Comparative study of modern credit risk assessment methods

التحليل المقارن المعاصر لأساليب تقييم مخاطر الإفلاس

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Abstract (وصف مختصر)

القدرة على التنبؤ بالافلاس هي ذات قيمة كبيرة للمستثمرين الدائنين وغيرهم من مستخدمي معلومات للشركة. وعلاوة على ذلك كما اتضح من الأزمة الأخيرة - الانخفاض المالي له أثر المضاعف على مستوى الاقتصاد الكلي وتكليف الحكومة والمجتمع غاليا. تنبئ نماذج متعددة بالإفلاس و هي وضِعت و عُرِضت من قبل العلم في السنوات الأخيرة.

في هذه الأطروحة قد أستعملت أساليب التقييم التالية:

أ) Merton's model
ب) KMV
ت) Z-score
ث) Binomial approach

في هذا العمل أستخدمت بيانات 14 شركة على مدى 3 سنوات (2007-2009) و هذه الشركات تمثل 4 مجالات:

أ) الصرافة
ب) صناعة السيارات
ت) صناعة الإلكترونيات
ث) النفط و الغاز

من بيانات 14 شركة – 12 منها ناجحة, و 2 منها أفلست في سنة 2007.

استنتاجات الأطروحة تتضمن مناقشة نتائج الاختبارات التي أجريت من أجل تحديد إحتمال الإفلاس. أيضا من هذه الأساليب المدروسة تم تحديد أفضل نموذج لاختبار احتمال الإفلاس.

Abstract

The ability to predict bankruptcy is of great value for investors, lenders and other stakeholders of the companies. Moreover as it has been shown by the recent crisis originated in sub-prime market the financial distress can have multiplicative macroeconomic effect and bring high cost to economy of countries and the society. Thus there are various models to forecast failure of the firms which have been developed and proposed by academics in the recent years.

This dissertation presents the basic framework and structure of four credit risk assessment models, namely (1) Merton's structural model, (2) KMV, (3) Z-score, and (4) Binominal approach. Then, work discusses limitations to practical usage of each model, and it also explains some necessary conditions before implementing these models.

Real historical data was used to examine the effectiveness of each model in early bankruptcy forecasting. Financial variables of fourteen companies from four different industries were analyzed with two companies in the sample which eventually went bankrupt. The following industries are under consideration in this study − (1) banking, (2) automobile, (3) electronics, and (4) oil and gas sector. Back testing simulation was run on company's financial data collected for 3-5 years pre sub-prime crisis time horizon. On purpose, sample from each industry contains one company that has really defaulted in subsequent years.

As a conclusion, work contains a discussion on results obtained to derive a conclusion on which risk assessment model(s) is (are) best in identifying pre-default companies.

Keywords: Merton, KMV, Z-score, Binominal, Distance-to-Default (DD), Expected Default Frequency (EDF), Implied Default Probability (IDP).

Contents

1. Introduction

Credit assessment is an acknowledged worldwide issue – the direct and indirect costs associated with financial distress of the companies are enormous and affect both shareholders and stakeholders. Recent failure of such large companies as General Motors, Lehman Brothers, Chrysler and many more, has demonstrated that even giant corporations are not insured against failure and distress caused by their bankruptcy can lead to both economic and social problems and trigger financial crisis on a worldwide level. Thus both corporates and regulators pay so much attention to this issue and research on predictability of bankruptcy has become so important.

A number of research papers have been devoted to the problem of credit assessment during last four decades. However two main methodologies and their extensions had gained the most popularity among both academics and practitioners. Specifically these are structural models, which are based on original approach proposed by Merton (1974) and reduced-form models which are based on model developed by Altman (1968). Data sample consist from financial variables of 14 companies for the time period of 2006-2009. Two companies in the selected pool have been classified as "defaulted" in this paper due to the fact that they have experienced to some extent or another troubles with meeting their financial obligations. The time horizon has been chosen deliberately so it would focus on recent turmoil in the World's economy, thus each model could be tested on its effectiveness in predicting the possible failure.

This paper compliments prior studies in several ways. First of all, the research objective of this study is to provide comparative analysis of the predictability powers of modern credit risk assessment methods: Altman's Z-score and Merton's structural models, commercialized version of Merton’s approach called Moody’s KMV and binomial model. To the knowledge of the author this is the first attempt to compare these four methods in one research paper. Secondly, the study concentrates on time period of recent Global crisis, which followed the sub-prime lending crisis in USA, a time when it is especially crucial for institutions and supervisory bodies to be able to forecast financial distress. Thus this paper estimates the efficiency in forecasting powers of the considered models under distress environment.

This part aims to provide background information on each of four selected modern credit risk assessment methods: (1) Merton's structural model, (2) KMV, (3) Z-score, and (4) Binominal.

Academic background section describes underlying theoretical frameworks that explain cause and effect relationships between financial characteristics of the corporation (borrowing entity) and probability of its bankruptcy. It contains discussion on assumptions within each model that make it possible to apply those theories to the real cases in practice. There is also a list of number of model limitations that affect reliability of outcomes.

Literature review section discusses number of previous academic works that studied selected four methods from different angles and findings of previous studies. It also provides a description of different deviations and extensions from these core models proposed by modern followers.

The **academic study section** provides details about the data, methodology used and research design.

Lastly, of the study **analysis** and **conclusions** of the work are presented on the testing of selected credit risk assessment methods and implications of the results are discussed together with practical usage and limitations.

1.1. Academic background

1.1.1. Credit risk assessment model − Merton's contingent-claims approach

One of the most commonly used credit risk assessment methods is based on the option pricing model introduced by Black and Scholes in 1973. Robert Merton in 1974 extended Black and Scholes's capital asset pricing model in his seminal paper on valuation of corporate debt. Merton developed his contingent-claims approach in which he looks on the corporate liabilities as on call option on the total value of the corporation. Merton formulated the model for assessing credit risk of a corporation by comparing company's equity to call options on its assets. Describing this idea from opposite perspective, the creditors of the corporation could be viewed as sellers of European put options on firm's assets and default occurs if the value of the company assets is lower than value of its debt obligations at the time of maturity. This asset valuation model, also known as option theoretic model, lies into group of Structural models. All structural models are derived from the core work of Black and Scholes and an extension of this idea developed by Robert Merton. The term "structural" implies that models are based on company's structural attributes − such as, capital structure and asset volatility. Based on these vital financial indicators structural models seek to assess the credit risk of the organization. Other relevant elements of the credit risk, as default or losses-driven default depend on these structural variables.

Robert Merton’s brief biography:

Robert Merton was born on 14 July, 1944. He is a well known American economist. He received bachelor degree in Mathematics from Columbia University. His first Master’s degree is from California University. And doctorate degree he received from MIT Sloan School of Management.

1970-1988	Professor of MIT University
1989	He received Master of Arts degree at Harvard University
1991	He received Doctor of Laws degree at University of Chicago
1997	He was awarded Nobel Prize in Economics
2005	The Merton Exhibit was launched at Harvard University
2010	Currently he is the Professor at Harvard University and professor at MIT Sloan School of Management

The Merton model allows estimating probability of the default of the particular company within the sample at particular point in time. Estimations are based on a snapshot of company financials at particular chosen moment in company's life. The probability is calculated as (1) an estimate of future value of the company *minus* (2) book value of the outstanding debt *divided* by (3) estimated volatility of the company (adjusted to the scale of forecasting time horizon). This calculated score, which in the Model is called − distance to default, is then applied into cumulative density function to calculate the probability of company value to be less than the book value of debt along selected time horizon. Here, market value of the company consists of market value of its debt and the market value of its equity. It would have been very easy to apply Merton's model to real companies if information on both of these components was easily obtainable. However, even through an estimate of company's equity value is usually a publicly available piece of information, good estimation of the debt's market value is often not that easy to find. To mitigate effect of this limitation Merton has made two especially valuable and important assumptions:

First, total market value of the company assumed to go in line with geometric Brownian motion. The basic cornerstone of Merton's model is that company's value is rising over time − assuming risk-neutral option pricing environment and increase of total value at risk-free interest rate − this over time decreases company's leverage.

The second important assumption of the Model is that the company has only one type of discounted bonds which mature at a time T (chosen time horizon during which model users want to measure the probability of company's default). Company is considered as defaulted if its predicted assets value upon maturity is less than the predicted debt repayment amount. As mentioned before, Model treats equity of the company as a European call option on company's assets. Maturity of these options is equal to the time T and option's strike price is taken as equal to the book value of the debt outstanding. To apply Merton model to real company in real environment the following inputs need to be collected and determined:

- current value of the assets;
- volatility of the assets;
- outstanding debt; and
- maturity of the debt.

In practice, Merton model can serve two important purposes, it can be applied to: 1) estimate the probability of company's default within chosen timeframe and 2) to estimate the credit spread on company's borrowings. However, empirical studies aiming to estimate the credit spreads using structural models, including Merton's model, are quite rare. They are hindered by the following limiting conditions:

- companies often have more than one issue of debt;
- the debt is not zero-coupon;
- there are call features and sinking funds;
- the firm value must be known in order to find its volatility, yet at the same time the volatility affects the value of the debt and hence the firm value;
- the presence of liquidity premium, transaction costs and taxes.

In particular the need to exclude companies which do not have very simple capital structures has led to small samples of bonds being available for testing the models. To be able to apply this model to real companies and to make results analytically meaningful analysts have to derive respective value and the volatility of the company's assets from the market value of company's equity and its at-the-moment volatility. Maturity of outstanding debt is chosen and if company has several loans with different repayment dates payment schedules are converted into one with a single repayment date.

Other unconditional assumptions to practical application of Merton model include:

- absence of transaction costs;
- bankruptcy costs;
- taxes or problems with indivisibility of assets;
- continuous time trading;
- unrestricted borrowing and lending at a constant interest rate;
- no restrictions on the short selling of the assets;
- and a few more of less influential implicit assumptions.

Another important point in assumption to Merton's method is that the term “probability” for purposes of these model does not refer to the real likelihood of default as a result of debt costs being above the value of company's assets at maturity or vice versa option's strike price being higher than assets value at expiration (a non-default situation). Since underlying asset is risky, its volatility does not perfectly correlate with risk free rate volatility pattern. Hence, for purposes of Merton formula, risk free interest rate can be replaced with the expected return on the asset's value or in other words with the volatility of asset's value. This measure will allow obtaining more objective estimate of company's probability of default. As per findings of Deliandes and Geske derived in their joined work issued in 2003 − risk neutral default probability is somewhere on the upper bound of a range of objective default probabilities. Even though the objective and risk neutral distributions of the company's value have the same variance (or similar distribution patterns), still the objective distribution will generally have a greater *mean* than those of the risk free rate, i.e. respected return on assets is generally higher than the risk free rate of return. As a result it can be concluded that risk neutral distribution shows higher default probability. However, it is general understanding that expected returns on equities is hard to estimate and that those estimated generally done with significant error. Here Model claims that since risk neutral probabilities of default can be estimated without calculating the company’s expected return, risk neutral probability of default is a more accurate estimate compared to objective default probabilities.

In general form **Merton's formula** can be derived in the following way. For simpler representation let us describe company's balance sheet can be viewed as:

Assets	**Liabilities**
A_t	D_t (T, F) E_t

Where,

A − is the asset value of a company;

D_t − is the value of company's debt at a time *t*;

E_t − is the value of company's equity at a time *t*;

A is determined by the firm’s future cash flows. As was mentioned before, *A* follows Brownian motion and its value at time *t*, given by A_t, satisfies the following equation:

$$\frac{dA_t}{A_t} = r_A(t)dt + \sigma_A(t)dz_t,$$

where, $r_A(t)$ and $\sigma_A(t)$ denote asset return rate − $r_A(t)$ can be broken down to the following components: risk-free interest rate r, asset risk premium λ and asset payment ratio δ − and volatility of asset value, z_t follows the standard Wiener process and dz_t has standard normal distribution. In Merton's work (1974), $r_A(t)$ and $\sigma_A(t)$ are taken as constants and considered to be non-stochastic. Plus Merton has made an assumption that the firm's capital structure depends only on two factors: (1) pure equity (no preference stocks is taken into consideration) and (2) an existence of only one single type of zero-coupon debt maturing at time t, with a face value D. The default event only occurs when the asset value, A, at maturity is less than D. In event of random default occurrence, the stock price of the defaulting firm is assumed to go to null. Thus, the following payment equations should hold true:

$$\begin{cases} \text{Receives of debt holders} = \min(A_t, B) \\ \text{Receives of equity holders} = \max(A_t - B, 0) \end{cases}$$

Again, Merton model implies that debt holder can be considered as a seller of European put option, whereas equity holder can be considered as a buyer of European call option, thus, asset value A can be considered as the price of underlying security. By assuming no pay-out of dividends, standard Black-Sholes option-pricing equation can be used to get a relation between the equity market value, E_t, and A_t and the bond market value, Y_t, and A_t. In general form, that will be:

$$\begin{cases} E_t = f_1(A_t, \overline{D}, r, \sigma_A(\overline{t}), T) \\ Y_t = f_2(A_t, \overline{D}, r, \sigma_A(\overline{t}), T) \end{cases}$$

where, r stands for short-term risk-free interest rate and other variables hold the same meaning which was set to them before. Variables that are marked with a "bar" sign above them are observable and exogenous. Here it should be noted that since statement of Modigliani-Miller Theorem I (1958) implies that for any capital structure $E_t + Y_t = A_t$, any one of the above two equations can be derived from the other. In practice this suggests that there is actually no need to solve the second equation. As proposed by Delianedis and Geske (1998), there is connection between observable stock volatility σ_t and unobservable asset volatility $\sigma_A(t)$, so if the form of this linkage is specified as

$$\sigma_E(t) = g\ (\sigma_A(t)),$$

[for example, many authors directly use $\sigma_E(t)$ to substitute for $\sigma_A(t)$], Y_t and A_t can be known.

