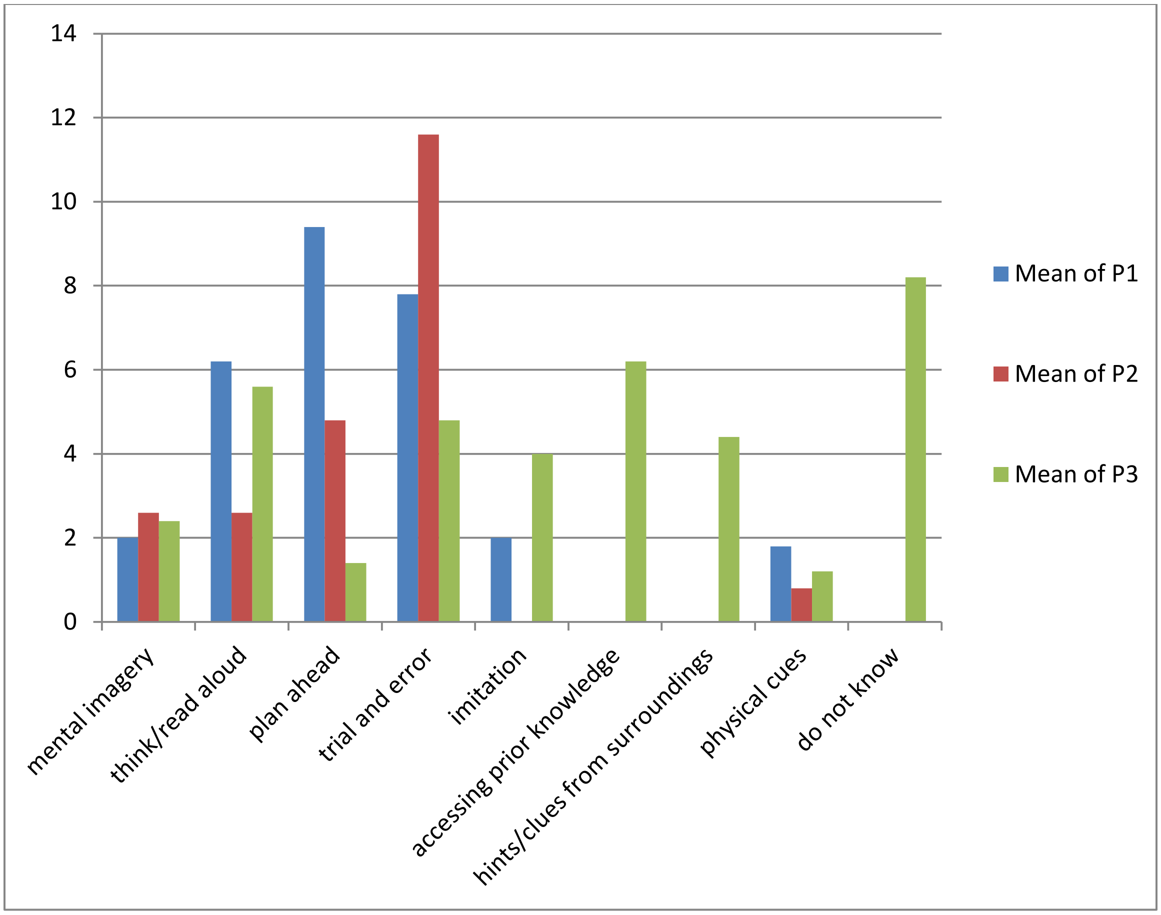

P3 - hints/clues from surroundings
90
80
70
60
50
40
30
20
10
0
third graders
eighth graders
eleventh graders
undergraduates
graduates
hints/clues from surroundings


P3 - physical cues
35
30
25
20
15
10
5
0
third graders
eighth graders
eleventh graders
undergraduates
graduates
physical cues



Thinking Strategies	Mean of P1	Mean of P2	Mean of P3
mental imagery	2	2.6	2.4
think/read aloud	6.2	2.6	5.6
plan ahead	9.4	4.8	1.4
trial and error	7.8	11.6	4.8
Imitation	2	0	4
accessing prior knowledge	0	0	6.2
hints/clues from surroundings	0	0	4.4
physical cues	1.8	0.8	1.2
do not know	0	0	8.2

Table 1.4. Means of P1 (Tower of Hanoi), P2 (River-crossing puzzle) and P3 (Find the pattern).