1.1.2. Credit risk assessment model − KMV[1] model

The whole range of structural credit risk assessment models became popular after their introduction by commercial firms such as KMV in 1990s. KMV Corporation is a company with major specialization in credit risk analysis. Since early 90's it has started to develop and master its own in-house credit risk methodology. KMV Model allows assessing probabilities of default and the loss distribution in relation to both default and migration risks. In addition to that, corporation has also started gathering an extensive historical database to estimate the empirical distribution of distances to default. Then, based on that distributions model became equipped to calculate default probabilities.

Generally KMV Model − or also known as Moody's KMV − is a more practical extension of Merton's model proposed in 1974. However, KMV Model contains certain substantial developments that differentiate it from underlying Merton's work.

- First important difference is that KMV Model employs a proprietary model that is called the VK model. Apparently the VK model is a generalization of the Merton model that currently embodies five classes of liabilities; short-term, long-term, convertible, preferred equity, and common equity.
- Second development is that Merton's model applies the cumulative normal distribution to convert distances to default into default probabilities, whereas Moody’s KMV uses its, above mentioned, historical database to estimate the empirical distribution of distances to default and to calculate default probabilities based on that distribution.
- Finally, KMV Model's functional allows making proprietary adjustments to the accounting information used to estimate the face value of the debt.

There is also number of other commercial credit risk assessment models that in principle are similar to KMV model pillars and some of which are also derived from core work of Merton (1974). Other most popular commercial models include:

[1] KMV is a trademark of KMV Corporation. Stephen Kealhofer, John McQuown and Oldrich Vasicek founded KMV Corporation in 1989. On February 11, 2002, Moody’s announced that it was acquiring KMV for more than $200 million in cash.

1) CreditRisk+

One of alternative commercial credit risk assessment models was introduced by Credit Suisse Financial Products at the end of 1997. This approach was named, CreditRisk+, and its distinction was in the fact that it only focused on default. CreditRisk+ assumes that default of individual bond or loan follows a Poisson process. Credit migration risk is not fully modeled in this analysis. Instead, CreditRisk+ allows for stochastic default rates which partially, although not strictly, rely on migration risk.

CreditRisk+ applies an actuarial framework for the derivation of the loss distribution of a loan portfolio. Only default risk is estimated with no reference to downgrade risk. Contrary to KMV, default risk here is not related to the capital structure of the company.

2) CreditPortfolioView

Another modern commercial credit risk assessment model was proposed by a business consulting company McKinsey. Firm's own model was introduced under the name, CreditPortfolioView, which, like CreditRisk+, intends to measure only a default risk. It is a discrete time multiperiod model, where default probabilities are functions of macroeconomic indicators like the growth rate of the economy, unemployment rate, interest rate level, volume of government expenses, foreign exchange rates. In other words macro variables which also determine credit cycles. Credit cycles heavily depend on movements on those indicators.

CreditPortfolioView is a model capturing and assessing influence of multiple macroeconomic factors on a firm. These factors are used to simulate the joint conditional distribution of default and migration probabilities for various rating groups in different economic sectors. It is logical and it was empirically observed that default probabilities, as well as credit migration probabilities, are linked to economical cycles. There is not much academic information available on this commercial model with most of available studies quite outdated. The reason of such limited information on recent developments to this model may be linked to the business confidentiality issue. Due to this possible reason McKinsey's technical improvements might not be disclosed to the public.

3) CreditMetrics

Finally, there is CreditMetrics developed by JP Morgan. It was first introduced and then well publicized in 1997. This approach is based on credit migration analysis, or in other words, on probability of firm's credit quality moving from one grade to another, including default credit ratings. Probability of such credit quality moves is studied within a given time horizon. Usually, there is a general market practice to take this time horizon as a one year arbitral period. CreditMetrics model builds the picture of forward values that any bond or loan portfolio may take in the future, say within one year time horizon, where future value movements are determined only in relation to credit migration, at the same time assuming that interest rates evolve in deterministic manner. Then, credit VaR of a portfolio is derived in a similar way as for market risk. It is basically the percentile of the distribution corresponding to the selected confidence level.

KMV methodology differs from CreditMetrics model as it uses the "Expected Default Frequency", or EDF, which is unique for each issuer and depends on its financials, rather than relying upon the average historical transition frequencies produced by the rating agencies for each credit grade. As oppose to the CreditMetrics, KMV does not base its estimations on Moody's or S&P's statistical data when assigning a probability of default. As such statistical data only takes into account the rating of the obligor. Classifying companies by credit ratings is not the best solution as one credit rating may include range of non-identical companies in terms of associated credit risk. Instead, KMV derives the actual probability of default and the Expected Default Frequency (EDF), individually for each borrower based on the Merton's model.

We have to note here that the methodology is not the major drawback of CreditMetrics, contrariwise methodology is rather appealing attribute of the model. The real problem of CreditMetrics is in the above mentioned reliance on transition probabilities that are based on average historical frequencies of defaults and credit migration. As a result the accuracy of final CreditMetrics outcomes rely upon two crucial assumptions:

first of them is that all companies that fall into one rating class supposed to have the same default rate, and second assumption is that the actual default rate is derived from the historical average default rate. These two assumptions will also apply to the other transition probabilities. Stated differently, changes of credit rating and credit quality are identical, and credit rating and default rates are taken as synonymous, meaning that the rating will change when the default rate is adjusted, and vice versa.

KMV has strongly challenged the above statement. Certainly one of the strongest arguments for overestimation of a given statement's accuracy lies in the fact that default rates have a continuous nature, whereas company's credit ratings are reconsidered and adjusted in discrete manner. Discrete nature of credit ratings adjustment is simply caused by the time lag required for rating agencies to estimate changes of company's financial health and to perform upgrade or downgrade of its rating. KMV has financed a simulation which has aimed to estimate how close are the historical transition probabilities and average default rates to the actual rates' values. As a result KMV discovered significant deviations. Moreover, KMV study has demonstrated that there are significant deviations of default rates among companies within the same credit rating boundary. And vice versa study found some overlapping of default probability ranges between, for example, BBB and AA rated borrowers − i.e. quite large number of companies with different ratings assigned by agencies used to have similar probabilities of default.

In the above mentioned serious and broad-based study KMV used Monte Carlo simulation to replicate for 50 thousand times Moody's default studies accumulated for 25-years period. The following are key assumptions that were applied by KMV during the study: (1) for each rating class KMV assumed fixed number of companies, which is approximately same distribution as was used in Moody's original studies. (2) For each rating class KMV assumed that probability of default was truly equal to the average default rate reported by Moody's over taken 25-years period. (3) KMV has run its simulation for four different levels of correlation with asset returns, chosen correlations levels were 15%, 25%, 35%, and 45%. Results of this study were described in details by M. Crouhy et al. in Journal of Banking & Finance 24th issue (2000), on pages 59-117. A typical result would show a range of 25-year average of historical default rates corresponding to a single actual default probability under 95% confidence level. The range of possible average historical rates turned to be quite broad and with skewed distribution, which suggests that mean default rate on average is usually higher than the median default rate within every credit class. Thus, study discovered that suggested by rating agencies average historical default probability rate is usually overstating the actual default rate of the typical borrower.

In practice this means that corporate customers might be adversely selected by lenders. Really, if loan pricing is based on the above studied average historical default rate, then for typical borrower this average credit rating would be overestimated (representing client's credibility worser that in reality), hence average client will be overcharged and may have an incentive to leave. Whereas, actual credit risk associated with the worst borrower in the class will be underestimates leading to advantageous pricing.

As oppose to CreditMetrics, KMV does not apply Moody's or Standard & Poor's historical data to determine the probability of default which will then depend only on a company's rating. Instead, KMV has managed to develop its own process of deriving the actual probability of default, so called − Expected Default Frequency, for each particular borrower. This process is based on a core model proposed by Merton (1974). Hence, in KMV the probability of default is a function of company's capital structure, the volatility of its asset returns, and the current value of its assets. The Expected Default Frequency is borrower-specific value, which can be applied into any rating system to obtain the equivalent obligor's rating. From different perspective, Expected Default Frequency can be viewed as a "quantitative ranking" of the borrower which is determined to its real credit/default risks, as oppose to more conventional "qualitative ranking" used by rating agencies which divide all borrowers into several risk categories, where borrowers with different risk levels associated fall into one category, denoted with letters like, for example, AAA, AA, and so forth.

Another crucial difference between models is that, contrary to CreditMetrics, KMV's model does not create specific relationships between credit ranking and transition probability. Accountability of the transition probability part in KMV's methodology is already embedded into Expected Default Frequency determination process. Indeed, each point on the Expected Default Frequency map corresponds to a specific point on a spread curve and an implied credit rating.

Similarities of the CreditMetrics and KMV's models are born from their common roots − both models are based on the option pricing approach to credit risk as originated by Merton [Vasicek (1997) and Kealhofer (1995, 1998]. Intimately, the changes in credit risk are essentially driven by the changes of the asset value of the borrower. Consequently, having (1) the current capital structure of the company − which is liabilities composition: equity, short-term and long-term debts, convertible bonds, etc., and (2) being able to specify the stochastic process for the asset value, it is then a straightforward process to determine the actual probability of default for any chosen time horizon (1 year, 2 years, etc.).

Both CreditMetrics and KMV approaches rely on the asset value model that was originally proposed by Merton (1974), however there is a significant difference between them which is caused by application of different simplifying assumptions aimed to make practical application of models for commercial purposes possible. The real extent to which these assumptions affect credibility of model's outcomes remains a topic for discussion. For sure it will attract new academic studies in the future.

KMV most easily can be applied to public companies whose debt is traded on the open market. For these companies the value of equity is determined by the market. The information available in the form of the company's stock price and its balance sheet can be transformed into an implied risk of default as shown in the Methodology section.

Advantages of KMV model:

a) Model uses a real value of equity in the market;
b) Main factors (expected default frequency) have minimum dependence on distributional assumptions.

Disadvantages of the KMV model:

a) Many of inputs necessary for KMV approach require complicated calculations;
b) Accuracy in predicting default probability of investments that have interest rate probability because interest rates used in the formula are deterministic.

1.1.3. Credit risk assessment model − Z-score

Another group of credit risk assessment models besides Structural credit models described in previous sub-sections is the Reduced-Form Models. This group of models contrasts sharply from considered earlier Structural models. In fact, all of the rest credit risk assessment models that do not fall into class of Structural models belong to the class of Reduced-Form Models.

Reduced-Form Model that is designed to assess credit risk of an individual company is usually called − credit scoring system or credit scoring model. The most well-known and widely used credit scoring model for predicting bankruptcy is Z-score model developed by Edward I. Altman in 1968. At that time Edward I. Altman was an assistant professor of Finance at New York University. Nowadays Dr. Altman is internationally recognized expert on corporate bankruptcy. He used to be a director of New York University's Stern Business School MBA program for 12 years and currently he is a vice-director of New York University's Salomon Centre.

Key points from E. Altman's biography:

1963	Received BA degree City College of New York
1965	He received MBA degree California University
1967	Received PhD degree California University
2001	Appointed as the President of Financial Management Association
2005	Was named one of the “100 most influential people in Finance”

However, Dr. Altman was not the first researcher exploring the field of Reduced-Form Models. Fitzpatrick in 1931 was, perhaps, the first one to presume the relationships between financial ratios and default probability. In his work we made a financial ratios analysis to compare two groups of companies − that went bankrupt and that did not. Fitzpatrick used a univariate analysis based on 13 ratios in attempt to find a relationship that would enable him to indicate upcoming failure of the company. However, the Fitzpatrick model failed to establish a statistically meaningful relationship between the values that ratios took and the upcoming failure.

The first successful and significant work in this area is generally considered to be the prediction model proposed by Beaver (1966 and 1968) in which he also used univariate analysis and did find a significant relationship between financial ratios and the default.

Basically, work of Altman (1968) is an expansion on the Beaver's work to the model with multiple discriminant analysis applied on various healthy and, on the contrary, non-performing groups of companies. Altman also proposed to use a variety of different groups of ratios to predict future financial difficulties of companies under study. Today, after more than forty years since model was first introduced, Altman's Z-Score model is still widely used by academic researchers and real practitioners to indicate company's financial health condition. In 1993, Altman has revised his model by incorporating new "four variable Z-Score" prediction model in it (Altman, 1993). Altman refers to his revised model as to "significantly improved", he claims that predictive ability of this revised model is enhanced, plus practical implementation has been made simpler.

The credit scoring model proposed by Dr. Altman in 1968 and revised by him in 1993 uses linear or Binomial models (like logit or probit) to run accounting/financial data through a regression and as a result obtain indication of defaulting companies. Dr. Altman's work concentrates on indentifying those variables (financial ratios) that have sufficient statistical explanatory power to differentiate defaulting companies from their well-performing peers. Basically, as a result of regressing statistically meaningful financial information model returns an estimate of specific coefficient assigned to a particular borrower, this coefficient if called a Z-score and it is used to classify credit worthiness of the company. Particular Z-score can be then jugged in comparison to critical values determined by Dr. Altman in his model – there is an area where company considered being financially healthy, another area indicates that company's probability of default is significant and the is also grey area where particular conclusion is hard to derive.

Below is the detailed description of stages and outcomes of initial Dr. Altman's work in which he developed his Z-score model. In 1968 Dr. Altman selected a list of twenty two possible financial ratios to examine which of them serve best in indicating financial problems of the firm. Dr. Altman compared twenty two ratios for their efficiency in predicting the bankruptcy by using a paired-sample approach. As a result of his studies

Dr. Altman managed to narrow down the list of ratios to five that were particularly precise in measuring the credit risk of the company when applied jointly. These five key ratios were selected during numerous tests for the discriminant function. They stood out as based on their combination it was possible to accurately predict the default for the time horizon up to one year, i.e. model proved that it could forecast the bankruptcy one year prior to the event. Resulting model predicted default correctly in 95 cases out of 100, and it predicted non-bankruptcy correctly in 80 percent of cases. The resulting function, famous under the name of Altman's Z-score, has been the benchmark for many later studies on Reduced-Form Models.

Initial Altman's sample was comprised of sixty-six companies, half of which were bankrupt firms and the rest are healthy corporations that still existed at 1966. The list of bankrupt companies was obtained from "bankruptcy petition under Chapter X of the National Bankruptcy Act" for the time period from 1946 to 1965. The mean asset size of defaulted group was $6.4 million, with minimal value at $0.7 million and maximum at $25.9 million. Author understood that defaulted group is not completely homogeneous due to variety of company's sizes and industry differences. That is why he attempted to compensate for this weakness by carefully selecting his non-defaulting group.

Non-defaulting group was compiled from a paired sample of manufacturing companies selected on a stratified random basis. Companies were stratified by industry and by total asset size. Total asset value was decided to be restricted to the range between 1 to 25 million US dollars. Financial statements collected for non-defaulting group corresponded by their issuance period to financial information available on defaulting group. To study bankrupt firms the data were derived from financial statements reported one year prior to firm's bankruptcy.

Next important issue was to determine the asset-size parameter of the sampled group. The decision was made to exclude from initial sample too small companies (under $1 million in total assets) and too large corporations. Too large corporations were excluded from well-performing group mainly due to the defaulting firms' asset range. In addition, bankruptcy cases among large-scaled companies were quite rare at the time of first Altman's study. Another frequent argument against incorporation of large companies in the sample is that the financial ratios in their inner nature have the effect of deflating statistics by the volumes and therefore a good deal of the size effect is eliminated with exclusion of outliers on size basis. Altman had to select his defaulting group with no regards to size range restrictions as it would have been impossible to do the reverse. It expected that both group have same selection criteria, however, choosing well-performing group of companies on a random basis would have been unwise. That is why non-defaulting group was chosen with regards to asset size scale of the companies. However, subsequent Altman's manipulations with the original sample did not use size as a parameter for stratification.

After both groups were defined and compiled from necessary number of companies, Altman started to collect their financial statements − balance sheets and income statements. Because previous studies have shown that large number of different variables might be important in determining corporate financial problems Altman had to compile his initial list of twenty-two potentially useful variables (or financial ratios). Dr. Altman has classified all ratios into five standard categories, which included: liquidity, profitability, leverage, solvency, and activity groups. Initial selection of variables has been done on the basis of their (1) number of appearance on the academic literature, (2) potential relevance to study objectives, plus some "new" ratios were proposed by Dr. Altman himself for this study.

From initial list of twenty-two variables Dr. Altman aimed to select key five ratios that will best predict the corporate bankruptcy when considered jointly. To narrow down the number of variables the following procedures were applied:

(1) The extent of statistical significance of each particular function, as well as the estimate of relative contribution from each single variable to forecasting the default;

(2) Observation of correlation between individual of variables that could benefit to default forecasting;

(3) Evaluation of the predictive accuracy of various variables combination; and

(4) Analyst judgments.

The final selection of five most powerful variables did not contain some of those ratios that seemed to be the most significant ones, among original group of the twenty-two variables, when their effects were measured independently. This is because Altman's objective was to create such a combination of variables that when taken jointly will comprise the optimal discriminant function. As a result, Altman got the function that does the best job in predicting the bankruptcy compared to alternative methods which include numerous software simulations analyzing different ratio-sets.

Original Z-score model combined selected five key financial ratios to determine the likelihood of company's default. The final discriminant function developed in 1968 was shaped as follows:

$$\mathbf{Z = X_1 + X_2 + X_3 + X_4 + X_5}$$

Where,

X_1 − Working Capital / Total Assets, (WC/TA);

X_2 − Retained Earnings / Total Assets, (RE/TA);

X_3 − Earnings Before Interest and Taxes / Total Assets, (EBIT/TA);

X_4 − Market Value Equity / Book Value of Total Liabilities, (MVE/TL); and

X_5 − Sales / Total Assets, (S/TA).

Altman suggest the following critical values based on which any result of the model can be mapped and assessed. Companies whose Z-score is less than 1.81 will most likely default. Although, the cut-off value was set at 2.675, Altman suggested and advocated that decreasing the lower bound of the zone-of-ignorance (to 1.81) is a more realistic cut-off Z-score. Hence, any result which is below cut-off value of 1.81 is considered to correspond to the company with a high probability of default. On the other hand, the company considered to be solvent, i.e. financial healthy, if its Z-score is above 1.81.

Altman has also defined the range between 1.81 and 2.99 as a "grey area". Companies that have Z-score in this range are considered to have uncertain credit risk, i.e.

represent the marginal cases and have to be watched with attention in subsequent accounting periods.

Some of the final users of the Model like, for example, private underwriting agents, credit and financial analysts, auditors and companies themselves indicated a concern that since Z-score model uses stock price as one of the input data (X_5), the model is only applicable to those companies which have floated their stocks on open public markets.

In 1993, when Dr. Altman revised his Z-score model with respect to latest financial innovations and developments, model has been modified by introducing factors that assigned different weights on the effects of each of five ratios brought to the final outcome. Modified Z-score model uses the same key variables but it multiplies them by different factors. It looks like:

$$\mathbf{Z = 1.2X_1 + 1.4X_2 + 3.3X_3 + 0.6X_4 + 1.0X_5}$$

Dr. Altman in 1993 named five most important valid reasons pro revising the successful and practically implicated model after twenty-five years since its first introduction. The reasons are:

1. With time, average size of the business failures, and perhaps their financial profiles, have changed dramatically. As a consequence of increased average size of bankrupt companies their public visibility also have changed − bringing more attention and concern from financial institutions, regulatory agencies and the general public. Most of the studies in the past comprised their samples of companies with relatively small asset size. The only exception is Altman's (1973) studies of railroad and commercial bank sectors. In his Z-score revising work Altman suggests that: "Any modern model should as relevant as possible to the population to which it will eventually be applied". His revising study was based on a bankrupt companies' sample with the average asset size for two subsequent annual reporting periods prior to the bankruptcy of approximately 100 million US dollars. Minimum asset size in the sample was 20 million US dollars.

2. In addition to the above reason, Altman aims to make his revised model "as current as possible with respect to the temporal nature of the data".

3. Previous Z-score model and other alike studies were concentrated on a sample that contained broad variety of manufacturers or some specific industries. In his modern work (1993) Dr. Altman shares his believes that: "with the appropriate analytical adjustments, retailing companies, a particularly vulnerable group, could be analyzed on an equal basis with manufacturers".

4. One important development of recent Altman's study is that underlying financial data including all relevant information from footnotes to financial statements have been scrupulously analyzed which has made it possible to incorporate the most recent developments in financial reporting standards and adopted accounting practices. In real study, in at least one case, a change in reporting standard which was planned to be implemented in the nearest future was included during data analysis. These modification allows to upgrade the Model from being relevant not only to past failures, but to also to the data that is planned to be disclosed in the future periods. Dr. Altman describes his new ZETA model's predictive ability as well as its classification accuracy as "implicit" due to the efforts that brought these changes.

5. The last of five reasons to revise Z-score model is to test and assess several of recent developments which were added to the knowledge on discriminant analysis, but remain controversial at the moment.

Advantages of Altman's z-score model in comparison with other modern credit risk assessment models available include:

1) Uses easily available data as an input;
2) Easy to understand and implement;
3) High level of popularity because of proven accuracy in predicting bankruptcy;
4) Modified, more recent and updated, structure gives better results as it incorporated major recent financial and economic developments.

The Z-score is a set of financial ratios in a multivariate context, based on a multiple discriminated model. It best serves in cases where a single measure is unlikely to predict the complexity of decision making or the scope of companies' entire activities.

Disadvantages of Altman's z-score model:

1) It is based on book values of assets and debt;
2) Does not include qualitative factors.

Professor Altman continues to take part in many researches on bankruptcy and default prediction analysis, which may bring more modifications to the current Z-score model in the future, enhancing its ability to determine the default probability.

1.1.4. Credit risk assessment model − Binomial option pricing model

Binomial option pricing model is a relatively simple but powerful technique which can be very handy in solving complex option pricing problems in absence of access to complex financial information about companies that are being studied. Binomial model uses small scope of financial data which is easily available from company's publicly shared financial statements. This fact makes the binomial model popular and widely applied in practice for purposes of credit risk evaluation.

As opposed to the Black-Scholes and alternative complex option-pricing models which need solutions to stochastic differential equations, the Binomial model (also known as two-state option-pricing model) is mathematically simple. It is easily understandable, numerical, and is built on the assumption of no arbitrage. No arbitrage assumption is defined as the state where: (1) every risk-free investment bears the risk-free rate of return, and (2) there is no investment opportunity which requires zero investment, but yields a positive return.

Binomial model was developed to price the American stock options by group of three researchers, Cox, Ross, and Rubinstein, in 1979. Model is classified as a "Discrete-Time" (lattice based) or Tree model. This is due to the graphical representation that gets a tree shape while depicting the stock and option prices over the large number of steps, for the time period between valuation points to the expiration of underlying financial instrument. There steps are used to estimate the option price. At each step, the stock price will either move up or down, probability of each direction depends on the stock volatility.

Brief biographies of the authors:

John C. Cox

1975	From 1975 John C. Cox is a PhD of the Wharton School (University of Pennsylvania)
1976	Cox-Rubinstein-Ross model was developed
1985	Term-structure model was published
1998	Received Financial Engineer of the Year Award

Steven A. Ross

1970	Received Doctorate degree in Economics from Harvard University
1976	Cox-Rubinstein-Ross model was developed
1988	Corporate Finance book was published
1988	Became the President of American Finance Association

1996	Won Financial Engineer Award of the Year
2005	Neoclassical Corporate Finance book was published

Ross nowadays is a well-known author of large number of financial publications, articles, books (especially he is famous among readers of the books of the following subjects: Corporate Finance, Financial Management, Mergers and Acquisitions, etc).

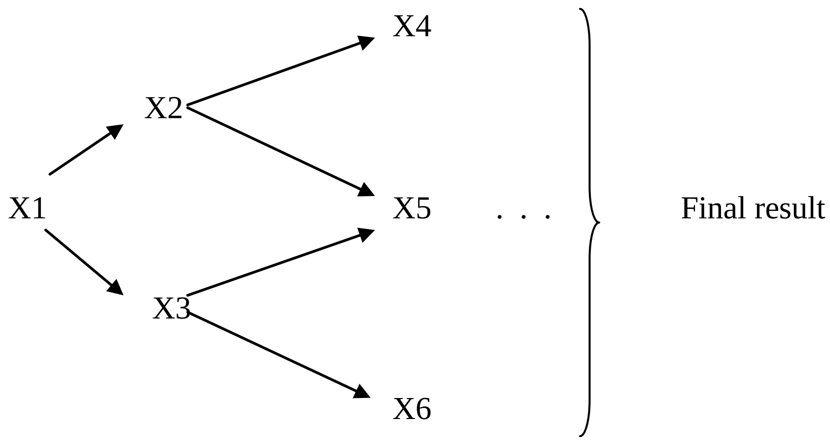
Mark Rubinstein

Before 1976	He received Bachelor degree in Economics (Harvard) and MBA in Finance (Stanford)
1976	Cox-Rubinstein-Ross model was developed
1993	Became the President of American Finance Association
1995	He received Financial Engineer of the Year Award
2003	He won Best Teacher of the Year Award
Currently	M. Rubinstein is a professor of Finance at California University

The binomial model is a good alternative to other methods because it demonstrates the analyzed instrument over a period of time (with descriptions of its behaviour during that period) as opposed to a single point in time. Also because of such quality it can be used for assessing both the American options which are exercisable at any time during between inception date and the maturity as well as the Bermudan options that are exercisable at specific predetermined moments in during their lives. There are some software solutions for this model available on the internet, which can facilitate practical application of the model.

Even though Binomial model is considered to be simpler than Black and Scholes option pricing model, it may take more time to compute the results if lots of steps or time intervals is used. For some types of derivatives this method is said to be less practical and it is recommended to use Monte Carlo simulations instead, but it is exclusion and the Binomial approach is widely used by lots of practitioners in the world.

Binomial method is based on constructing a tree which shows the evolution of certain instrument during time period with chosen time intervals. Binomial tree demonstrates the sequence of values which are changed upwards or downwards due to different choices (as a result of circumstances).



Upward ratio = $e^\wedge \sigma \sqrt{t}$, this coefficient shows the number to multiply to the previous value to get an increase of the value. Downward ratio = 1/Upward ratio, this number shows the movement of value down or decrease of the value. To get the final result the analyst will have to calculate weighed average of all the final values.

Advantages of Binomial model:

a) Easy to understand the approach and its components;

b) Easy to apply for time periods with up to 4 intervals;

c) All the data used in calculations are easy to get;

d) There are lots of freely available software based on this model;

e) It is said to be more simplified than Black and Scholes method, however, it has same precision if all the inputs are used properly.

Disadvantages of Binomial model:

a) Usually there are lots of steps/intervals and it takes quite a long time to get the final result for, say, case with 120 steps incorporated (that is why Monte Carlo simulations in some cases is a good alternative);

b) This model can be used only if there are no expectations of rapid growth or drastic downturn in the market, which model is not able to account for;

c) It considers only two possibilities that an instrument increases or decreases for certain amount after certain time lag.

1.2. Literature review

As mentioned previously, financial failure forecasting tools are very important for all parties involved: from stakeholders of the companies (i.e. owners, shareholders, creditors, suppliers) to government and society in general. Thus academics have been focusing on identifying the most reliable model for bankruptcy prediction.