Detailed Results for Groups 1-5

Students	mental image	think/read aloud	plan ahead	trial and error	imitation	accessing prior knowledge	hints/clues from surroundings	physical cues	do not know
third graders	3	11	1	18	6	0	0	9	0
eighth graders	1	3	13	6	0	0	0	0	0
eleventh graders	2	2	8	9	0	0	0	0	0
undergrad.	4	1	12	1	0	0	0	0	0
Graduates	0	14	13	5	4	0	0	0	0
Total	**10**	**31**	**47**	**39**	**10**	**0**	**0**	**9**	**0**

Puzzle One- Tower of Hanoi.

students	mental image	think/read aloud	plan ahead	trial and error	imitation	accessing prior knowledge	hints/clues from surroundings	physical cues	do not know
third graders	0	6	6	20	0	0	0	4	0
eighth graders	0	1	5	9	0	0	0	0	6
eleventh graders	2	0	1	10	0	0	0	0	7
undergrad.	4	1	7	6	0	0	0	0	3
graduates	7	5	5	13	0	0	0	0	1
Total	**13**	**13**	**24**	**58**	**0**	**0**	**0**	**4**	**17**

Puzzle Two-River-crossing puzzle.

students	mental image	think/read aloud	plan ahead	trial and error	imitation	accessing prior knowledge	hints/clues from surroundings	physical cues	do not know
third graders	8	14	0	10	20	20	17	6	0
eighth graders	0	3	2	0	0	1	1	0	14
eleventh graders	0	2	1	1	0	3	0	0	14
undergrad.	0	2	4	1	0	2	1	0	11
graduates	4	7	0	12	0	5	3	0	2
Total	12	28	7	24	20	31	22	6	41

Puzzle Three-Find the Pattern.

Summarized Results for Groups 1-5

students	mental image	think/read aloud	plan ahead	trial and error	imitation	accessing prior knowledge	hints/clues from surroundings	physical cues	don't know
3rd p1	3	11	1	18	6	0	0	9	0
3rd p2	0	6	6	20	0	0	0	4	0
3rd p3	8	14	0	10	20	20	17	6	0
8th p1	1	3	13	6	0	0	0	0	0
8th p2	0	1	5	9	0	0	0	0	6
8th p3	0	3	2	0	0	1	1	0	14
11th p1	2	2	8	9	0	0	0	0	0
11th p2	2	0	1	10	0	0	0	0	7
11th p3	0	2	1	1	0	3	0	0	14
Sop p1	4	1	12	1	0	0	0	0	0
Sop p2	4	1	7	6	0	0	0	0	3
Sop p3	0	2	4	1	0	2	1	0	11
Gr p1	0	14	13	5	4	0	0	0	0
Gr p2	7	5	5	13	0	0	0	0	1
Gr p3	4	7	0	12	0	5	3	0	2
total	**35**	**72**	**78**	**121**	**30**	**31**	**22**	**19**	**58**
mean	**2**	**5**	**5**	**8**	**2**	**2**	**1**	**1**	**4**

QUESTIONNAIRE

Gender: ___

Age: ____

Have you done any of these puzzles before? ____

Puzzle 1: Tower of Hanoi

Did you solve it?

Yes ____

No ____

No. of moves: _________

How did you solve it?

Puzzle 2: Cannibals and Missionaries

Did you solve it?

Yes ____

No ____

How did you solve it?

Puzzle 3: Find the Pattern

Did you solve it?

Yes ____

No ____

How did you solve it?