Morris (1998) demonstrates different types of financial distress prediction models which were generated during time period of 30 years prior to his study. Author divides the previous theoretical framework into two broad classes of univeriate and multivariate analysis. The results are presented in the table below:

Table 1. Types of bankruptcy forecasting models by Morris (1998)

Derivation	Univariate	Multivariate
Iterative (simulation)		a) Experimental (i.e. credit scoring) b) Recursive partitioning c) Artificial intelligence d) Neural networking
Statistical	a) Conventional ratio analysis b) Systematic ratio analysis c) Balance sheet decomposition d) Gambler's ruin	a) Discriminant analysis b) Regression analysis c) Logit/Probit analysis d) Expanded logit e) Survival analysis
Behavioural reaction		a) Share prices b) Laboratory experiments
Case studies		a) Purely descriptive b) Analysis of common factors

McKee (2000) provides his classification of the available failure prediction methodologies and techniques conducted by previous studies. Author identifies the following ten main methods used to forecast bankruptcy:

- Linear probability models;
- Univariate ratios;

- Multiple discriminant analysis;
- Multivariate conditional probability models (i.e. logit and probit models);
- Recursive partitioning models;
- Proportional hazard model (i.e. survival analysis);
- Models based on expert systems;
- Mathematical programming;
- Neural network models;
- Rough sets approach;

As the main focus of this work has been on analyzing the predictability powers of four bankruptcy forecasting methodologies, the below section provides literature review of the past academic research and theoretical framework on:

- Merton's structural model;
- KMV;
- Z-score;
- Binominal approach.

1.2.1. Extensions of Merton's model and First passage time models

Empirical usage of the Merton's model is restricted by the number of simplified assumption used in it. Thus number of authors has extended the Merton's model over the years focusing of changing the basic shortcomings and the most notable papers in this area are: Black and Cox (1976), Geske (1977), Mason and Bhattacharya (1981), Kim et al. (1993), Leland (1994), Longstaff and Schwartz (1995), Leland and Toft (1996), Briys and de Varenne (1997) and Collin-Dufresne and Goldstein (2001).

Black and Cox (1976), often referred to as to first-passage model, unlike Merton (1974) assumes that default can occur not only on maturity of the debt, but anytime prior to this date. In their model default event can happen in case entity's asset value falls below some pre-defined threshold level. In other words default occurs by the first passage of the company's value to some critical barrier below which no additional equity can be sold. This critical level might be defined by shareholders during the determination of the company's optimal capital structure. Authors explored the situation in when company can sell its assets in order to meet the obligations of making interest payments. In addition authors consider specific attributes of the debt instruments which exist in practice, such as subordination arrangements, safety covenants and limits on financing of interest and dividend payments.

In the study authors present the model in which safety covenant provides bondholders with opportunity to force bankruptcy whenever predefined condition is met. The safety covenant is presented as barrier which is dependent on time and this barrier is expressed as exponential function in the following form:

$$V^d\,(t) = ke^{-r(T-t)}$$

with k and r are constants. The company is forced into default whenever the company value equals $V^d(t)$, and the debt owners take company's assets $V = V^d$.

Geske (1977) expands the Merton's model by assuming that company can have a range of debt maturities thus effectively equity holders can be seen as holders of compound option. In this regard author assumes that company has the option to issue new equity for servicing the debt. The value of this compound option can be presented by multivariate normal distributions.

Mason and Bhattacharya (1981) extend Black and Cox's first passage model by introducing jump process, which helps to model value of the company by randomizing of default time. Authors find out that this process can have large impact on the value of risky bonds.

In the model by Kim et al. (1993) the interest rates are assumed to follow stochastic process proposed by Cox et al. (1985). Contrary to Black and Cox (1976) the default barrier is assumed to be constant. The default boundary is equal to coupon payments during lifetime of the bond and in case it is not reached until maturity this barrier becomes the actual value of firm. Thus according to the model default event can happen prior to maturity if company misses the payment of coupon or at the date of maturity in case the value of the company is below the bond value. Another interesting finding proposed by authors is that stochastic interest rate lead to generation of higher credit spreads comparing to other models.

Model proposed by Leland (1994) follows the original framework proposed by Black and Scholes and accounts for taxation, cost of bankruptcy and protection covenants which make the model more realistic. Author explains that bankruptcy itself is not costless due to legal and reorganization costs. Interestingly, paper mentions that default costs are fully covered by existing bondholders and thus does not have affect on company's equity. The tax advantages which are associated with financing of debt are presented as a separate security with constant coupon payments. However this is applicable only until company is not defaulted, as tax benefits can not be claimed after the default.

Similarly to Black and Cox's model Longstaff and Schwartz (1995) assume that default event may occur prior to debt maturity and default occurs when company's value reaches some specified level.

Authors extend previous studies by assuming that interest rate is stochastic, follow the Vasicek (1977) process and correlate with entity's value.

The proposed model is based on the following formula:

$$dV/V = rdt + \sigma_V dZ$$

in which r is the rate of return, V is a value of assets, σ is constant and dZ is standard Wiener process.

The short-term rates follow the following stochastic process:

$$dr = (\varsigma - \beta r)dt + \eta dZ_r$$

in which ς, β, and η are the constants and dZ_r is one more standard Wiener process.

Author find out that credit spreads for corporate debt instruments are driven by two main factors – asset value factor and interest rate factor. In addition authors claim that recovery rate is not necessarily equal to threshold value after the first passage and introduce seniority classes of debt where senior debt has higher recovery rate on a coupon and assume static structure of debt. In addition they investigate firm's optimal capital structure and value of the debt taking into assumption similar to Leland (1994) the cost of bankruptcy and taxes. By doing so authors examined the driving factors for firm's choice of certain capital structure.

Briys and de Varenne (1997) indicate that the purpose for development of new model was to address the main shortcomings of some previous models (e.g. Longstaff - Schwartz model), i.e. the fact that payments to bondholders can be higher than the actual asset value of the company upon default event. Thus authors suggest default barrier and recovery level which guarantee that payoff to bondholders can not exceed the value of the firm. In contrast to static default trigger described in Longstaff-Schwartz model, bankruptcy-triggering barrier introduced by Briys and de Varenne stochastically follows the interest rates. In addition authors distinguished and reviewed two concepts of default: default event prior to maturity of the bond and default event at the maturity of the bond.

Model suggested by Collin-Dufresne and Goldstein (2001) captures the fact that entities tend to raise additional debt whenever company's assets are growing, in other words each firm has desirable leverage ratio which they tend to constantly maintain. Thus in the long term the default threshold is remaining relatively flat, whereas in the short term default barrier constantly changes, i.e. whenever firm increases the debt book default threshold raises as well and visa versa. Furthermore authors point on correlation between risk-free interest rates and default threshold of the company – the higher the rates the higher the default threshold.

1.2.1.1. Empirical analysis of Merton's Model

Empirical test on Merton’s model and other structural models which followed it is limited. One of the most notable empirical study is paper by Eom et al. (2004) which compares five models of corporate bond pricing using the sample of 182 non-callable bonds from entities with simple structures of their capital for the period form 1986 to 1997. Analysis includes the following models: Merton (1974), Geske (1977), Longstaff and Schwartz (1995), Leland and Toft (1996) and Collin-Dufresnc and Goldstein (2001). Eom et al. (2004) find that Merton and Geske models tend to underestimate bond spreads, whereas rests of the models tend to overvalue them. Interestingly authors find that newer and more sophisticated models tend to significantly overestimate the problem for bonds with high volatility or high leverage and understate the spread for low risk bonds.

1.2.2. KMV model

The academic literature on KMV model is very limited which can be partly explained by commercialization of the model, i.e. restrictions of the information on improvements or developments of the model due to confidentiality.

Crouhy et al. (2000) provide comparative analysis of four commercial models in their paper: CreditRisk+, Credit Metrics, CreditPortfolioView and KMV. Authors observed that KMV model is able to produce very accurate analysis of the firm's credit quality. The model shows the best results when applied to public firms, i.e. which shares are openly traded on the stock market. The information which is included in the company's balance sheet and the share value is reflected and computed in the firm's probability of default calculation.

Results obtained by Kealhofer and Kurbat (2002) show that KMV outperform Moody's ratings and number of different accounting ratios in predicting the company's default. In addition authors find that KMV model captures all data used by rating agencies or accounting ratios.

Oderda et al. (2003) observe that credit risk models (KMV-based Credit Monitor and RiskCalc by Moody's) outperform rating agencies by predicting default events around ten months earlier. In addition these models add appropriate data which is not taken into account by credit ratings.

One of the most notable papers on this subject published in recent time is the work by Hillgeist et al. (2004) which is comparing model based on Merton-KMV (distance to default model) with classical Altman's discriminant analysis model and Ohlson's logit model. Empirical results obtained by authors have proven that values of probability of bankruptcy based on Merton's model carry more data than Z-score or O-score models. Authors identified that the main "weakness" of traditional models was dependence on single observation per each entity, i.e. the company's probability to default depended on last available set of observations irrespective of it's current status.

Duan et al. (2004) argue that although Moody's KMV is a very popular tool among both practitioners and academics little is known about statistical properties used in the methodology. Moreover author claim that it is not completely apparent if the model is statistically reliable or not. Among the main findings authors report that KMV fails to generate appropriate estimates for structural models with unknown parameters of capital structure. The estimates generated by KMV method are similar to the maximum likelihood estimations when computed in relation to Merton's (1974) model.

Agarwal and Taffler (2008) provide comparison for the bankruptcy prediction performance between accounting ratios based models and market based models. Using UK data they demonstrate that there are little differences in accuracy of prediction between the two mentioned models.

1.2.3. Z-score model

Altman et al. (1977) have developed second generation of the original Z-score model called ZETA model. ZETA model was build upon sample of 58 non-bankrupt and 53 defaulted entities both from manufacturing and retailing sectors. It should be noted that inclusion of retailing companies did not have negative impact on the results of the model. Authors studied larger companies comparing to original study - average asset value of entities was around $ 100 million versus around $ 25 million previously. New approach used seven variables (Profitability, Stability of earnings, Debt service capability, cumulative profitability, Liquidity, Capitalization and Size) and included recent developments in accounting and financial reporting of that time (e.g. capitalization lease change in reporting). Model proved to be very effective in predicting which companies would go bankrupt: with around 70 percent accuracy up to five years prior to default and 90 percent accuracy one year before default. According to Altman (1993) the core objective for modifying Z-score model was the desire to reflect recent changes in the profile and sizes corporate defaults, requirement to have model applicable to different industries including retailing and test recent developments in discriminate analysis.

Springate (1978) provides modification of the original Altman's Z-score model so it can be tested to identify the default companies on the Canadian market. Author used multiple discriminate analysis to select 4 financial ratios out of 19 variables widely used in practice at that time. The four financial variables which author has used are as following: Working Capital/Total Assets; Net PBT/Total Assets; Net PBT/Current Liabilities and Sales/Total Assets. The testing of new model showed that formula achieved 92.5% accuracy rate on a sample of 40 companies from Canadian market. Subsequently, Botheras (1979) and later Sands (1980) tested Springate's model on a samples of 50 firms and 24 respectively. Botheras concluded for the set of observed firms that the accuracy rate for prediction of defaults was 88%, while Sands reported this rate to be around 83.3%. It should be noted that average asset size of companies in the Botheras model was around $2.5 million, whereas this value for Sands sample was in the region of $63.4 million.

Altman et al. (1995) introduced so called emerging markets scoring system (EMS model) which is enhancement of the classical Z-score model, so it can be applied to non-US corporate bonds. Unlike the Z-score, EMS model is relevant for both manufacturing and non-manufacturing companies as well for as public and private entities. In the original paper EMS model has been applied to Mexican companies with quite accurate results. As mentioned by Altman (2005) EMS model later has been applied to analyze entities from numerous countries such as Argentina, Brazil and Southern Asian countries and the results were impressive. However author recommends building models which could take into account peculiarities of individual countries. Especially taking into account that EMS model is quite flexible and allows future adjustments to be made depending on country's sovereign risk and financial environment.

Altman and Sabato (2005) based on adaptation of Z-score build model for Small and medium size enterprises (SME). Model uses five financial ratios (Liquidity, Leverage, Profitability, Account and Coverage) and focuses on impacts of introduction of Basel II on Capital requirements for banks based on SMEs data from USA, Italy and Australia.

In their paper Brigham and Gapenski (1996) discuss practical usage of the Altman's Z-score model. Authors indicate that several investment banks (e.g. Morgan Stanley, Salmon brothers and other) used Z-score model to assess the quality of junk debt instruments.

Among critics of Altman's traditional model couple of papers can be noted down. Scott (1981) provide comparison of several the leading models, including Beaver (1966), Altman (1968), Deakin (1977), Wilcox (1971) and Altman et al. (1977) and challenges Altman and other authors in terms of their selection of financial variables for developing models. Author indicates that there is quite large selection of possible financial ratios available for building the models, however researchers tend to apply those which support the developing model. If the same set of financial variables to be applied to different sample of companies the result often proves to be quite different from the one obtained to support the model.

Grice & Ingram (2001) state possible bias in the accuracy rates projected by Altman and other researchers because of the following reasons: time period in estimation and hold-out samples do not differ substantially; hold-out sample is too small to be compared to real bankruptcy rates on the market; estimation sample and hold-out sample are restricted to companies from the similar industries.

1.2.4. Binomial approach

There are several methods on how binomial model is used to evaluate the default risk on practice. The most notable approach is so called Binomial expansion technique (BET) which was introduced by Moody's in 1996 mainly to evaluate default rates and analyze cash inflows for Collateralized debt obligations (CDOs). Tool became the benchmark on the market in that time and one of the reasons behind it's popularity was simplicity of the approach and transparency of principles used in it. Although it should be noted that according to some authors BET model is not that widely used in recent time, for example Xie and Witt (2005) report that Moody's moved away from binomial approach and have adopted Monte Carlo model instead, a number of papers has studied this technique and brief review of those researches is provided below.

Cifuentes and O'Connor (1996) provide the overview of the BET methodology. The main principal behind BET model is to utilize the diversity score to build a synthetic pool of homogeneous and uncorrelated assets with similar probabilities of default and face values to estimate the behaviour of original portfolio and compute the "expected loss" parameter. This approach would help to forecast expected losses in the hypothetical pool which would follow binomial distribution. Computation of the following variables is prerequisite for BET estimation: weighted average rating factor, the probability to default, recovery rate and diversity score.

Cifuentes and Wilcox (1998) offer adaptation of BET model to special cases of CDOs. The modified method, called double binomial approach, proposes to divide the pool is divided into dissimilar portfolios with different probabilities to default and which do not have correlations between them. The methodology is best applied for samples of distinct group of assets with low diversity.

Yoshizawa (2003) provide review of further extension of the BET methodology, which is called multi binominal method and mainly applicable for synthetic CDOs. Author suggests using this methodology to perform estimation for portfolios with dissimilar probabilities of default.

2. Academic study

2.1. Hypothesis

The main objective of the bankruptcy prediction models is to forecast the default occurrence as early as possible. Thus it is logical to assume that each model in this study would be bale to identify and forecast the failure of the firms which eventually went bankrupt. From this perspective the first hypothesis is formulated in the following form:

H1: Back-testing results for all four studied models should show high probability of default for companies that actually experienced financial difficulties.

As has been shown by previous studies the accuracy rate of bankruptcy predictability models has been typically in the range of 60% and 90%. Thus it is fair to assume that not all methods would show similar results. Based on this the second hypothesis is the following:

H2: Back-testing results will show that one/several risk assessment methods failed to indicate upcoming financial problems or indicated high probabilities of default for those companies which historically proved well financial health in future periods.

We utilize hypothesis testing to evaluate the significance of difference in the performance of studied models in the prediction of bankruptcies.

2.2. Definition of terms

Lambda (λ), the market price of risk;

SP, the size premium for a given borrower;

RSQ, a measure of the amount of undiversifiable credit risk in a given borrower;

LGD, the expected loss in the event of a borrower's default;

EDF and CEDF refer to the probability of default over a time period, expressed in annual and cumulative terms. For example, looking at a time frame of 3 years, the overall probability of a default sometime during that period (CEDF) might be 1.5%, which would correspond roughly to an annualized default probability (EDF value) of 0.5%;

CQDF refers to a *quasi* cumulative default probability. This probability is generally higher than the *actual* probability (CEDF) and is the input into CreditMark valuation. CQDFs are risk-neutral default probability measures.

2.3. Methodology

Each of the sampled companies was analyzed by applying 4 credit risk methods KMV, Merton, Z-score and binomial approach.

2.3.1. The methodology employed to measure Merton model estimates

Merton formula used in calculations of probability of default (PD):

PD = N1*V+N2*V

Where,

PD = probability of default

N1= (Ln(TA-TL)/TA+R+ σ ^2)/ σ

N2 = (Ln(TA-TL)/TA-(R- σ ^2))/ σ

V = Value of a company (or total assets)

TA = Total assests

TL = Total liabilities

R = risk-free rate (or 1 year T-bill rate)

σ(sigma) = volatility of stocks

The lower value resulting from Merton's model calculations shows higher probability of default whereas higher value means reduction of frequency of default event.

2.3.2. The methodology employed to measure KMV estimates

For purposes of the current study we cannot perfectly replicate the method of Moody's KMV because some part of this model consists of proprietary information or programs, and subscribing to these databases is prohibitively expensive for us. Thus, an adopted KMV approach is used in this paper. We apply several assumptions necessary to cope with data limitations. Then it becomes possible to obtain approximations to KMV estimates that will be used for Hypothesis testing in later Sections.

One of the main components of KMV model is the Expected Default Frequency™, EDF™. This term refers to the probability of default over a time period, expressed in

annual (EDF) and cumulative terms (Cumulative Expected Default Frequency, CEDF). For example, looking at a time frame of 3 years, the overall cumulative probability of a default sometime during that period (CEDF) might be 1.5%, which would correspond roughly to an annualized default probability (EDF value) of 0.5%. EDF is a registered trademark. That is why outputs found in this paper to substitute for corresponding EDF values should only be treated as approximations to EDF that were obtained through calculation techniques intended to replicate the original model owned by Moody's.

First step in implementing KMV method would be to determine Expected Default Frequency. To do so analyst will need to follow the following three steps:

A. Assess market value of underlying assets and its volatility;
B. Assess distance-to-default − debt to asset ratio adjusted by asset volatility;
C. Substitute found values into EDF table to find out annual EDF figure for corresponding Distance to Default.

Let us closely examine each of these three steps.

A. Estimating asset value and volatility

To understand how asset value and volatility can be measured let us consider a simplified case where there is only one class of debt and equity. The equity holders have the right, but not the obligation, to pay off the debt holders and take over the remaining assets of the firm. Thus, equity is the same as a call option on the firm's assets with a strike price equal to the book value of the firm's liabilities. For public firms, we know the equity prices, and can infer the asset values. For practical application of KMV model, two following relationships needs to be solved simultaneously:

[Equity Value] = *OptionFunction* ([Asset Value],[Asset Volatility], [Capital Structure],[Interest Rate])

[Equity Volatility] = *OptionFunction* ([Asset Value],[Asset Volatility], [Capital Structure],[Interest Rate])

Market Value of asset if influenced by the following factors:

- Market value of equity, measured as total market value of outstanding shares = share price * number of shares outstanding;
- σ − Volatility of share price;
- K − Leverage coefficient (Debt/Equity ratio);
- C − Average annual repayment amount to service the debt;
- R − risk-free rate (1 year T-bills rate).

B. Estimating Distance-to-Default

The following parameters influence Distance-to-Default value:

- Current asset value
- Distribution of asset value at time T
- Volatility of future asset value at time T
- Default point
- Expected rate of growth in asset value over the horizon
- Length of horizon, T

Distance-to-Default is measured as the number of SDs the asset value is away from default, i.e. the time left to theoretically estimated default point, for cases with very high expected default frequency approximation.

To calculate distance-to-default the following equation can be applied:

$$\textbf{[Distance to Default]} = \frac{\textbf{[Market Value of Assets] - [Default Point]}}{\textbf{[Market Value of Assets] * [Asset Volatility]}}$$

The Distance to Default measure combines three key credit issues:

- The value of the firm's assets;
- Business and industry risks associated with the company and reflected in the asset volatility. Volatility is expressed as percentage; and
- Company's leverage which is reflected in default point value.

Moreover, via the asset value and volatility Distance to Default also incorporates the effects of industry, geography and a firm's size.

Default point is calculated as:

$$DP = STD + 1/2\ LTD$$

Where,

STD = Short Term Debt

LTD = Long Term Debt

C. Determining Expected Default Frequency

After Distance to Default has been estimated, finding Expected Default Frequency is an easy task for which a researcher would have to use proprietary information of Moody's - KMV's database that includes over 400,000 company-years of data and over 4,900 incidents of default or bankruptcy. Researcher will need to lookup in the frequency table, that relates various levels of Distance to Default to the likelihood of default, the level of Distance to Default corresponding to his/her case. It needs to be pointed out here that to determine the default probability model uses the empirical data, not the assumptions.

A study run by KMV has discovered that: (1) the relationship between Distance to Default and EDF is constant across industry, assets size, time horizons; (2) the relationship is invariant across countries and regions.

2.3.3. The methodology employed to measure Altman's Z-score model estimates

Less than 1.23 high probability of default to take place. Closer to 0 and lower than 1.23 – very high credit risk. 1.23<X<2.9 - grey zone, more than 2.9 - safe zone

Z-score formula :

Z=1.2*WC/TA+1.4*RE/TA +3.3*EBIT/TA +0.6*Equity/TA +0.999*Sales/TA

Where :

WC - working capital;

TA - Total assets;

EBIT - Earnings before interest and taxes.

2.3.4. The methodology employed to measure Binomial approach estimates

The binomial option pricing model creates simulation of possible ways in which the key option's underlying variables may evolve in a discrete-time. During this process a model draws a binomial lattice (or tree), which describes different "ways" in which option price may change during discrete time periods, each period starting at valuation point and ending at chosen expiration date. Each node of the binomial tree depicts possible movement of underlying asset's price at a given period of time. Then, option price valuation is done iteratively. As the starting points of the valuation computations model takes all of the final nodes (usually nodes that represent time piece corresponding to expiration date). Valuation model then moves from these starting points back towards the first node (which represents time piece at valuation date) following all nodes of the tree. The resulting value at each node is the price of option at corresponding point in time.

Option valuation approach applied by this method can be broken down to three-step process. The following are the steps:

A. Generation of the price lattice;
B. Calculation of price that option takes at each of the final nodes;
C. Cyclical calculation of the option's value at each node moving from final to starting node.

A. Build the binomial price tree

Tree of all possible price movements is generalized by starting from valuations dates value of the option and calculating possible values option price may take further while moving in time towards the expiration date.

At each next step (or node) the assumption is that option's price may either go up or down. The extent of price movement is determined by a specific factors u and d. By definition $u \geq 1$, and $0 < d \leq 1$. So, if current price at valuation date is equal to S, then next period's price can possibly take two different values: one grater that a current price, i.e. $S_{up} = S * u$, another one smaller that a current price level, i.e. $S_{down} = S * d$.

Factors u and d are determined using the underlying volatility, σ, and the time length of particular node taken is number of years, t. Here, to determine the corresponding time length of a node model suggests using the day-count convention of the underlying instrument. From the condition that the variance of the Lg of the price is $\sigma^2 t$, we have:

$$\boldsymbol{u = e^{\sigma \sqrt{t}}}$$

$$\boldsymbol{d = e^{-\sigma \sqrt{t}} = 1/u.}$$

The process described and formulas derived above represent the original Cox, Ross, & Rubinstein method. However, other techniques for creating the lattice tree exist, such as, for example "the equal probabilities" tree.

The very nature of the The Cox, Ross, & Rubinstein method presumes that the resulting tree will be recombinant, meaning that when the underlying asset's price increases and then decreases the final price is the same if at the first stage price had first moved down and then up, i.e. the sequence was reverse. Being recombinant binomial tree's paths merge over time or in other words they - recombine. This condition of the model reduces the number of nodes in the tree, and hence accelerates the computation process.

This property also facilitates quick calculation of the value of the underlying asset by deriving and using formula, instead of wasting lot of time to construct complicated and extended trees. Formula that enables to calculate option price at each node is describe below:

$$\boldsymbol{S_n = S_0 * u^{Nu - Nd}}$$

Where:

N_u: Number of up moves
N_d: Number of down moves

B. Finding the price that option takes at each final node

At each final node of binomial tree, in other words at options expiration date, the option value is equal to its intrinsic value, which is the same as its exercise value.

Max [($S_n - Y$), 0], for a call option

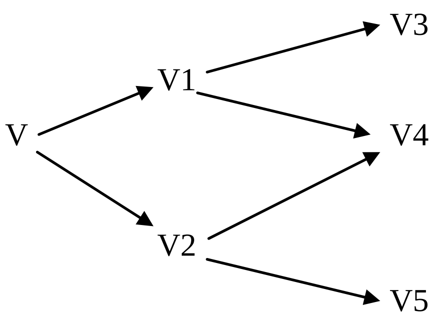
Max [($Y - S_n$), 0], for a put option:

Where: Y is the Strike price and S_n is the spot price of the underlying asset at the n^{th} period.

C. Find Option value at all previous ticks

Once the second step is complete, next thing is to calculate the option value for each of the previous node ends in sequence. When calculations are worked back to the very first node (when time period is equal to the valuation date) the resulting figure will be the current value of the assessed option.

For purposes of the current study the following scheme of calculations was used. Calculations of the binomial method were based on the formula extracted from paper of Robert Conroy who is a professor at University of Virginia and who wrote an article called "Binomial option Pricing model" in 2003. In the calculations was implemented only two-step analysis due to research purposes of this paper, while in practice analysts usually use more than 120 steps to get practically applicable results.



Where V's are the market values of the stock within one step

V - beginning value of total stock

Between V-V1 and V1-V3, V2-V4 was used upward ratio = Vx*e^-sigma/2

Where Vx - Previous steps' value of stock.

Between V-V2 and V2-V5, V1-V4 was used downward ratio= 1/upward ratio

Ending value of total stock = ¼*V3+1/2*V4+1/4*V5

Ending value of total stock is weighed average of all results received during calculations.

2.4. Data sampling

The studied sample includes the accounting statements (balance sheet and profit and loss accounts) of 2 bankrupt companies, for up to 3 years prior to bankruptcy. It also includes the corresponding data for 12 healthy companies, thus overall data pool for 14 companies was analyzed.

2.4.1. Definition of defaulted company

It is not easy to define term bankruptcy or "failure" as it has different meanings according to the context it is being used. For example, from legal point of view the company is classified as bankrupt whenever all juridical formalities are passed, like submission of required forms, passage of bankruptcy case in court etc. Typically by the time the formal bankruptcy is being announced all stakeholders are already aware about this. In contrast, from economical point of view firm is described as bankrupt whenever it fails to meet it payment obligations (e.g. bond coupon payment). Thus previous researches on prediction of financial distress use the economic definition of the financial failure term. This study will follow this trend and the following definitions of "failure" and "success" are used to complete this work:

i. A company considered failed in the case if it had default event (it couldn't meet its' liabilities at least one time during 2007-2009) and it had to receive bailout package or to be nationalized .

ii. A company considered successful if it did not experienced default event during time period of 2007-2009.

2.4.2. Selection of financial variables and market data

As for the selection of the appropriate financial ratios the following has been decided:

a) Set of ratios which are commonly used in similar studies were taken into consideration;

b) In addition set of ratios were selected, which focus on the liquidity, profitability and firm's structure of capital in the studied pool and which have proved to be effective variables in other researches.

List of components and financial parameters considered in the paper:

#	**Name of a parameter**
1	Current Assets
2	Current Liabilities
3	Long Term Debt
4	Working Capital
5	Total Assets
6	Total Debt
7	Total Equity
8	Market value of Equity
9	Retained Earnings
10	Revenues
11	Earnings Before Income Taxes
12	Expected Return
13	Volatility/ Standard Deviation
14	Historical returns on stocks

The accounting ratios used in the thesis were taken from the financial statements of sampled companies (balance sheets and income statements), which were taken from the annual reports. Stock prices were used to calculate were calculated volatilities and expected returns. Historical stock prices of the sample companies were collected from Reuters and Bloomberg databases.

2.4.3. Selection of companies

Companies from two main sectors of economy were analyzed (financial sector companies and real sector companies) and each of these sets has a company that experienced default event during the period of 2007-2009. Both of defaulted companies (Northern Rock Plc and Ford Motor Company) were studied in time range of 2006-2008. This is due to the fact that to both these companies were defaulted in 2008, thus analysis for these two companies is based on financial variables from 2006-2008 time period.

All the companies were segmented by:

a) Industries where they operate their businesses (there are 4 different industries with 3-4 companies in each industry)
b) Sizes of the companies (all the sampled companies are among the largest in their sectors)

This diversification was done to compare companies from different industries, sizes and the level of success which those firms went through the crisis period.

As mentioned above there are two main sectors of analysis of the companies:

a) Financial sectors industry (large international public financial institutions were compared with 4-th largest bank in UK as of 2007)
b) Real sector of global economy (represented by leaders in electronics , oil and gas and automotive manufacturers against defaulted large multinational automaker from US)

List of sampled companies (total number of companies is 14):

Name of a company	**Industry**
Royal Bank of Scotland	Financial services
Barclays	Financial services
Standard Chartered	Financial services
Northern Rock Plc	Financial services
British Petroleum	Oil and gas
Royal Dutch Shell	Oil and gas

Chevron	Oil and gas
Toshiba Corp	Electronics
Fujitsu Group Corp	Electronics
Hitachi Corp	Electronics
Hyundai Motor Company	Automotive
Nissan Motor Company	Automotive
Mazda Motor Company	Automotive
Ford Motor Company	Automotive

As it is possible to see the studied companies in the sample pool are from four different industries: three companies represent oil and gas sector, four corporations from automobile industry, four institutions operate in financial services industry and three companies are manufacturing electronic goods.

Industries under consideration are discussed below.

2.4.4. Banking industry

Global banking sector consist of a large number of multinational corporations. Table below presents the list of World’s Top 10 largest banks ranked by total assets as of end of 2009:

Table 2. List of 10 largest banking institutions in the world.

Names of banks	**Total assets**
BNP Paribas (BNP)	$ 3.21 trillion
Royal Bank of Scotland (RBS) Group	$2.99 trillion
Barclays PLC (Barclay’s)	$2.54 trillion
Deutsche Bank	$2.43 trillion
HSBC Bank	$2.42 trillion
Credit Agricole	$2.3 trillion

Bank of America	$2.25 trillion
Mitsubishi UFJ Financial Group	$2.07 trillion
J.P. Morgan Chase	$2.02 trillion
UBS AG	$1.80 trillion

As it is possible to see the list of largest banks is dominated by banking institutions headquartered in European countries at the moment. This is in contrast to situation which was common in the beginning of 21st century when the global financial markets have been dominated by US-based corporations.

Global banking industry in 2007-2009 had a dramatic downturn. USA and Europe financial institutions were affected by the crisis quite severely. These regions experienced reduction in GDP rates, exports volume and internal consumption due to the corporations and individuals had less money to spend due to financial distress. Many financial companies of banking sector reported substantial losses during that period. Consequently government had to undertake special measures of bail-out packages. Stock market of US and EU had a dramatic decrease and significant volatility due to uncertainty and banks financial troubles. That situation led to liquidity problems of many large western banks (Northern Rock Plc, Lehman Brothers, Citibank) they all had to solve severe financial troubles by use of governments' bailout packages.

Financial data of one of such banks - Northern Rock Plc - was added to the studied sample to evaluate the ability of bankruptcy prediction models to forecast distress among financial institutions. In September 2007 UK based mortgage lender requested Bank of England to provide financial aid due to inability to raise fund on the interbank money markets. As a consequence these lead to mass panic among retail customers which withdrew about £18 billion of deposits. After two failed take over attempts bank has been nationalized in February of 2008 by the British government. As of end of 2010 bank remained under control of the government although it is understood that several bidders are ready to over take over deals.

2.4.5. Oil and Gas industry

Oil sector has also experienced negative effects from the recent global recession. However due to favourable prices on commodities markets this sector might be the least affected among those considered in this paper. Oil production had a peak in 2006 which was followed by constant decrease at a rate of several percent per year. This can be the explanation for extraordinary high oil prices in 2007-2008 when limited supply coupled with growing demand as consumption went up by China, India, Brazil and some other developing countries.

One the main problems which companies from the oil and gas sector would need to tackle in the nearest future (in 20-30 years according to some estimates) is the fact that these resources are non-renewable. Nevertheless oil and gas remains main source energy for humanity. Forecasted reductions in future supply stimulated some automobile, electronics and energy companies to develop alternative types of energy sources for industries and individuals. Chevron, Shell and BP are very well-known for their research in alternative fuels developments. Most popular types of this energy sources are solar panels, photovoltaic facilities, wind power stations, sea wave and sea flow power stations, hydrogen engines.

There are four major companies which operate in petrochemical industry are Royal Dutch Shell ($292 bln), British Petroleum ($257 bln), ConocoPhillips ($182 bln) and Chevron($161 bln). Three of the mentioned companies are studied in this research.

2.4.6. Automobile industry

The global financial turmoil had large impact on the automotive industry. Car manufacturing construction peaked in 2007 when 73.3 million cars were produced worldwide having grown by average rate of 2.3% annually in preceding decade. However this peak has been followed by decline of 3.7% in 2008 and even larger drop of 13.5% in 2009 – this has brought back the worldwide card manufacturing volumes to the level of 2003. Interestingly the car sales in developing countries have shown some growth in the reported period, e.g. there was 45% increase in new car sales in China in 2009.

During last few years world has seen several cases when automobile giants have gone bankrupt. One of such companies was included in the studied sample as a “failed” institution. It is Ford Motor Company, which together with other two firms from “Detroit Big Three” General Motors and Chrysler applied for government financial aid in November 2008. The financial crisis for American car manufacturers coupled with high petroleum prices in 2007-2008 which discouraged customers from buying new cards, especially large automobiles (e.g. four wheel drive, SUVs etc.) which were very popular during pre-crisis time. After recognizing total loss of $14.6 billion in 2008 (which became the worst ever year in corporation’s history), firm started to shown signs of recovery and from 2009 Ford Motor Company has recorded profits in each consequent quarter.

In addition to Ford three other companies from car manufacturing industry were included in the studied pool – Hyundai, Honda and Mazda.

2.4.7. Electronics industry

Electronics industry is one of the fastest growing sector of global economy with rapid increase during last 30 years. Analytics predict that in next 20 years electronics industry will grow by 4 times with most of factories located in Asia. China and India has seen tremendous growth of factory relocation of electronics companies relocated there due to stable economic growth in these countries and the fact that operating expenses are significantly lower than in developed economies.

Currently the main production centres of electronics in world are:

Europe	22%
Asia	56%
America	37%

As other industries electronics business has experienced major setbacks by the dropdown in the global economy in recent years, although there were none of the global players went bankrupt during the latest crisis. The main driver for this fall was decrease of demand as a result of overall drop in consumption worldwide. Most companies had to cut their expenses by selling non-electronic businesses or inefficient plant lines. However it should be noted that observed extension of several global industries like medical services, telecommunications industrial manufacturing and automobile sectors would play increasing role in development of electronics segment as support producer of parts and equipment and will accelerate the industry growth in the nearest future.

Three companies from the electronics industry were incorporated in the pool of studied firms. Specifically it is Toshiba Corporation, Hitachi Corporation and Fujitsu Corporation Group – all the observed companies are headquartered in Japan however have multiple division located around the globe.

2.5. Data limitations

In this paper all the data was used in it is a real financial and stock market data and it has the following limitations (due to reasons described below):

a) Two companies' Northern Rock Plc and Ford Motor Company data was used of time periods which is different from the majority companies' time ranges used in the research (For Northern Rock Plc was taken 2005-2007 and for Ford Motor Company was taken period of 2006-2008) due to default events took place in both companies during that time frames;

b) For comparison purposes of results in real manufacturing sector was chosen Ford Motor Company because:

    i. In oil and gas sector there were no defaulted public companies of large size due to oil and electronics industries' corporations had strong financial positions and their total assets amounts were supported by revenues consisted of valued real product items (in tonnes or units);

    ii. Negative impact of financial turmoil in petroleum and electronic sectors was lower that's why no companies from these sectors had liquidity problems;

    iii. Most of electronic sectors' largest companies' factories are located in China and India which had positive GDP growth slightly lower than in previous years what positively affected their credit risk levels;

    iv. Oil and gas sector had a substantial reserve of financial resources due to accumulating profits in 2006-2007 when oil prices were very high and there was lower level of bankruptcy among public oil companies;

v. Electronics sector had significant investments in R&D (due to consumption levels and high demand for their products in 2006-2008) that allowed the companies to have some innovative solutions and new products at the beginning of the crisis and there were almost no defaulted companies;

c) Use of Ford Motor Company's data has a number of following limitations:

i. Ford Motor Company's size is considerably higher than of other automotive companies;

ii. Most of Ford Motor Company's facilities are in North America;

iii. This company is mainly targeted American customers while others are working on global level;

iv. Ford Motor Company's strategic policy is linked to continuous growth of American economy (and its' GDP levels);

v. For the last 5 years Ford Motor Company is gradually losing its positions among the largest automakers while other considered companies (Nissan MC , Mazda MC and Hyundai MC) are expending their businesses on the global scale;

vi. Ford Motor Company's products' quality is lower than of Nissan MC or Mazda MC;

vii. Ford Motor Company's headquarters is in US and there are many internal and external factors affecting American automakers that are different from other parts of the world automotive sectors;

viii. Ford Motor Company defaulted at the same time with General Motors and Chrysler what could be an effect of counterparty risks and other common parameters that affected American autos market in 2007-2008;

ix. Production process of Ford Motor Company is based on high qualifications and responsibilities of managers while in Japanese and Korean automakers main role is given to engineers and they are the major drivers of manufacturing operations;

d) Use of Northern Rock Plc in the paper has some limitations because of the following reasons:

i. Its' size is not as large as other sampled banks;

ii. It had most of its operations in the UK while other sampled financial institutions are multinational public banks;

iii. Northern Rock Plc had more aggressive policy to extend its' market share while other banks were more concentrated in international markets (Americas , continental Europe , Middle East);

iv. Northern Rock Plc is mainly a mortgage bank while other sampled companies are international financial companies with majority of operations in consumer and corporate banking , financial services , credit cards , investment banking.

3. Data Analysis

Processed data of failed and successful companies were categorized by use of application of all four methods to assess whether those methods forecasted default events properly and to estimate what was the difference in measures of used models in predictions of default. After this the findings should be compared by method used and by industry or sector of economy of a company. All the components and changes in these components of models' formulas (were shown in methodology part of the thesis) will be analyzed in details in Analysis Part of the dissertation .The main elements of the formulas are so called Key Financial Indicators (Working Capital, EBIT, Retained Earnings, Total Assets, Total Debt etc.) and main parameters of stock performance (historical returns, Expected Return, Volatilities).

The following tables contain all the results of the processed data (financial ratios and numbers and descriptions of share price movements):

Z-score Table

	Standard Chartered			RBS			Barclays		
	2007	2008	2009	2007	2008	2009	2007	2008	2009
z-score	0.52	0.58	0.49	0.67	0.63	0.61	0.66	0.82	0.77
	Fujitsu			Toshiba			Hitachi		
z-score	1.70	1.63	1.44	1.68	1.64	1.37	1.58	1.62	1.47
	Shell			BP			Chevron		
z-score	1.91	1.82	1.44	1.66	1.61	1.51	1.74	1.84	1.45
	Hyundai			Mazda			Nissan		
z-score	1.55	1.24	1.50	2.36	2.67	2.70	1.42	1.64	1.14
	Ford Motor company(2006-2008)			Northern rock(2006-2008)					
z-score	1.20	1.41	1.38	0.068	0.506	0.511			

KMV model's table of results

	Standard Chartered			RBS			Barclays		
	2007	2008	2009	2007	2008	2009	2007	2008	2009
DD	0.49	1.26	2.32	6.37	1.66	0.95	3.59	0.90	0.41
	Fujitsu			Toshiba			Hitachi		
DD	5.23	10.74	4.58	14.81	10.85	9.03	6.25	2.99	1.11
	Shell			BP			Chevron		
DD	29.75	12.33	22.31	39.75	18.99	40.14	47.29	18.58	43.57
	Hyundai			Mazda			Nissan		

DD	12.15	4.42	7.03	4.44	8.18	9.08	17.94	10.68	15.55
	Ford Motor company(2006-2008)			Northern rock(2006-2008)					
DD	7.36	9.39	1.61	1.75	5.27	4.55			

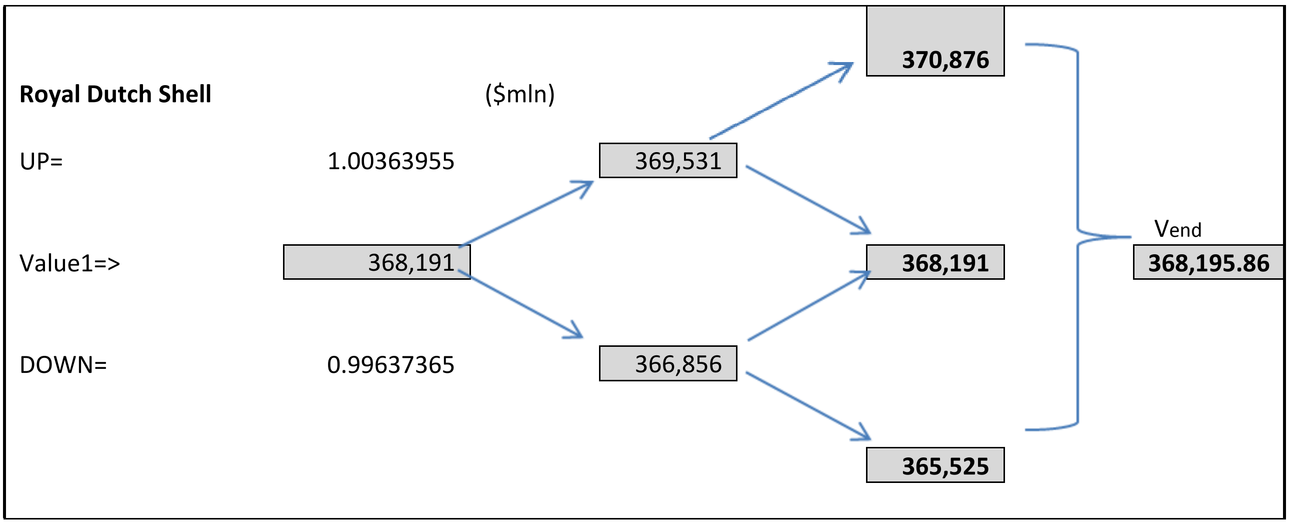
Merton model's table of results

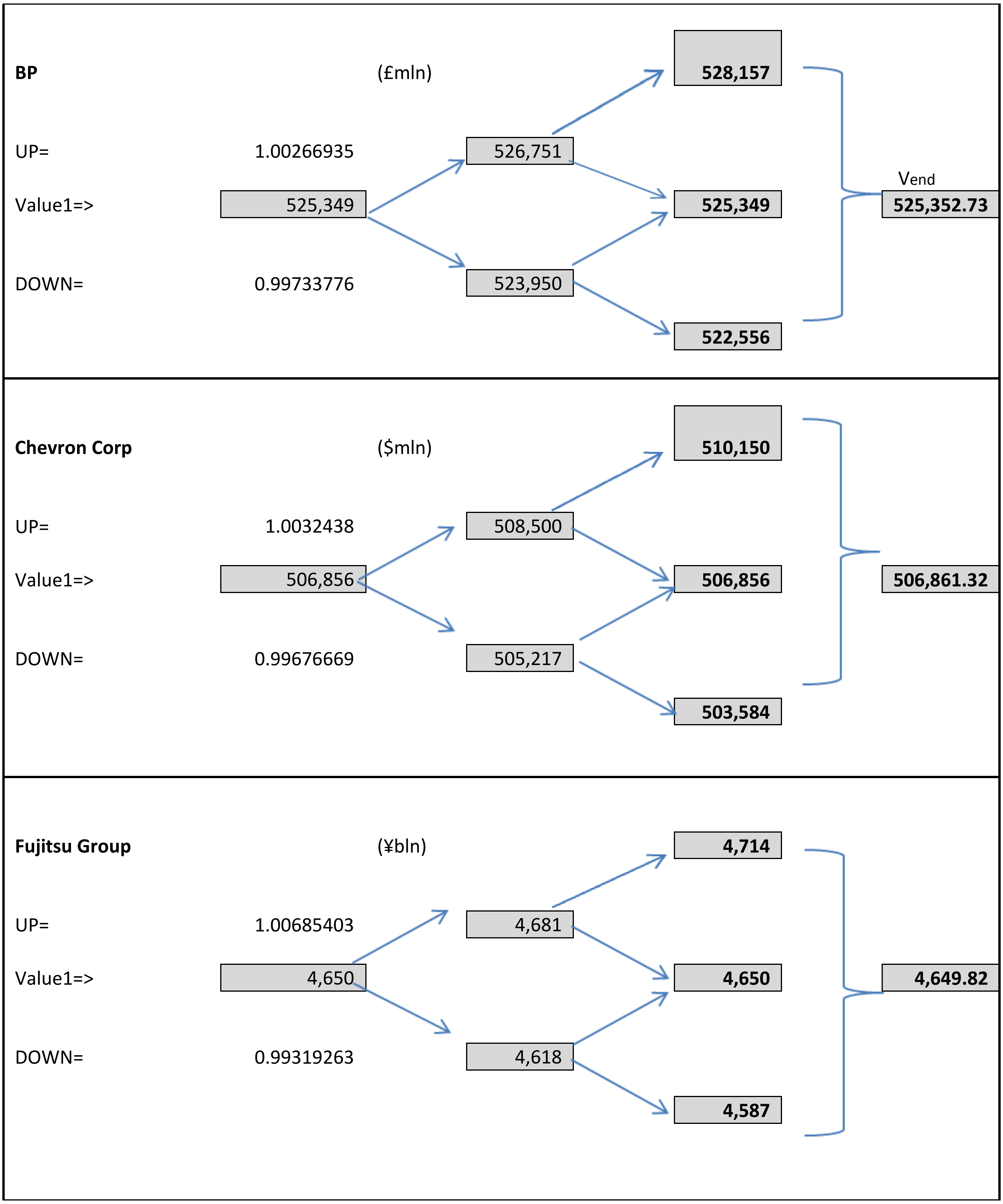
	Standard Chartered			**RBS**			**Barclays**		
	2007	2008	2009	2007	2008	2009	2007	2008	2009
PD	32 591.25	42 984.72	43 141.32	181 873.9	237 283.2	161 684.8	121 263	202 834	136 238
	Shell			**BP**			**Chevron**		
	2007	2008	2009	2007	2008	2009	2007	2008	2009
PD	26 623.64	27 901.22	29 454.35	23 324.31	22 549.91	23 313.64	14 700.1	15 923.1	16 264.6
	Fujitsu			**Toshiba**			**Hitachi**		
	2007	2008	2009	2007	2008	2009	2007	2008	2009
PD	389 639.9	377 609.9	318 331.8	586 077.8	586 440.9	538 810.9	1.051.65 3	1 040 448	929 086
	Hyundai			**Mazda**			**Nissan**		
	2007	2008	2009	2007	2008	2009	2007	2008	2009
PD	8 284 136	10 196 731	10 208 503	1 225 338	1 179 621	1 011 667	188 486	196 174	177 937
	Ford(2006 -2008)			Northern rock(2006-2008)					
	2006	2007	2008	2006	2007	2008			
PD	21570.81	27601.76	33694.75	9979.788	10800.91	1022.185			

Binomial model

Where Value 1=first value of a company $\qquad$ V_{end}=resulted value

Upward ratio=e^(-sigma/2) $\qquad$ Downward ratio= 1/Upward ratio

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(£mln)
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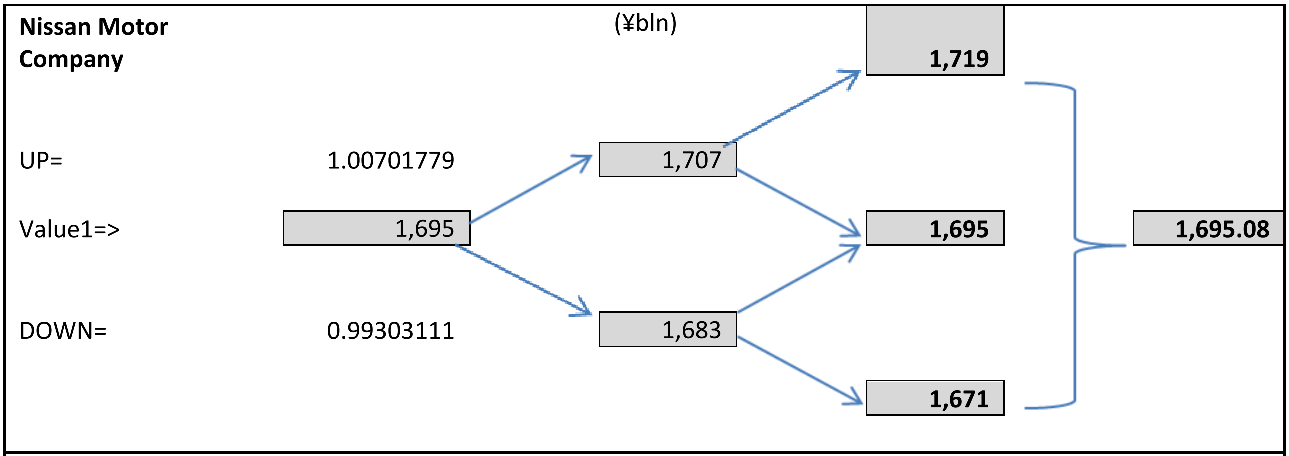
Value1=>
525,349
DOWN=
0.99733776
526,751
523,950
528,157
525,349
522,556
Vend
525,352.73
Chevron Corp
($mln)
UP=
1.0032438
Value1=>
506,856
DOWN=
0.99676669
508,500
505,217
510,150
506,856
503,584
506,861.32
Fujitsu Group
(¥bln)
UP=
1.00685403
Value1=>
4,650
DOWN=
0.99319263
4,681
4,618
4,714
4,650
4,587
4,649.82

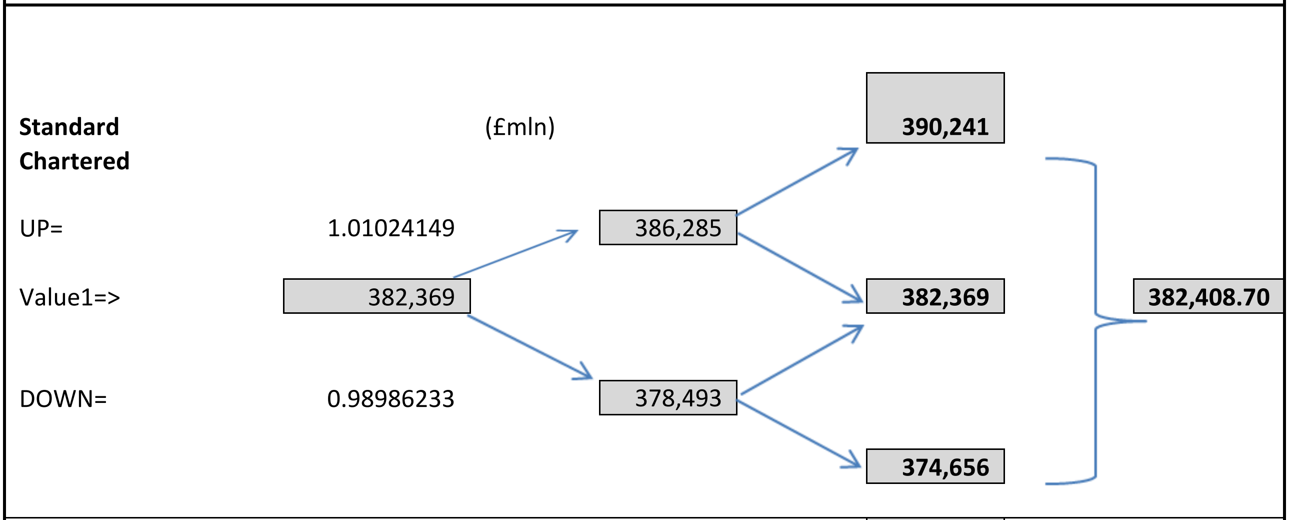
Toshiba Corp		(¥bln)	**8,263**	
UP=	1.00339278	8,235		
Value1=>	8,208		**8,208**	**8,207.69**
DOWN=	0.99661869	8,180		
			8,152	

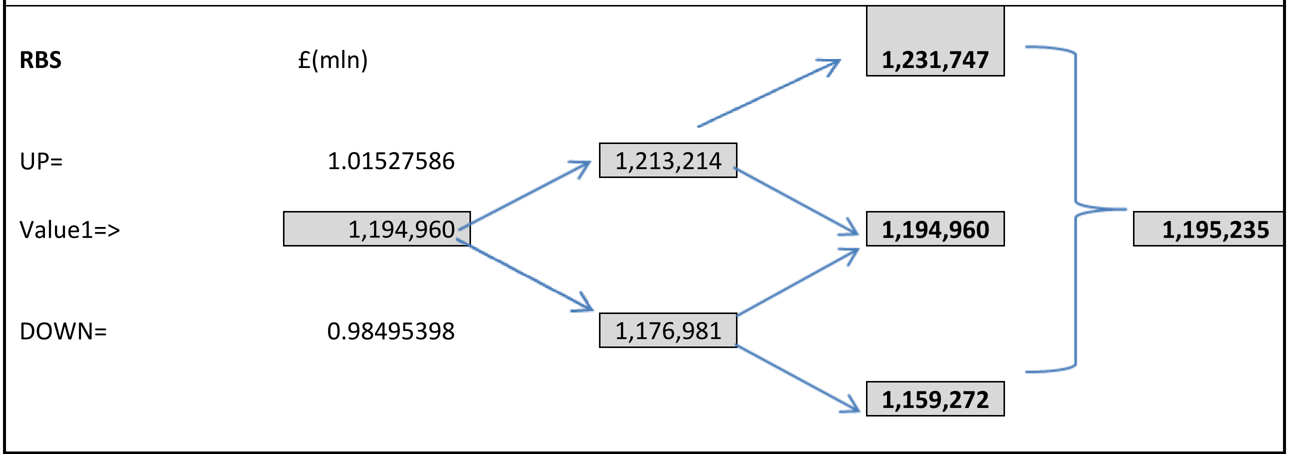
Hitachi Group		(¥bln)	**8,801**	
UP=	1.00362695	8,769		
Value1=>	8,738		**8,738**	**8,737.67**
DOWN=	0.99638616	8,706		
			8,675	

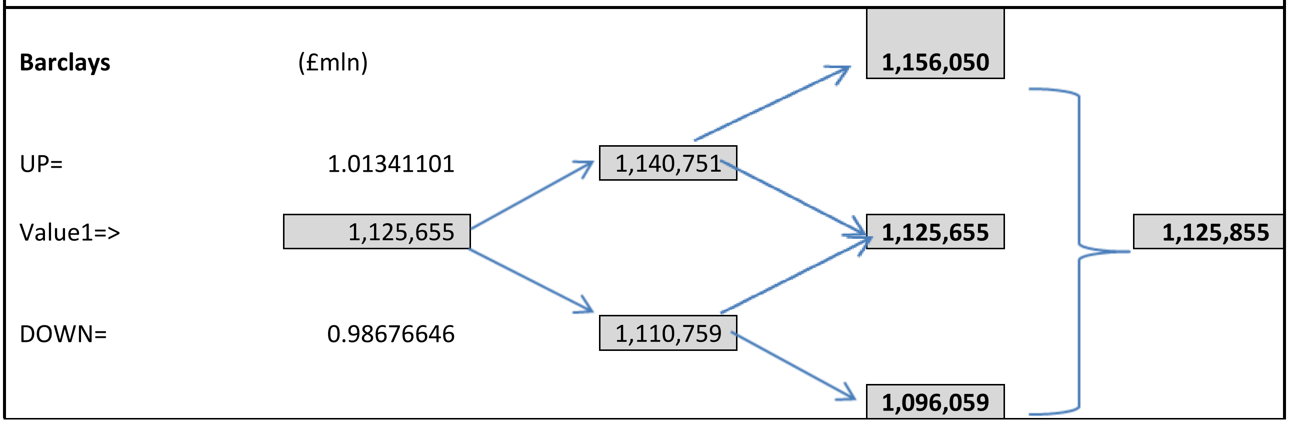
Hyundai Motor company		(*w*bln)	**7,957**	
UP=	1.00647676	7,906		
Value1=>	7,855		**7,855**	**7,855.33**
DOWN=	0.99356491	7,804		
			7,754	

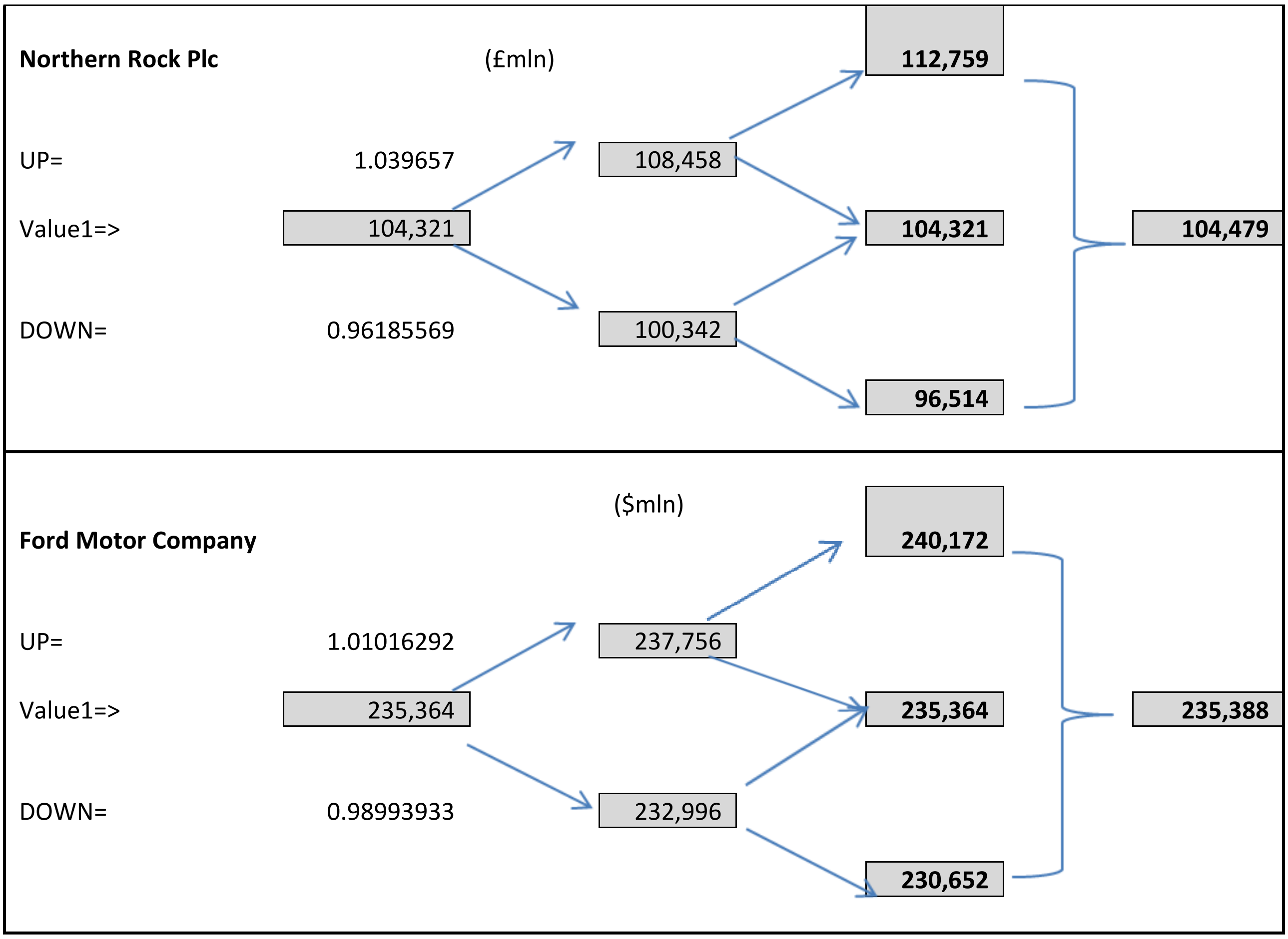
Mazda Motor Company		(¥bln)	**4,399**	
UP=	1.00405284	4,382		
	4,364		**4,364**	**4,364.07**
DOWN=	0.99596352	4,346		
			4,329	


Nissan Motor Company
(¥bln)
UP=
1.00701779
Value1=>
1,695
DOWN=
0.99303111
1,707
1,683
1,719
1,695
1,671
1,695.08


Standard Chartered
(£mln)
UP=
1.01024149
Value1=>
382,369
DOWN=
0.98986233
386,285
378,493
390,241
382,369
374,656
382,408.70


RBS
£(mln)
UP=
1.01527586
Value1=>
1,194,960
DOWN=
0.98495398
1,213,214
1,176,981
1,231,747
1,194,960
1,159,272
1,195,235


Barclays
(£mln)
UP=
1.01341101
Value1=>
1,125,655
DOWN=
0.98676646
1,140,751
1,110,759
1,156,050
1,125,655
1,096,059
1,125,855



This table demonstrates all results of 4 credit risk models applied to sampled companies' data:

Name of method	RBS	Barclays	Standard Chartered	Northern Rock	Toshiba	Fujitsu	Hitachi	BP	Shell	Chevron	Ford	Hyundai	Nissan	Mazda
KMV	High	High	High	Very high	Medium	Medium	Medium	Low	Low	Low	High	Medium	Low	Medium
Merton	High	Medium	High	High	Medium	Medium	Medium	Low	Low	Low	High	Low	Low	Low
Z-score	High	High	High	Very high	Medium	Medium	Medium	Medium	Medium	Medium	High	Medium	Medium	Low
Binomial	Low level of probability of default occurrence for all companies													

In this table:

a) **high** stands for high level of probability of default;

b) **Low** stands for low level of occurrence of default event;

c) **Medium** stands for level of default risk is medium;

d) **Columns with grey colours** for companies which had default event in 2007-2009.

4. Conclusions

All the considered credit risk models shown different results in predicting default events and by level of differences between processed sampled companies results when using these methods with real or actual default events.

Hypothesis 1 (H1) was strongly confirmed by KMV approach and Z-score method. Both methods predicted high level of credit risks for Northern Rock Plc and Ford Motor Company which experienced default events in 2007-2009. KMV, Merton and z-score models demonstrated appropriate level of confidence in credit risk parameters of considered real sectors of global economy (oil, electronics and automotive industries).

Binomial method confirmed Hypothesis 2 (H2) due to the fact that it did not forecasted credit risk problems for defaulted companies and because its results were the same as for successful companies as well as for failed ones. The explanation for this can be due to the fact that there were not many steps of changes in values were used in calculations and because of some subjective factors - for instance, binomial model considers mainly share price fluctuations while when expectations of analysts are too optimistic/pessimistic these analysts could be misled.

By looking at that table an analyst could see that only binomial method did not recognized properly the occurrence of default events while for successful companies it shown good results, this could be due to the fact that only two step-method was used (because the paper was written with research purposes) while many practitioners recommend to process at least one hundred twenty steps with values of the companies fluctuations every step (one upward and one downward).

Merton, KMV, Z-score models forecasted defaulted companies with quite high level of accuracy while there are too high assessments of credit risks among financial sector companies. Especially good results in predicting default event for Northern Rock Plc were done by z-score model which predicted high level of bankruptcy all three years of analyzed period (2006-2008) with z-score equal zero in 2008 when Northern Rock bank was nationalized by British government. All this three methods shown high level of credit risk for Ford Motor Company, what could be a signal for this company long way before the beginning of problems with liquidity.

Financial services sector was severely affected by negative economic environment during the period 2007-2009 and this explains why all models (except binomial approach) gave almost the same results for all sampled companies from banking industry (Barclays, Northern Rock, RBS, Standard Chartered) . In banking sector's sampled financial institutions there were significant decreases of Sales and Total assets volumes and there were drastic increase of losses what led to number of bankruptcies in this sector.

In real manufacturing sector (consists of electronics, oil and gas and automotive industries) all the models shown appropriate level of default risks while for Ford Motor Company all the approaches demonstrated high level of frequency of default . That exception could took place as to economic environment of automobile sector in USA and differences of economic conditions affecting automakers on country level in US with economic situation in South East part of Asia as well as to differences in selling and operations policies between Asian automakers (Nissan, Mazda, Hyundai) and American automotive giants (Ford Motor Company). Oil and gas and electronics segments of considered real manufacturing sector assessed as comparatively stable. And this stability of results confirmed by absence of defaulted large, public companies from petroleum and electronic industries. There were observed some reduction in volumes of Revenues and Total assets and there were higher levels of volatilities of sampled companies from electronics and oil industries in 2007-2009 while these facts did not worsen credit risk levels for these corporations.

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Appendix 1.

The tables with variables for calculation of Z-score values:

	Standard Chartered			**RBS**			**Barclays**		
	2007	2008	2009	2007	2008	2009	2007	2008	2009
CA	113207	178446	141838	951382	1340826	792946	638002	1371389	833151
CL	231085	368963	357444	1233372	1094367	908950	983695	1786899	1146051
WC	-117878	-190517	-215606	-281990	246459	-116004	-345693	-415510	-312900
RE	2841	3241	3380	7549	-23710	-2672	4417	4382	9393
EBIT	4035	4568	5151	9832	-40836	-2595	6223	5136	4585
MV of Eq	110768	138782	382365	2211828	2007593	1194960	1498266	1523934	1125655
Sales	10777	11794	12818	28110	17882	20067	18277	16083	19985
TL	308648	412928	409313	1787791	2342773	1618750	1204070	2016362	1331652
TA	329871	435068	436653	1840829	2401652	1636486	1227361	2052980	1378929
Z-score	**0.52781**	**0.58758**	**0.49968**	**0.67169**	**0.63438**	**0.61695**	**0.66771**	**0.82900**	**0.77316**

	Fujitsu			**Toshiba**			**Hitachi**		
	2007	2008	2009	2007	2008	2009	2007	2008	2009
CA	50.43%	49.35%	49.89%	54.06%	56.78%	58.58%	51.05%	51.29%	53.87%
CL	47.39%	50.31%	56.25%	45.83%	42.16%	48.12%	43.85%	45.13%	49.15%
WC	3.03%	-0.95%	-6.37%	8.23%	14.62%	10.46%	7.20%	6.16%	4.72%
RE	1.92%	1.64%	-5.05%	2.01%	0.90%	-2.39%	-0.32%	-0.52%	-7.87%
EBIT	4.16%	3.41%	-4.11%	4.21%	2.05%	-2.41%	1.97%	2.89%	-2.90%
BV of Eq	18.68%	17.22%	8.20%	24.58%	24.81%	23.25%	22.95%	20.61%	11.17%
TL	81.32%	82.78%	91.79%	75.42%	75.19%	76.75%	77.05%	79.39%	88.83%
Sales	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%
TA	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%

	Fujitsu			Toshiba			Hitachi		
	2007	2008	2009	2007	2008	2009	2007	2008	2009
CA	2132023	2169937	1887537	2991207	2929382	2720631	5434135	5401755	5065399
CL	1807565	1611340	1550422	2811291	2985987	3067773	4667544	4752899	4621904
WC	324458	558597	337115	179916	-56605	-347142	766591	648856	443495
RE	102415	48107	-112388	137429	127413	-343559	-32799	-58125	-787337
EBIT	214495	109444	-113314	298460	265049	-279252	202338	324782	-289871
MV of Eq	3221168	3776290	4649645	5296482	9182615	8207688	8321706	10587940	8737564
TL	2974202	2873759	2473015	4823641	4913372	5005879	8201462	8360235	8353758
Sales	5100163	5330865	4692991	7168845	7779038	6801441	10247903	11226735	10000369
TA	3943724	3821963	3221982	5931962	5935637	5453552	10644259	10530847	9403709
Z-score	**1.70088**	**1.63620**	**1.44250**	**1.68088**	**1.64968**	**1.37353**	**1.58750**	**1.62530**	**1.47838**

	Shell			BP			Chevron		
	2007	2008	2009	2007	2008	2009	2007	2008	2009
CA	115397	116570	96457	80202	66384	67653	39377	36470	37216
CL	94384	105529	84789	77231	69793	59320	33798	32023	26211
WC	21013	11041	11668	2971	-3409	8333	5579	4447	11005
RE	31331	26277	12518	20845	21157	16578	10483	23931	18688
EBIT	51576	50820	21020	31611	34283	24124	32274	43057	18528
MV of Eq	637415	351449	368191	658650	214041.2	552349	331218	683751	506856
TL	145510	155116	155750	142386	136935	134355	71698	74517	72707
Sales	355782	458361	278188	290684	366317	245346	214091	264958	167402
TA	269470	282401	298121	236076	228238	235968	148786	161165	164621
Z-score	**1.91725**	**1.82315**	**1.44551**	**1.66313**	**1.61216**	**1.51277**	**1.74495**	**1.84099**	**1.45398**

The spine

Comparative study of modern credit risk assessment methods	Yerzhan Narzhanov